SU(3)$_F$ in nonleptonic charm decays$^*$

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We present updated results of an SU(3)-flavor analysis of $D \to PP$ decay data including linear breaking and no further assumptions. The global fit is consistent with nominal ($\sim 30\%$) SU(3)$_F$ breaking and returns enhanced penguin (triplet) contributions. Their size is driven, in addition to $\Delta A_{CP}$, by the CP asymmetries of $D_0 \to K_S K_S$, $D_s \to K_S \pi^+$ and $D_s \to K^+ \pi^0$ decays. It is therefore especially important to improve these measurements.

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1. Introduction

In 2011 and 2012, spectacular results indicated large CP violation in $D$ decays [1, 2]. Before the Moriond conference in March 2013, the world average for the difference of the CP asymmetries $\Delta a_{CP}^{\text{dir}}$ of $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ was 4.6$\sigma$ away from zero. This situation triggered a lot of theory papers [3, 4, 5, 6, 7, 8, 9, 10, 11, 12], especially trying to explain the effect in new physics models, e.g., [13, 14, 15, 16, 17]. Recently, LHCb updated the previous measurement of $\Delta A_{CP}$ and performed a further measurement in an additional channel, resulting in

$$\Delta A_{CP} = -0.0034 \pm 0.0018,$$  \hspace{1cm} (D$^*$ decay channel [18]) \hspace{1cm} (1.1)

$$\Delta A_{CP} = 0.0049 \pm 0.0033,$$  \hspace{1cm} (semileptonic $B$ decay channel [19]) \hspace{1cm} (1.2)

where we added quadratically the statistical and systematic uncertainties. There is a tension of $2.2$\,$\sigma$ [19] between both results, which differ in sign. The current world average including these results is $\Delta a_{CP}^{\text{dir}} = -0.0033 \pm 0.00120$ [1, 2, 18, 19, 20, 21, 22], only 2.8$\sigma$ away from zero.

In the Standard Model (SM), one can write $\Delta a_{CP}^{\text{dir}} \sim |P/T| \sin \delta \sin \gamma$ [6, 23] with the penguin over tree ratio $P/T$, the CKM angle $\gamma$ and a strong phase $\delta$. With respect to the tree amplitude, the penguin one is CKM suppressed by $\lambda^3 \sim 10^{-3}$ and a “naive” loop factor $\alpha_s/\pi \sim 0.1$. Altogether, this gives the rough estimate $\Delta a_{CP}^{\text{dir}} \lesssim 10^{-4}$. The latter is however not reliable because $\alpha_s(m_t)$ is large and the expansion in $\Lambda_{QCD}/m_b$ is not expected to converge fast. Consequently, it is unclear if a large $\Delta A_{CP}$ indicates new physics or is a QCD effect. In order to approach this problem we include additional observables which we relate by a symmetry principle.

2. SU(3)$_F$ approach to nonleptonic $D$ decays

Besides $\Delta a_{CP}^{\text{dir}}$, many other observables of $D \rightarrow PP$ decays have been measured. It is important to obtain a complete picture of all singly Cabibbo suppressed (SCS) CP asymmetries as well as SCS, Cabibbo-favored (CF) and doubly Cabibbo suppressed (DCS) branching ratios. In a data-driven way we use the approximate SU(3)$_F$ symmetry of QCD including linear breaking to relate several decay modes of $D \rightarrow PP$. For that, we analyze the operators of the effective Hamiltonian as well as initial and final states on their SU(3)$_F$ representations. Subsequently, we use the Wigner-Eckart theorem in order to obtain the corresponding reduced matrix elements, all of which we fit from data only. The amplitudes of SCS, CF and DCS amplitudes can then be written as

$$\mathcal{A}(d) = \Sigma \left( \sum_{i,k} c_{d,ik} A^k_i + \sum_{i,j} c_{d,j} B^j_i \right),$$ \hspace{1cm} (SCS) \hspace{1cm} (2.1)

