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Prospects for Lattice Calculations of Rare Kaon Decay Amplitudes

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Since the previous kaon conference in 2009, there has been very significant improvement in the precision which can be reached in lattice simulations for *standard* physical quantities, such as quark masses, the leptonic decay constants, B_K and the form factors for $K_{\ell 3}$ decays. In this talk I focus on quantities which have not been previously computed in lattice computations and for which the RBC-UKQCD collaborations are developing the necessary theoretical and computational framework. There are rare-kaon decays and the long-distance contributions to the K_L - K_S mass splitting, Δm_K. For Δm_K exploratory numerical studies have been performed to test the techniques and to gain some understanding of the precision which might be reached. For rare kaon decay amplitudes the exploratory work is now beginning.

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1. Introduction

At this conference we are seeing that there has been very significant progress since Kaon 2009 in the precision which can be reached for important quantities in flavour physics in general and kaon physics in particular [1]. This improvement has been made possible largely because of new algorithms and increased computing resources. It is now both necessary and possible to extend the range of physical quantities which can be studied using lattice simulations and the RBC-UKQCD collaborations are leading on several important extensions in kaon physics. In this talk I discuss the prospects of computing the decay amplitudes $K\to\pi\ell^+\ell^-$ and $K\to\pi\nu\bar\nu$ as well as reporting on the recent RBC-UKQCD exploratory study of the K_L - K_S mass difference [2], considering in particular the evaluation of long-distance effects in these quantities, i.e. contributions which cannot be written in terms of matrix elements of local operators.

Standard physical quantities which are studied using lattice simulations include those in which non-perturbative QCD effects can be written as matrix elements of local composite operators between single-hadron states or between a single hadron and the vacuum:

$$\langle 0\,|\,O(0)\,|\,h\rangle \quad \text{and} \qquad \langle h_2\,|\,O(0)\,|\,h_1\rangle\,, \tag{1.1}$$

where h, h_1 and h_2 represent hadrons. An important recent extension is the evaluation of $K\to\pi\pi$ decay amplitudes with the goal of understanding the $\Delta I=1/2$ rule and the numerical value of ε'/ε [3–7]. Here we have two hadrons in the final state, in which the final-state interactions and non-exponential finite-volume effects have to be accounted for.

2. The K_L – K_S Mass Difference [2]

In this and the following section I will discuss our early attempts to extend the range of physical quantities which can be computed in lattice simulations by computing long-distance contributions. These are not given in terms of matrix elements of local operators but require the evaluation of integrals of non-local products of operators of the form

$$\int d^4x \int d^4y\;\langle h_2\,|\,T\{O_1(x)\,O_2(y)\}\,|\,h_1\rangle\,, \tag{2.1}$$

where $O_{1,2}$ are local composite operators. For $\Delta m_K = m_{K_L}-m_{K_S}$, the relevant integral is

$$\int d^4x \int d^4y\;\langle \bar K^0\,|\,T\{H_W(x)\,H_W(y)\}\,|\,K^0\rangle\,, \tag{2.2}$$

where H_W is the $\Delta S=1$ weak Hamiltonian given explicitly in eqs. (2.6) and (2.7) below. In both equations (2.1) and (2.2) T represents time-ordering.

Weak perturbation theory leads to the following expression for Δm_K:

$$\Delta m_K \equiv m_{K_L}-m_{K_S} = \frac{1}{2m_K}\,2\mathscr{P}\sum_\alpha \frac{\langle \bar K^0\,|\,H_W\,|\,\alpha\rangle\,\langle\alpha\,|\,H_W\,|\,K^0\rangle}{m_K-E_\alpha} = 3.483(6)\times 10^{-12}\,\text{MeV}\,, \tag{2.3}$$

where the numerical value is the experimental result. Here the sum over α includes the integrals over the relevant phase-space and $\mathscr{P}$ indicates that the principal value prescription is to be used for the poles at $E_\alpha = m_K$.

