PROCEEDINGS OF SCIENCE

Radiative Decays, including $K^+ \to \pi^+\pi^0 ee$, $K_S \to \mu\mu$

Giancarlo D'Ambrosio[*†]
Istituto Nazionale di Fisica Nucleare, Sezione di Napoli, Complesso Universitario di Monte S. Angelo, Via Cintia Edificio 6, 80126 Napoli, Italy
E-mail: gdambros@na.infn.it

We discuss first the decays $K_{S,L} \to \mu^+\mu^-$, which have received recently some attention due to the measurement by LHCB. Short distance (SD) can also be extracted from other decays after a careful analysis of the background long distance contributions (LD); therefore we will study also $K_L \to \pi^0 e^+e^-$,$K^+ \to \pi^+\pi^0\gamma$, $K^+ \to \pi^+\pi^0 e^+e^-$ and related channels

2013 Kaon Physics International Conference, 29 April-1 May 2013 University of Michigan, Ann Arbor, Michigan - USA

PoS(KAON13)021

[*]Speaker.

[†]This work was partially supported by MIUR under project 2010YJ2NYW.

1. $K_{L,S} \to \mu^+\mu^-$

Recent LHCB measurement on $K_S \to \mu\overline{\mu}$ [1] is very interesting even if it is still smaller than the SM prediction

$$B(K_S \to \mu\overline{\mu})_{LHCB} < 9 \times 10^{-9} \text{ at } 90\ \%\ \text{CL} \qquad B(K_S \to \mu\overline{\mu})_{SM} = (5.0 \pm 1.5) \times 10^{-12}. \tag{1.1}$$

It represents an important milestone since it has improved the previous limit, $< 3.2 \times 10^{-7}$ at 90 % CL, lasted 40 years [2]. It is based on a production of 10^{13} K_S per fb^{-1} inside the LHCB acceptance and it is obtained using 1.0 fb^{-1} of pp collisions at $\sqrt{s} = 7$ TeV collected in 2011.

Historically $K_{S,L} \to \mu^+\mu^-$ have been important to the discovery of GIM mechanism. Two photon exchange generates the dominant contribution for both K_L and K_S decays to two muons [3]. The structure of weak and electromagnetic interactions entails a vanishing CP conserving short distance contribution to $K_S \to \mu^+\mu^-$. We can write generally

$$A(K^0 \to l^+l^-) = \overline{u}_l(iB\ +\ A\gamma_5)v_l \tag{1.2}$$

and if no polarization is measured A- and B-amplitudes do not interfere:

$$\Gamma(K_{L,S} \to \mu^+\mu^-) = \frac{m_K\ \beta_l}{8\pi}\left(|A|^2\ +\ |B|^2\right) \qquad \beta_l = \sqrt{1 - \frac{4m_\mu^2}{m_K^2}} \tag{1.3}$$

However polarization studies can be interesting as shown by Peter Heczeg in Ref.[4] and later references. The question of the relevant contributions to $K_{L,S} \to \mu^+\mu^-$ has been addressed in Refs.[3], even before the powerful and elegant approach of effective field theory had been fully exploited; already in these papers it was established very generally that no short distance CP conserving contributions to $K_S \to \mu^+\mu^-$ were allowed but only long distance contributions. Indeed the SM short diagrams in Fig. 1 lead to the SM effective hamiltonian [5]

$$\mathscr{H}_{eff}^{SM} = -\frac{G_F\,\alpha_{\rm em}(M_Z)}{2\sqrt{2}\pi\sin^2\theta_W}\left[(V_{ts}^*V_{td})Y(x_t) + (V_{cs}^*V_{cd})Y_{NL}\right](\bar{s}\ d)_{V-A}\ (\overline{\mu}\ \mu)_{V-A} + \quad h.c.\ . \tag{1.4}$$

where Y_{NL} and $Y(x_t)$ are the Inami-Lin functions [5]. The LD contributions to $K_S \to \mu^+\mu^-$ in Fig. 1 have been computed reliably in CHPT in Ref. [6]. CHPT predicts a non-vanishing, finite and unambiguous contribution at $\mathscr{O}(p^4)$ for the decay $K_S \to \gamma\gamma$ [7], small NLO CHPT contributions have been evaluated in Ref. [8] to match the experimental value for $B(K_S \to \gamma\gamma)$.