$$\mathcal{A}(d) = V_{cd}^{*}V_{ud} \left( \sum_{i,k} c_{d,ik} A^k_i + \sum_{i,j} c_{d,j} B^j_i \right),$$ \hspace{1cm} (CF) \hspace{1cm} (2.2)

$$\mathcal{A}(d) = V_{cd}^{*}V_{us} \left( \sum_{i,k} c_{d,ik} A^k_i + \sum_{i,j} c_{d,j} B^j_i \right),$$ \hspace{1cm} (DCS) \hspace{1cm} (2.3)

with $\Sigma \equiv (V_{us}^{*}V_{us} - V_{cd}^{*}V_{ud})/2$, the SU(3)$_F$ limit matrix elements $A^k_i$, the SU(3)$_F$-breaking matrix elements $B^j_i$ and Clebsch-Gordan coefficients $c_{d,ik}$. CP violation is induced by the interference of
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3. Extraction of SU(3)$_F$ breaking and penguins from present data

In order to quantify the SU(3)$_F$ breaking and the size of the penguins we introduce the following complementary measures [24]:

\[
\delta_X = \frac{\max_{ij}|B|^2}{\max(|A_{15}|, |A_{6}|, |A_{8}|)} , \quad \delta'_X = \max_d \left| \frac{\sigma_X(d)}{\sigma'(d)} \right| , \quad \delta_3 = \frac{\max(|A_{15}|, |A_{6}|, |A_{8}|)}{\max(|A_{15}|, |A_{6}|, |A_{8}|)} , \quad \delta'_3 = \max_d \left| \frac{c_{d:13}A_{15}^3 + c_{d:83}A_{8}^3}{c_{d:2715}A_{2715}^1 + c_{d:86}A_{8}^6 + c_{d:815}A_{815}^5} \right| .
\]

\[ (3.1) \]

\[ (3.2) \]
Here, \( \mathcal{A}(d) \) is the \( SU(3)_F \)-breaking part of the amplitudes, i.e., \( \mathcal{A}(d) \) with \( A^d_T = 0 \). In the definition of the measures \( \delta_X \) and \( \delta'_X \) we do not take into account the decay \( D^0 \to K_S K_S \). The latter would introduce a bias, since, in contrast to all other considered decays, its \( SU(3)_F \) limit is CKM-suppressed \( \propto \Delta \). The ratios of \( SU(3)_F \) matrix elements \( \delta_X \) and \( \delta'_X \) do not take into account effects of small Clebsch-Gordan coefficients in front of the respective matrix elements. On the other hand, the ratios of parts of amplitudes, \( \delta_X \) and \( \delta'_X \) do not take into account possible large cancellations between different summands. It is therefore essential to study both measures to achieve the full information on the \( SU(3)_F \) breaking and the penguin enhancement.

The truncation of the perturbative \( SU(3)_F \) expansion at the next to leading order is only sensible if the \( SU(3)_F \) breaking is not too large. Therefore at first we validate our ansatz. In Fig. 1(a), we show our fit results for the \( SU(3)_F \) breaking with present data as summarized in Table II of [24] and updated in Table 1. Already an \( SU(3)_F \) breaking of \( \sim 30\% \) can describe the data reasonably well, while larger \( SU(3)_F \) breaking cannot be excluded from data only. This is true independent of the measure \( \delta_X \) or \( \delta'_X \). Therefore, the \( SU(3)_F \) ansatz is justified for nonleptonic charm decays. Consequently, we can proceed with our analysis and study the size of the penguins.