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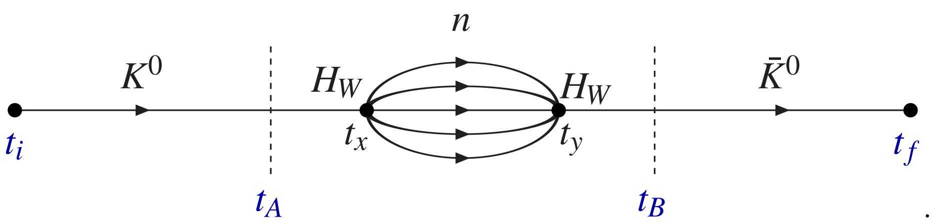

Figure 1: Schematic diagram of the evaluation of long-distance contributions to Δm_K. The integrals in (2.2) are performed over the *fiducial volume* $t_A \le t_{x,y} \le t_B$. n labels a generic state between the two H_W.

In lattice calculations of long-distance effects, it is not so much the discrete (i.e. lattice) nature of space time which presents difficulties for the evaluation of Δm_K as the fact that the calculation is necessarily performed in a finite volume. In a finite volume, the practical way to isolate the initial and final states correctly while still performing the time integrals is to integrate over a large subinterval in time, $t_A \le t_{x,y} \le t_B$, and to create the K^0 and annihilate the $\bar{K}^0$ well outside of this region (see Fig. 1). This is the natural modification of standard field theory for which the asymptotic states are prepared at $t \to \pm\infty$ and then the operators are integrated over all time. The corresponding 4-point correlation function is then

$$C_4(t_A,t_B;t_i,t_f) = |Z_K|^2 e^{-m_K(t_f-t_i)} \sum_n \frac{\langle \bar{K}^0\,|\,H_W\,|\,n\rangle\,\langle n\,|\,H_W\,|\,K_0\rangle}{(m_K-E_n)^2}\left\{e^{(m_K-E_n)T}-(m_K-E_n)T-1\right\}, \tag{2.4}$$

where, in units of the lattice spacing a, $T = t_B - t_A + 1$ and Z_K is the matrix element of the kaon interpolating operator between the vacuum and a kaon at rest. From the coefficient of T we can obtain

$$\Delta m_K^{\rm FV} \equiv 2\sum_n \frac{\langle \bar{K}^0\,|\,H_W\,|\,n\rangle\,\langle n\,|\,H_W\,|\,K_0\rangle}{(m_K-E_n)}\,. \tag{2.5}$$

In order to evaluate Δm_K itself we need to be able to:
1. Evaluate the four types of diagram illustrated in Fig. 2. The four-quark current-current operators in the weak Hamiltonian are represented by two small filled circles, each representing a bilinear quark current operator. The type of diagram is defined in terms of the quark flow between the interpolating operators for the mesons and the insertions of H_W. Other diagrams, in addition to those in Fig. 2, contribute to each type.
2. Use the T dependence of C_4 in (2.4) to isolate the coefficient of T. Note that in general there may be states $|n\rangle$ with $E_n < m_K$ implying that there will be terms which grow exponentially with T which may be difficult to subtract. The contributions from the vacuum and single pion states can be explicitly removed by respectively adding the pseudoscalar density $\bar{s}\gamma^5 d$ and the scalar density $\bar{s}d$ to the weak Hamiltonian with coefficients c_P and c_S such that $\langle 0|\,H_W + c_P(\bar{s}\gamma^5 d)\,|K^0\rangle = 0$ and $\langle \pi^0|\,H_W + c_s(\bar{s}d)\,|K\rangle = 0$ [1]. As the volume of the lattice is increased there are an increasing number of two-pion states with energies smaller than m_K, making the practical problem of isolating the linear term in T in the integrated correlation function more difficult. For the foreseeable future however, we anticipate that the available volumes will be such that there will be at most one such

[1] I am grateful to Guido Martinelli for discussions on this point.

