

Neutrino masses from Large Scale Structures - including theoretical errors

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We show how to estimate in a robust and conservative way the sensitivity of an Euclid-like survey to the parameters of a minimal Λ CDM and the total neutrino mass for a Euclid-like survey. To include as much information as possible, and use the mildly non-linear scales in a meaningful way, we present a method for including theoretical errors. Two sets of errors are considered: (i) one to account for uncertainties in the modelling of the effect of neutrinos on the non-linear galaxy power spectrum and (ii) one to parametrise the overall residual uncertainties in modelling the non-linear galaxy power spectrum at small scales. We discuss how the inclusion of these errors impact the error on M_ν by comparing different ansatz, and we mention the effect of taking into account data from smaller scales considering our current understanding of non-linearities. We also discuss how this method can be extended to other probes to have a reliable estimate of the current and achievable sensitivity to the total neutrino mass.

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1. Introduction

Even though in the Standard Model of Particle Physics, the neutrino particles do not have a mass, it has been proven experimentally since a long time that at least two out of the three have a non-zero mass. At the cosmology level, however, the impact of considering neutrinos massive or massless was mostly neglected when performing parameter extraction. The situation is evolving, though, with the advent of precision cosmology, with the release of Planck data [1] in March, on the Cosmic Microwave Background (CMB) side, and the upcoming Euclid experiment that will measure Large Scale Structures (LSS) with unprecedented precision.

Indeed, taking a model with a total neutrino mass of 0.06 eV shifts the central value of the posterior distribution of H_0 by as much as 0.6 km/s/Mpc, compared to a model with massless neutrinos. For the LSS, the neutrinos will stream-out of overdensities due to their high peculiar velocities, impacting the total amplitude of the power spectrum at small scales by around 5%[2]. This turns also into a new opportunity: one can use cosmology as a complementary probe to laboratory experiments, to put a bound on the total neutrino mass. Current result from Planck [1] indicates a 2σ upper bound of $\sum m_\nu < 0.23$ eV, whereas laboratory measurements indicate $\sum m_\nu \geq 0.06$ eV.

One of the most alluring goal of the Euclid experiment is to explore fully this range of allowed values, through the analysis of the washing-out of structures. Indeed, N-body studies [3] showed that this effect of streaming-out stays at the nonlinear level, with a slightly different shape (as seen on fig. 1). This will slowly remove power from too small over-densities, effectively suppressing structures below a certain scale, depending on the total neutrino mass.

To capture this behaviour without simulating the complete system, M. Viel and collaborators [3] expanded on the existing HALOFIT method to include correction for massive neutrinos (see fig. 1). HALOFIT is a fitting formula adjusted on N-body simulations, so that given an input cosmology, it produces a matter power spectrum, that should approximate as well as possible the simulated one. This part will prove crucial for a realistic analysis of upcoming experiments.

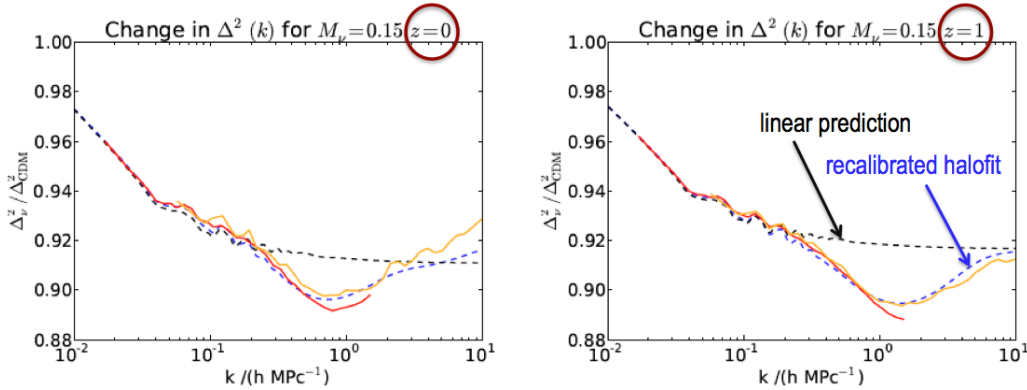


Figure 1: Relative impact of massive neutrinos on the matter power spectrum as a function of wavenumber, against a scenario with massless neutrinos, at redshifts 0 and 1 [3]. The solid red and yellow lines shows N-body prediction, whereas black dotted represent linear prediction, and blue dotted the recalibrated HALOFIT method.