Our fit results to the current data for the relative size of the penguins, or more specifically, the triplet matrix elements which come with \( \Delta \), are shown in Fig. 1(b). The generic SM expectation is \( \delta_3^{(r)} \sim P/T \sim 0.1 \), while values of \( \delta_3^{(r)} \sim 1 \) are generally regarded as an enhancement. As can be seen from Fig. 1(b), the allowed 68\% C.L. region is located at very high values of \( \delta_3^{(r)} \). The border of the 95\% C.L. region can be identified in the zoom of the \( \delta'_3 - \delta_3 \) plane which is shown in Fig. 2(a) and which shows the fit result \( \delta_3^{(r)} > \mathcal{O}(1) \). The reason is the following: besides \( \Delta a_\text{dir}^{\text{CP}} \), there are additional CP asymmetries in the global analysis that are measured with large central values and generate the need for a penguin enhancement. This is illustrated in Fig. 2(b), where we take for comparison the observables \( a_\text{dir}^{\text{CP}}(D^0 \to K_S K_S) \), \( a_\text{CP}(D_s \to K_S \pi^+) \) and \( a_\text{dir}^{\text{CP}}(D_s \to K^+ \pi^0) \) out of the fit. Without them, also \( \delta_3^{(r)} < \mathcal{O}(1) \) becomes allowed in the fit. The tendency towards large values for the penguins in the global analysis highlights the importance of future improved measurements of these asymmetries. These will enable us to better constrain the size of the penguins.

<table>
<thead>
<tr>
<th>Observable</th>
<th>Average Nov 2013</th>
<th>Average Nov 2012 [24]</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta a_\text{dir}^{\text{CP}}(K^+ K^- \to \pi^+ \pi^-) )</td>
<td>(-0.00333 \pm 0.00120 )</td>
<td>(-0.00678 \pm 0.00147 )</td>
<td>[20],[1, 2, 18, 19]</td>
</tr>
<tr>
<td>( \Sigma a_\text{CP}^{\text{dir}}(K^+ K^- \to \pi^+ \pi^-) )</td>
<td>(+0.00008 \pm 0.00228 )</td>
<td>(+0.0014 \pm 0.0039 )</td>
<td>([1, 2, 22, 30, 32] )</td>
</tr>
<tr>
<td>( a_\text{CP}^{\text{dir}}(D_s \to K_S \pi^+) )</td>
<td>(+0.011 \pm 0.007 )</td>
<td>(+0.028 \pm 0.015 )</td>
<td>([33, 34, 35, 36] )</td>
</tr>
<tr>
<td>( a_\text{CP}^{\text{ind}} )</td>
<td>(+0.00015 \pm 0.00052 )</td>
<td>(-0.00027 \pm 0.00163 )</td>
<td>[20]</td>
</tr>
<tr>
<td>( \delta_{K \pi} )</td>
<td>((1.17 \pm 10.2)^\circ )</td>
<td>((21.4 \pm 10.4)^\circ )</td>
<td>([\uparrow 20] )</td>
</tr>
<tr>
<td>( \mathcal{A}(D_s \to K_S K^+) )</td>
<td>((1.50 \pm 0.05) \cdot 10^{-2} )</td>
<td>((1.45 \pm 0.05) \cdot 10^{-2} )</td>
<td>([\uparrow 37, 38] )</td>
</tr>
</tbody>
</table>

Table 1: Updates of averages of experimental measurements compared to the status in Nov 2012 given in [24]. \( ^\dagger \)Our average where we added systematic and statistical error quadratically. \( ^\uparrow \)Uncertainties calculated by symmetrization. Table adapted from [24] and [25].
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4. Future data

There are plans to measure the CP asymmetry $a_{CP}^{dir}(D^0 \rightarrow \pi^0 \pi^0)$ in the future with a $\sim 10$ times smaller uncertainty than is the case at present [20]. We study therefore a hypothetical future data scenario, assuming that $a_{CP}^{dir}(D^0 \rightarrow \pi^0 \pi^0)$ is measured as

$$a_{CP, future}^{dir}(D^0 \rightarrow \pi^0 \pi^0) = 0.000 \pm 0.006,$$

(4.1)

where the uncertainty is motivated by the prospect given by Belle [20]. The corresponding 95% C.L. contour of the penguin enhancement differs hardly from the one shown in Fig. 2(a). This result can be understood from counting the degrees of freedom in the fit: in order to determine the two complex penguin matrix elements that induce CP violation, at least 4 significant measurements of SCS CP asymmetries are needed. This means that, while present uncertainties leave much room for possible enhancements, even in the pessimistic scenario of no CP violation showing up in $D^0 \rightarrow \pi^0 \pi^0$ decays, the search for CP violation in SCS decays remains interesting.