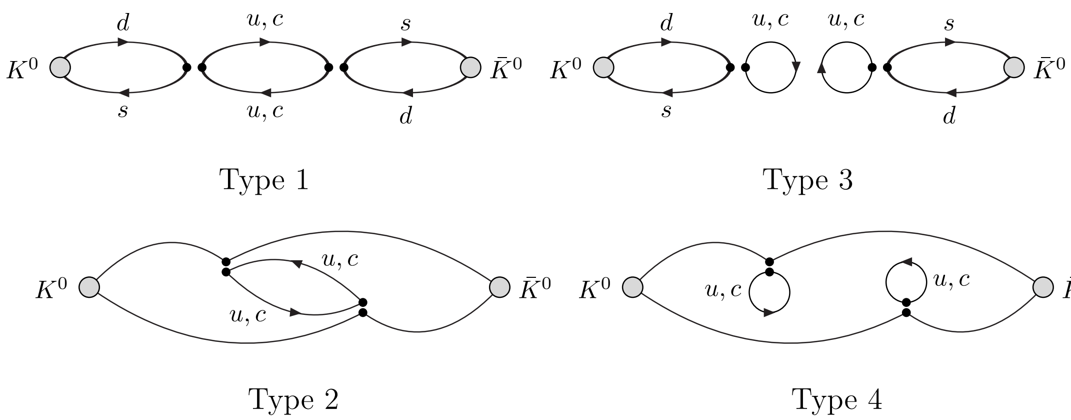

Figure 2: Four types of diagram which have to be evaluated for the calculation of Δm_K.

term. Indeed, in our exploratory studies at unphysical quark masses there are no two-pion states with energies below m_K.

3. Relate Δm_K and Δm_K^{FV}. Following the pioneering papers of Lüscher [8] concerning two-pion states in a finite volume, it is known that in general finite-volume effects are not exponentially small in the volume, but decrease only as powers. The evaluation of finite-volume corrections for Δm_K requires a significant extension of the theory of finite-volume effects developed previously for the two-pion spectrum [8] and for $K \to \pi\pi$ amplitudes [9, 10]. Results for the case for which the volume is tuned so that one of the two-pion states has energy equal to m_K were presented in [2, 11] and a study of the general case is being prepared for publicationl [12]. I do not present the details of the derivation here other than to observe that the complete term on the right-hand side of Eq. (2.4) does not have a pole as one of the $E_n \to m_K$ which confirms that the finite-volume corrections to the correlation function itself are exponentially small as expected [10]. The term linear in T by itself however, does have a pole and hence power-like corrections which need to be subtracted to obtain a precise result.

4. Control the additional ultraviolet divergences as the weak Hamiltonians come close together. Power counting implies that these corrections are quadratic in the ultraviolet cutoff (a the lattice spacing) and this is indeed the case. This divergence is cancelled by the GIM mechanism which requires the presence of charm quarks. We also find that after the GIM subtraction of the power divergences there are no remaining logarithmic ones [2].

The $\Delta S = 1$ effective weak Hamiltonian including four flavours is

$$H_W = \frac{G_F}{\sqrt{2}} \sum_{q,q'=u,c} V_{qd} V^*_{q's} (C_1 Q_1^{qq'} + C_2 Q_2^{qq'}) \tag{2.6}$$

where

$$Q_1^{qq'} = (\bar{s}_i d_i)_{V-A} (\bar{q}_j q'_j)_{V-A} \quad \text{and} \quad Q_2^{qq'} = (\bar{s}_i d_j)_{V-A} (\bar{q}_j q'_i)_{V-A}\,. \tag{2.7}$$

In our exploratory study on the 16^3 ensembles with $m_\pi = 420\,\mathrm{MeV}$, we only evaluated the type 1 and 2 graphs. The development of techniques necessary to evaluate disconnected diagrams effectively is an area of worldwide active research and in this study we focused instead on learning