The Euclid mission [4] has been already approved by ESA, and is scheduled for launch on

2019. It is a space telescope, consisting of several probes. The main surveys will be a galaxy redshift one, and a weak-lensing one. It will probe matter distribution down to very small scale ($k \simeq 1h/\text{Mpc}$) with a high sensitivity. This should provide, as stated previously, a huge lever arm on the total neutrino mass. The necessary condition to extract all this information, however, is to also control our prediction up to these scales. I will detail in sec. 2 how to describe these errors, and present the results of a conservative analysis in sec. 3. I will finally conclude in sec. 4 with a discussion on the implications of the study.

2. Theoretical Errors

To extract cosmological parameter values from an experiment like Euclid, one usually utilise a statistical tool, for instance a Markov Chain Monte Carlo (MCMC) technique. This will explore randomly the parameter space, following the regions of higher likelihood. At each time-step, a model is selected randomly, and the cosmological observables are produced by a Boltzmann code. Given these observables, the likelihood to observe some data is computed through a non-analytic function, provided by the experiment. Depending on the number of free parameters in the analysis, the typical number of steps needed to achieve convergence of this process is of the order of 10^4 to 10^5 . It is thus evident that both the Boltzmann evolution and the likelihood call must be as fast as possible.

With the data being not yet available, it is nonetheless to predict the sensitivity of an experiment given its design requirements. By following strictly the recommendations in [cite](#), we devised a fake study. This consists in choosing arbitrarily a fiducial value of the parameters, which will then act as “observed” best-fit parameters. By placing ourselves in the less favorable case (*i.e.* with an exactly minimal total neutrino mass), one can then derive conservative estimate of the experiment sensitivity.

This observation rules out in practice any use of a N-body simulation. Indeed, the only possibility would be to run several simulations for each value on a grid in parameter space, and proceed by interpolation between the points. However, to test a new parameter, one would have to restart the grid all over again. This forces us to choose between two less satisfactory but feasible methods: (i) stop our parameter extraction for wave numbers below the nonlinear scale ($k < 0.1 h/\text{Mpc}$) or (ii) take also mildly nonlinear scales ($k < 0.6 h/\text{Mpc}$), but with a rather rough estimate of the theory. In this study, we opted for the second solution, while keeping in mind this uncertainty. This way, we try to recover as much information as possible from the experiment.

In [3], an estimate of the uncertainty associated to the presence of massive neutrino is worked out to be linked to the total neutrino mass (the more massive, the more error), and is a correlated error in Fourier space. By the latter, I mean that the shape of the error in Fourier space is assumed to be known, and the only choice is the amplitude of the error. However, using only this uncertainty assumes that the underlying method, namely HALOFIT is completely accurate. This is of course not the case, and lead to an extremely unphysical estimate of Euclid sensitivity.

These first assumptions were extremely crude, especially considering that we really observe the galaxy power spectrum in redshift space, and so that in addition to our uncertainty on the matter power spectrum prediction, one should add our uncertainty on the bias, and on the redshift-space distortions.

All these considerations led us to develop a more robust description of the error, and we implemented it directly in the likelihood formula for both Euclid probes, as described in [5], in appendices A and B. We estimated the global uncertainty of HALOFIT with respect to N-Body simulations, using for instance fig. 2. As an optimistic estimate, we took a functional dependence that saturates at 5%, given that the experiment will take place only in 2019. Given this error envelope, we allow the data to lie anywhere inside - by giving such a data no more than a $\Delta\chi^2 = 1$. We think that this accounts to a realistic error prescription. Note that other propositions exist, that use instead form filling functions [7].