5. Conclusion

The global SU(3)$_F$ fit of two-body charm decays to kaons and pions shows that an SU(3)$_F$ breaking of $\sim 30\%$ suffices to describe the data. Furthermore, it reveals a tendency towards large values for the penguins. The penguin enhancement has decreased somewhat with respect to an earlier study [24], especially due to new experimental results on $\Delta a_{CP}^{dir}$. Nevertheless, taking all observables into account, the overall characteristics found in [24] persist. The observed penguin enhancement is driven, in addition to $\Delta a_{CP}^{dir}$, by the CP asymmetries $a_{CP}^{dir}(D^0 \rightarrow K_S K_S)$, $a_{CP}^{dir}(D_s \rightarrow K_S \pi^+)$ and $a_{CP}^{dir}(D_s \rightarrow K^+ \pi^0)$. These are not measured significantly on their own, so that at present the situation is inconclusive regarding the validity of the Standard Model. It is therefore very important
to improve these measurements. Furthermore, the tendency to large triplet matrix elements remains assuming a hypothetical but realistic measurement of the CP asymmetry in $D^0 \to \pi^0 \pi^0$ at the few-permille level but consistent with zero.

We summarize here the following characteristics of $D \to PP$ decays, that can serve as a guide for future measurements:

- The CKM-leading contribution to the decay $D^0 \to K_S K_S$ comes first into play when taking SU(3)$_F$ breaking into account. Therefore its CP asymmetry is enhanced with respect to $\Delta a_{\text{dir}}^{\text{CP}}(D^0 \to \pi^0 \pi^0)$ in the SM as well as in generic new physics models [24, 12].

- Due to isospin symmetry in the SM to very good precision $a_{\text{dir}}^{\text{CP}}(D^+ \to \pi^0 \pi^+)$ = 0. The violation of this rule would be a smoking gun for $\Delta I = 3/2$ new physics [24, 39].

- The $\mathcal{O}(1)$ breaking of the U-spin relations
  \begin{align}
  a_{\text{dir}}^{\text{CP}}(D^0 \to K^+ K^-) + a_{\text{dir}}^{\text{CP}}(D^0 \to \pi^+ \pi^-) &= 0, \\
  a_{\text{dir}}^{\text{CP}}(D^+ \to \bar{K}^0 K^+) + a_{\text{dir}}^{\text{CP}}(D_s \to K^+ \pi^0) &= 0,
  \end{align}
  \begin{equation}
  (5.1)
  \end{equation}
  \begin{equation}
  (5.2)
  \end{equation}
beyond the amount of SU(3)$_F$ breaking would be a sign of new physics [24]. Especially with regard to Eq. (5.2) there is much space left for an improvement in experimental precision.

- The present tendency towards a penguin enhancement is driven by the CP asymmetries $\Delta a_{\text{dir}}^{\text{CP}}$, $a_{\text{dir}}^{\text{CP}}(D^0 \to K_S K_S)$, $a_{\text{dir}}^{\text{CP}}(D_s \to K_S \pi^+)$ and $a_{\text{dir}}^{\text{CP}}(D_s \to K^+ \pi^0)$ [24].

SU(3)$_F$ analysis allows improved $D \to PP$ measurements to provide more precise information on the borders of the SM and new physics in $|\Delta C| = 1$ processes.

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References

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