$$B(K_S \to \mu\overline{\mu})_{SM}^{LD} = (5.0 \pm 1.5) \times 10^{-12} \quad \text{from} \quad B(K_S \to \gamma\gamma)_{p^4} \tag{1.5}$$

Still too small compared to the LHCB result in eq. (1.1) but larger than SM short distance contribution and interestingly smaller than possible allowed new physics contributions (NP)[9]

$$B(K_S \to \mu\overline{\mu})_{SM}^{SD} = 1 \times 10^{-5}|\Im(V_{ts}^*V_{td})|^2 \sim 10^{-13} \qquad \text{vs} \qquad B(K_S \to \mu\overline{\mu})_{NP} \leq 10^{-11} \tag{1.6}$$

The short distance hamiltonian in eq.(1.4) will contribute to $K_L \to \mu\overline{\mu}$, through a CP conserving amplitude, $\Re(A_{\rm short})$, that can be extracted but requires a careful study of the dominant LD two-photon exchange, $A_{\gamma\gamma}$:

$$A(K_L \to \mu\overline{\mu}) = \left[A_{\gamma\gamma} + \Re(A_{\rm short})\ \right]\ \overline{\mu}\ \gamma_5\ \mu \tag{1.7}$$

PoS(KAON13)021

Figure 1: Left figure: $K_{S,L} \to \mu\mu$ SM short distance contributions, CP violating (conserving) for K_S, (K_L). Right figure: Long distance (LD) contributions to $K_S \to \mu\mu$, evaluated in CHPT in Ref. [6], $K_S \to \gamma\gamma$ is finite at leading order in CHPT [7].

and $A_{\gamma\gamma}$ generates the only source of absorptive contribution; then we can write explicitly

$$\Gamma(K_L \to \mu\overline{\mu}) = \frac{2\alpha_{\rm em}^2 r_\mu \beta_\mu}{\pi^2}\left[R_{abs} + R_{disp}\right]\Gamma(K_L \to \gamma\gamma) \tag{1.8}$$

where $r_\mu = \dfrac{m_\mu^2}{m_K^2}$, $\beta_\mu = \sqrt{1-4r_\mu}$, the model independent absorptive contribution is

$$R_{abs} = \left[\frac{\pi}{2\,\beta_\mu}\ln\frac{1-\beta_\mu}{1+\beta_\mu}\right]^2 = 27.14. \tag{1.9}$$

A careful analysis of several K_L decay modes ($\pi\pi$, $\gamma\gamma$..) [10] leads to the precise determination

$$\left[R_{abs} + R_{disp}\right] = (1.238 \pm 0.024)\cdot 10^{-5}. \tag{1.10}$$

which turns in a relevant determination

$$R_{disp} = |\chi_{\gamma\gamma}(M_\rho) + \ \chi_{\rm short} - 5.12|^2 = (0.98 \pm 0.55) \tag{1.11}$$

where $\chi_{\gamma\gamma}(M_\rho) - 5.12$ is the LD dispersive model dependent contributions and $\chi_{\rm short}$ is the short distance one, generated in the SM from the SD effective hamiltonian in (1.4); then [9, 11]

$$\chi_{\rm short}^{\rm SM} \ = \ \kappa\left[\frac{-\Re(V_{ts}^*V_{td})Y(x_t) - \Re(V_{cs}^*V_{cd})Y_{NL}}{4\times 10^{-4}}\right] = \kappa(1.11 - 0.92\overline{\rho})\ , \tag{1.12}$$

$$|\kappa| \ = \ 4\times 10^{-4}\left[\frac{m_K}{16\pi\Gamma(K_L \to \gamma\gamma)}\right]^{1/2}\frac{\sqrt{2}G_F m_K F_K \alpha_{\rm em}(M_Z)}{\sin^2\theta_W\,\alpha_{\rm em}} = 1.96\ , \tag{1.13}$$

To extract $\chi_{\gamma\gamma}(M_\rho) + \ \chi_{\rm short} - 5.12$ several models have been proposed [12, 13, 14]; here we describe mainly Refs. [9, 11] where a low energy parameterization of the $K_L \to \gamma^*\gamma^*$ form factor that includes the poles of the lowest vector meson resonances ($m_V \sim m_\rho$):

$$f(q_1^2, q_2^2) = 1 + \alpha\left(\frac{q_1^2}{q_1^2 - m_V^2} + \frac{q_2^2}{q_2^2 - m_V^2}\right) + \beta\,\frac{q_1^2 q_2^2}{(q_1^2 - m_V^2)(q_2^2 - m_V^2)}\ . \tag{1.14}$$