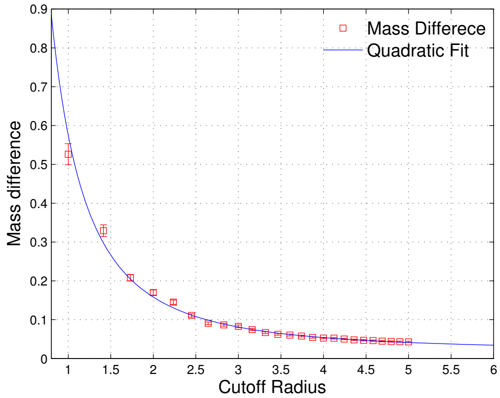

Figure 3: Contribution to the mass difference from the correlation function of $Q_1 - Q_1$ as a function of the cut of radius R as described in the text. The curve is a fit to the function $b/R^2 + c$, where b, c are constants. The quadratic divergence is cancelled when the GIM mechanism is applied.

how to control the remaining systematics. The physical value of Δm_K is $3.483(6) \times 10^{-12}$ MeV, whereas we obtain values in the range $\{5.81(28) - 10.58(75)\} \times 10^{-12}$ MeV as m_K is varied from 563 to 839 MeV. In spite of the unphysical masses and neglecting type 3 and 4 diagrams we are nevertheless able to obtain a result reasonably close to the physical value and are encouraged to proceed to a full physical calculation. We are currently completing a study on a larger lattice (with a spacial volume of 24^3 points) at the same value of the lattice spacing with a pion with mass 330 MeV and evaluating all 4 types of diagram.

The details of our first study are presented in [2]. As an example of the investigations, consider the ultraviolet behaviour of the $Q_1 - Q_1$ correlation function (i.e. taking the Q_1 component of each H_W in (2.2)) without the GIM subtraction but with an artificial lower cut-off, $R = \sqrt{\{(t_y - t_x)^2 + (\vec{x} - \vec{y})^2\}}$, on the separation of the two Q_1 insertions. The plot in Fig.3 exhibits the quadratic divergence as the two operators come together. This divergence is canceled by the GIM mechanism.

3. Rare Kaon Decays

Rare kaon decays which are dominated by short-distance FCNC processes, $K \to \pi \nu \bar{\nu}$ decays in particular, provide a potentially valuable window on new physics at high-energy scales. The decays $K_L \to \pi^0 e^+ e^-$ and $K_L \to \pi^0 \mu^+ \mu^-$ are also considered promising because the long-distance effects are reasonably under control using ChPT [13]. They are sensitive to different combinations of short-distance FCNC effects and hence in principle provide additional discrimination to the neutrino modes. A challenge for the lattice community is therefore to calculate the long-distance effects reliably. The existing phenomenology on rare kaon decays is based largely on $\mathrm{SU(3)_L \times SU(3)_R}$ chiral perturbation theory (ChPT) and lattice calculations will also provide the opportunity for checking the range of validity of ChPT and evaluating the corresponding Low Energy Constants.

As an example consider the decay $K_L \to \pi^0 \ell^+ \ell^-$ which has three main contributions to the amplitude [13],

(i) short distance contributions corresponding to matrix elements of the local operators $(\bar{s}\gamma_\mu d)(\bar{\ell}\gamma^\mu \ell)$ and $(\bar{s}\gamma_\mu d)(\bar{\ell}\gamma^\mu\gamma_5\ell)$. The hadronic (non-perturbative QCD) contribution is simply given by the form-factors of semileptonic $K\to\pi\ell\bar{\nu}_\ell$ decays, which are known to much better precision than the remaining contributions.

(ii) long-distance indirect CP-violating contribution from the CP-even component of K_L, $A_{ICPV}(K_L\to\pi^0\ell^0\ell^-)=\varepsilon A(K_1\to\pi^0\ell^+\ell^-)$ and

(iii) the two-photon CP-conserving contribution $K_L\to\pi^0(\gamma^*\gamma^*\to\ell^+\ell^-)$.