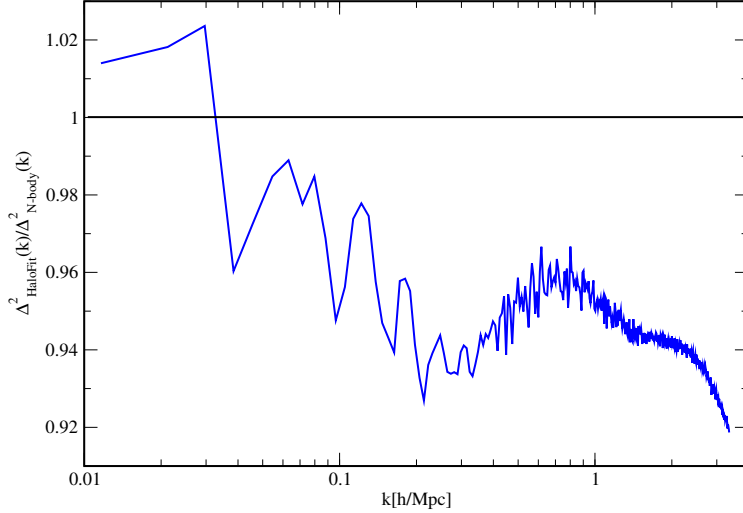


Figure 2: Extracted from [6]. Error shape of HALOFIT method compared to N-body simulations, at a given redshift

Our solution corresponds then to adding a nuisance parameter per data point, and marginalize independently over this one, within the given error envelope. It is a much more severe error than simply allowing the prediction to move up or down with some amplitude: it there accepts any shape the power spectrum might take, as long as it stays in the envelope. To guide the eye, one can see on fig. 3 the shape of our assumed theoretical error in solid green. In solid red is represented the observational error from Euclid, that saturates to a very small but non-zero value as k grows. This is due to the fact that, at small scales, the experiment will measure sensibly more galaxy, reducing the shot noise. Within the green band, we allow for any shape of the power spectrum to be equally likely than a straight line touching one of the edge of the envelope.

It is clear that the result of the analysis will then depend on the functional shape of the error, as well as the maximum amplitude chosen. It seems however fair to say that controlling the amplitude to a reasonable value is of the utmost importance, regardless of the exact shape taken. A more detailed assessment of the exact error shape will not change quantitatively our results.

3. Results

Our results for the Galaxy Redshift Survey are shown in table 1. We tested two setups: a $k_{\max} = 0.1$ h/Mpc (top) and $k_{\max} = 0.6$ h/Mpc (bottom). In each of these two cases, we then

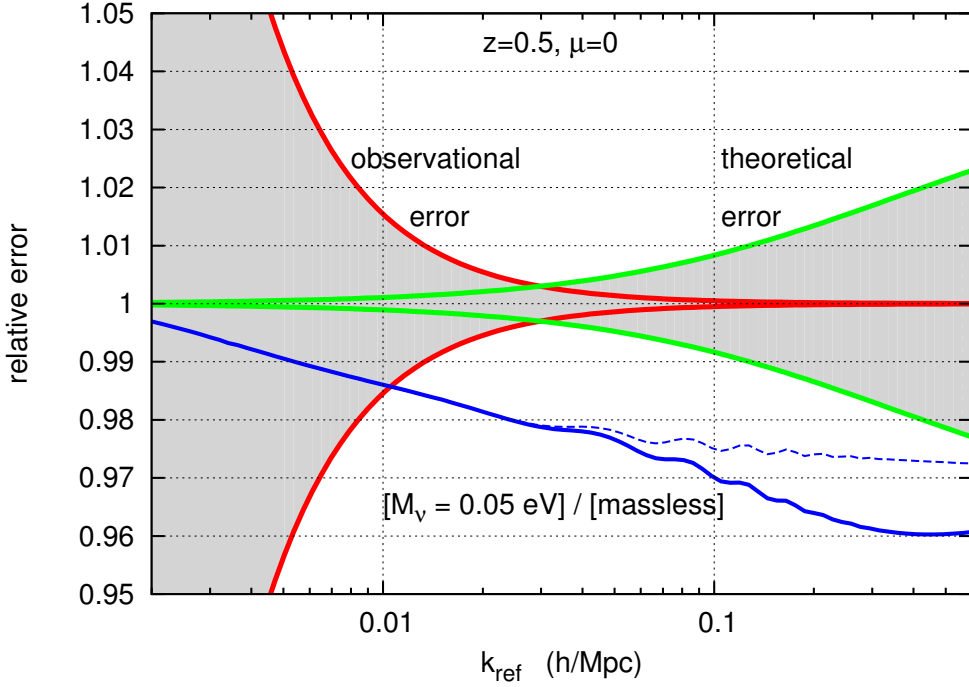


Figure 3: Relative error for the Galaxy Redshift Survey - both observational and theoretical error. In solid and dotted blue are guides for the eye, to remember the effect we are trying to measure: the impact of massive neutrinos compared to massless.

did a parameter inference and wrote the marginalised 1σ errors for all cosmological parameters, considering five cases: from no error at all (first line) to both uncorrelated and correlated error (fifth line).

Focusing on the last column, one can see that, without the inclusion of the theoretical errors, going from 0.1 h/Mpc to 0.6 h/Mpc shrinks the error bar on the total neutrino mass by a factor of 3. However, once the errors are taken into account, both predictions are degraded (even the one stopping at 0.1 h/Mpc, though less drastically). With the full error considered, actually, going to mildly nonlinear scales does not bring any improvement over stopping at linear scales.

A similar result is achieved for the Weak Lensing Survey, though the degradation due to the theoretical error is less noticeable. Indeed, Weak Lensing probe the matter at a larger redshift, which is less sensitive to nonlinear effects for similar scales. The degradation of the expected sensitivity compared to a study without errors is about 20%.

4. Discussion

This result, though disheartening, simply highlights the current state of our knowledge. We simply do not know well enough how to predict the matter power spectrum at mildly nonlinear scales, at least within our current understanding. Reaching a precision of around 0.2% at scales of $k = 0.6$ h/Mpc would allow us to make good use of Euclid precise measurement, but otherwise, stopping the analysis at bigger scales is enough.

k_{\max} (h/Mpc)	un. err.	co. err.	$10^4 \omega_b$	$10^4 \omega_c$	$10^3 n_s$	$10^{11} A_s$	$10^3 h$	z_{reio}	$3m_\nu = M_\nu$ (meV)
0.1	–	–	1.2	6.2	2.8	3.0	4.1	0.38	18
0.1	1/10	–	1.2	6.9	2.8	3.1	4.5	0.39	18
0.1	1/2	–	1.3	9.5	3.2	3.5	6.1	0.39	23
0.1	•	–	1.3	11	3.4	3.6	6.7	0.40	25
0.1	•	•	1.3	11	3.4	3.6	6.7	0.40	25
0.6	–	–	0.86	2.1	0.37	1.2	0.40	0.23	5.9
0.6	1/10	–	1.1	4.8	2.5	2.7	3.0	0.37	14
0.6	1/2	–	1.2	8.6	3.2	3.4	5.7	0.39	22
0.6	•	–	1.3	10	3.4	3.6	6.7	0.39	25
0.6	•	•	1.3	10	3.4	3.6	6.7	0.39	25

Table 1: Marginalized 1σ error for each model parameter, in a fit of *Planck* + *Euclid*-like galaxy survey data. The different lines correspond to different choices of k_{\max} , to the inclusion or not of the global uncorrelated theoretical error (un. err.), divided by ten (1/10), by two (1/2), or full (•), to that of the specific neutrino-related correlated error (co. err.), and to the use of the non-linear or linear power spectrum. The models with correlated error have one more nuisance parameter e_ν not shown here, with unit 1σ error.

This clearly shows two things. First, even current estimates that stop the analysis at supposedly linear scales of $k = 0.1$ h/Mpc might be overly optimistic - even to these scales, we can not claim a precision of better than 1%. Second, in order to make use of the future wealth of data that *Euclid* will provide us, and extract meaningful constraints on the total neutrino mass - amongst other physical effect - we will need to have a much better control of the systematics of our precision. Improving the schemes to predict the matter power spectrum is one thing, but controlling the exact error shape, see its evolution in the parameter space, characterise its robustness, all this will make our forecasts, and future parameter extractions, much more reliable. This will slowly lead to Large Scale Structure Cosmology being also part of precision cosmology.

The positive outcome of this study is that, despite not being able to extract data from the nonlinear scales, the detection of the total neutrino mass at at least 2.4σ is a guaranteed output of *Euclid*. This will finally put an upper bound on the individual masses, and help us uncover the veil on these elusive particles.

Acknowledgements

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