A summary of the corresponding phenomenology is presented in Ref. [14]. For example the branching ratios for the CP-violating (CPV) component are written as:

$$\mathrm{Br}(K_L\to\pi^0e^+e^-)_{\mathrm{CPV}} = 10^{-12}\times\left\{15.7|a_S|^2\pm 6.2|a_S|\left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right)+2.4\left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right)^2\right\} \quad (3.1)$$

$$\mathrm{Br}(K_L\to\pi^0\mu^+\mu^-)_{\mathrm{CPV}} = 10^{-12}\times\left\{3.7|a_S|^2\pm 1.6|a_S|\left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right)+1.0\left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right)^2\right\}, \quad (3.2)$$

where a_S is the (unphysical) amplitude for the decay $K_S\simeq K_1\to\pi^0\ell^+\ell^-$ at momentum transfer $q^2=0$. Using ChPT based phenomenology, $|a_S|=1.06^{+0.26}_{-0.21}$ but the sign of a_S is unknown [14]. One goal of future lattice calculations is the determination of a_S, together with similar other quantities. In addition however, we will be able to vary the external momenta and study the behaviour of the amplitude as a function of q^2. Using partially twisted boundary conditions [15, 16], this has been done very successfully for the form factors in $K_{\ell 3}$ decays [17]. See [18] for a review of the current status of the calculations of $K_{\ell 3}$ decay amplitudes.

From the above discussion we wish to compute the amplitudes for the CP-Conserving decays $K_S\to\pi^0\ell^+\ell^-$ and $K^+\to\pi^+\ell^+\ell^-$ and start by considering

$$T_i^\mu=\int d^4x\,e^{-iq\cdot x}\,\langle\pi(p)\,|\,\mathrm{T}\{J^\mu_{\mathrm{em}}(x)\,Q_i(0)\,\}\,|\,K(k)\rangle\,, \quad (3.3)$$

where Q_i, $i=1,2$ is an operator in the effective Hamiltonian and J^μ_{em} is the electromagnetic current. Electromagnetic gauge invariance implies that T_i^μ takes the form

$$T_i^\mu=\frac{\omega_i(q^2)}{(4\pi)^2}\left\{q^2(p+k)^\mu-(m_K^2-m_\pi^2)\,q^\mu\right\}. \quad (3.4)$$

It is the form factor $\omega(q^2)$ which will be the output of the calculation. The computation will proceed in a similar way to the evaluation of Δm_K, by inserting the interpolating operators for the initial kaon and final pion states at times which are sufficiently far from the "fiducial volume", i.e. the range of integration over which the time of the insertion of the current is integrated.

Although the lattice computation does not rely on ChPT nevertheless, since most of the existing phenomenology is performed in the ChPT framework, it may be useful to compute the necessary low energy constants. The LECs a_+ and a_S are defined by

$$a=\frac{1}{\sqrt{2}}V_{us}^*V_{ud}\left\{C_1\omega_1(0)+C_2\omega_2(0)+\frac{2N}{\sin^2\theta_W}f_+(0)C_{7V}\right\}$$

where $Q_{1,2}$ are the two current-current GIM subtracted operators and the C_i are the Wilson coefficients, (C_{7V} is the coefficient of $\bar{s}\gamma_\mu d)(\bar{l}\gamma^\mu l)$) [19]. An interesting target for the lattice calculations is to check the validity of the phenomenological values: $a_+ = -0.578 \pm 0.016$ and $|a_S| = 1.06^{+0.26}_{-0.21}$, as well as to determine the sign of $|a_S|$.

The generic non-local matrix elements which we need to evaluate are

$$X \equiv \int_{-\infty}^{\infty} dt_x\, d^3x\, \langle \pi(p)\,|\,\mathrm{T}\left[J_\mu(0)\,H_W(x)\right]\,|K(\vec{0})\rangle \tag{3.5}$$

$$= i\sum_n \frac{\langle \pi(p)\,|\,J_\mu(0)\,|n\rangle\,\langle n\,|H_W(0)\,|\,K(\vec{0})\rangle}{m_K - E_n + i\varepsilon} - i\sum_{n_s} \frac{\langle \pi(p)\,|\,H_W(0)\,|n_s\rangle\,\langle n_s\,|J_\mu(0)\,|\,K(\vec{0})\rangle}{E_{n_s} - E_\pi + i\varepsilon}\,. \tag{3.6}$$

J_μ represents a vector or axial, electromagnetic or weak, current and $\{|n\rangle\}$ and $\{|n_s\rangle\}$ represent complete sets of non-strange and strange states. In Euclidean space we envisage calculating correlation functions of the form

$$\int_{-T_a}^{T_b} dt_x\,\langle \phi_\pi(\vec{p},t_\pi)\,\mathrm{T}\left[J_\mu(0)\,H_W(t_x)\right]\,\phi_K^\dagger(\vec{0},t_K)\,\rangle \equiv \sqrt{Z_K}\,\frac{e^{-m_K|t_K|}}{2m_K}\,X_E\,\sqrt{Z_\pi}\,\frac{e^{-E_\pi t_\pi}}{2E_\pi}\,, \tag{3.7}$$

where ϕ_π and ϕ_K are interpolating operators for the pion and kaon respectively and

$$X_{E_-} = -\sum_n \frac{\langle \pi(p)\,|\,J_\mu(0)\,|n\rangle\,\langle n\,|H_W(0)\,|\,K\rangle}{m_K - E_n}\left(1 - e^{(m_K-E_n)T_a}\right) \quad \text{and} \tag{3.8}$$

$$X_{E_+} = \sum_{n_s} \frac{\langle \pi(p)\,|\,H_W(0)\,|n_s\rangle\,\langle n_s\,|J_\mu(0)\,|\,K\rangle}{E_{n_s} - E_\pi}\left(1 - e^{-(E_{n_s}-E_\pi)T_b}\right)\,. \tag{3.9}$$

We use the time dependence to subtract the exponential terms in a similar way to the corresponding subtraction for Δm_K.

Sample diagrams which have to be evaluated to determine the amplitudes for $K \to \pi \ell^+\ell^-$ decays are presented in Fig. 4.

The authors of ref. [20] investigated the ultraviolet behaviour as the current J_μ approaches H_W. For illustration consider the diagram of type 2 shown in Fig. 5, redrawn using the Fierz identity. Dimensional counting allows for a quadratic divergence in such diagrams but conservation of the vector current suggests that the degree of divergence is reduced by 2 to result in a logarithmic divergence. For this to be the case the conserved lattice vector current must be used in the simulations. This was checked in an explicit one-loop perturbative calculation for Wilson and Clover fermion actions in [20]. This absence of power divergences does not require the use of the GIM mechanism and for a chiral symmetric formulation of lattice QCD, such as DWF, the same applies for the axial current. If the calculations are performed in the four-flavour theory, i.e. with charm quarks, then the GIM mechanism also cancels the logarithmic divergence in this diagram.

4. Summary, Conclusions and Prospects

The goal of the lattice flavour-physics community is to develop a programme with an ever increasing precision and a growing range of physical quantities which can be studied. Precision flavour physics complements the large $p_\perp$ approach to testing the limits of the standard model and searches for new physics. Standard quantities, such as quark masses, mesonic decay constants,

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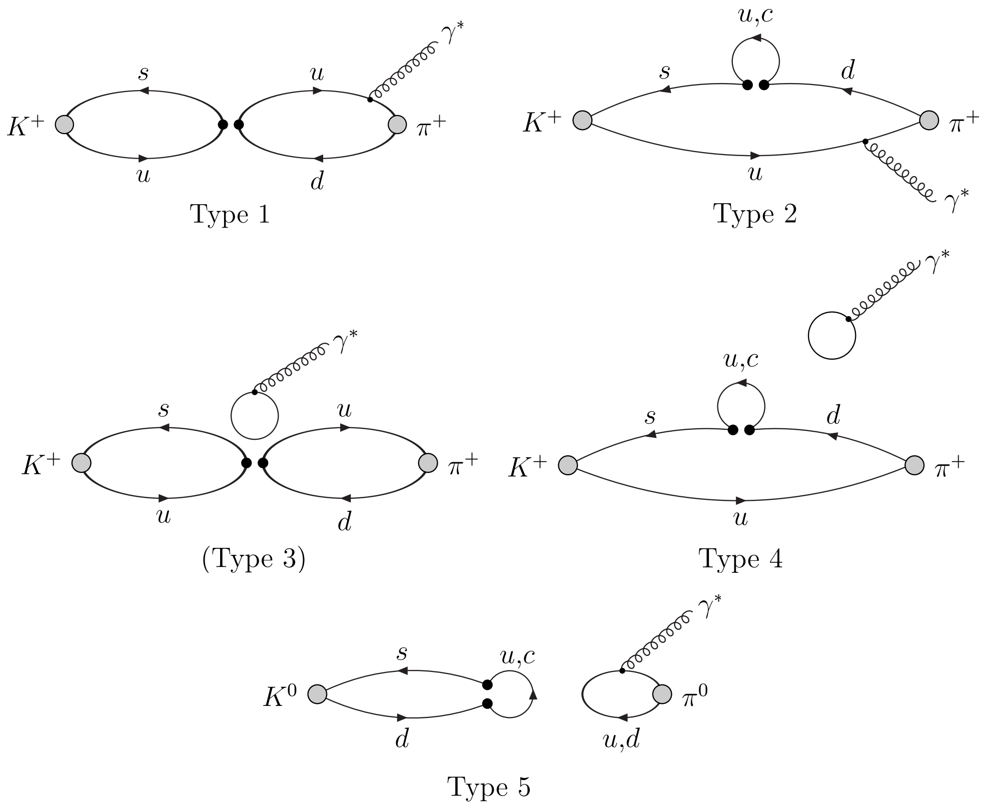

Figure 4: Sample diagrams which need to be evaluated to determine the amplitudes for $K \to \pi \ell^+ \ell^-$ decays. For diagrams of type 1, 2 and 5 the photon can be emitted of any internal quark line. Diagrams of type 1 - 4 contribute to both $K^+ \to \pi^+ \ell^+ \ell^-$ and $K^0 \to \pi^0 \ell^+ \ell^-$ decays. The diagrams of type 5 only contribute to K^0 decays.

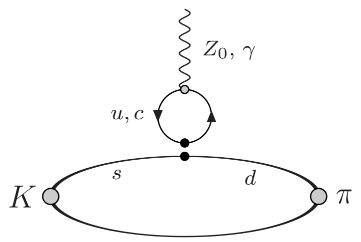

Figure 5: Contribution in spite of power counting is logarithmically divergent in the ultraviolet, in spite of naïve dimensional counting. H_W is represented by the two small filled circles.

the B_K parameter of neutral kaon mixing and $K_{\ell 3}$ form factors are now calculated with excellent precision [21]. In this talk I have reviewed some recent ideas from the RBC-UKQCD collaboration, focussing on new quantities in kaon physics which we are learning to compute. Norman Christ at this conference has explained how we have performed the first direct calculation of the $K \to (\pi\pi)_{I=2}$ decay amplitude A_2 and how we are well on our way to computing A_0 [3]. In this talk I have described the progress towards evaluating long-distance effects in Δm_K, the procedure for which is now largely tested, and our preparation for the similar computations of the amplitudes for rare kaon decays.

It is our hope that by discussing the prospects for calculations of rare kaon decay amplitudes at this conference we will encourage a discussion with the wider community with the aim of optimising plans for our future research programme at an early stage.

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