The ansatz is that, since we are able to describe the relevant resonances fully, this is the proper form factor to high energy up to the charm scale and in fact by comparing it with the short-distance result in [15] we constrain the parameters α and β: the form factor in Eq. (1.14) goes as $1+2\alpha+\beta$ for $q_i^2 \gg m_V^2$ and thus the logarithmically divergent $\chi_{\gamma\gamma}(M_\rho)$ can be phenomenologically compared with the known perturbative QCD calculation [15] leading to $|1+2\alpha+\beta|\ln(\Lambda/m_V) < 0.4$ (Λ is the ultraviolet cutoff); since α is fixed experimentally this turn in a value for β leading to [9, 11]

$$\chi_{\gamma\gamma}(M_\rho) = 5.83 \pm 0.15_{exp} \pm 1.0_{th} \tag{1.15}$$

Then from (1.11)

$$\chi_{\text{short}} = \begin{cases} -1.7 \pm 1.4 \\ +0.3 \pm 1.4 \end{cases} \qquad \text{or} \qquad -3.1 < \chi_{\text{short}} < 1.7 \tag{1.16}$$

If we make no assumption on the the sign of $A(K_L \to \gamma\gamma)$ only the second inequality holds.

2. $K_L \to \pi^0 e^+ e^-$, the related channels $K \to \pi\gamma\gamma$ and $K_S \to \pi^0 e^+ e^-$

The electroweak short distance contribution to $K_L \to \pi^0 e^+ e^-$, analogously to $K_L \to \pi^0 \nu\overline{\nu}$, violates directly CP violation; however there is long distance contamination due to electromagnetic interactions: i) a CP conserving contribution due to two-photon exchange and ii) an indirect CP violating contribution mediated by one photon exchange, i.e. the contribution suppressed by ε in $K_L \sim K_2 + \varepsilon K_1 \to \pi^0 e^+ e^-$ determined by the CP conserving $A(K_S \to \pi^0 e^+ e^-)$[16, 17, 8].

The CP-conserving decays $K^\pm(K_S) \to \pi^\pm(\pi^0)\ell^+\ell^-$ are dominated by the long-distance process $K \to \pi\gamma^* \to \pi\ell^+\ell^-$ [16, 17]. Our ignorance in the long distance dominated $A(K_S \to \pi^0 l^+ l^-)$ can be parametrized by one parameter a_S to be determined experimentally. NA48, finds in the electron [18] and muon final state [19]

$$|a_S|_{ee} = 1.06^{+0.26}_{-0.21} \pm 0.07 \qquad |a_S|_{\mu\mu} = 1.54^{+0.40}_{-0.32} \pm 0.06 \tag{2.1}$$

These results allow us to evaluate the CP violating branching

$$B(K_L \to \pi^0 e^+ e^-)_{CPV} \; = \; \left[15.3 a_S^2 - 6.8 \frac{\Im \lambda_t}{10^{-4}} a_S + 2.8 \left(\frac{\Im \lambda_t}{10^{-4}} \right)^2 \right] \times 10^{-12} \,, \tag{2.2}$$

The first term is the indirect CP violating contribution while the last term is the direct CP violating contribution; the second one is the interference, expected constructive. This allows a stronger signal [8]. This prediction is not far from the the present bound from KTeV [20]

$$\mathscr{B}(K_L \to \pi^0 e^+ e^-) < 2.8 \times 10^{-10} \quad \text{at} \quad 90\% \quad \text{CL}. \tag{2.3}$$

which also sets the interesting limit $\mathscr{B}(K_L \to \pi^0 \mu^+ \mu^-) < 3.8 \times 10^{-10}$ [21]. Still we have to show that we have under control the CP conserving contribution generated by two photon exchange. The general amplitude for $K_L(p) \to \pi^0 \gamma(q_1)\gamma(q_2)$ can be written in terms of two Lorentz and gauge invariant amplitudes $A(z,y)$ and $B(z,y)$. where $y = p(q_1 - q_2)/m_K^2$ and $z = (q_1+q_2)^2/m_K^2$. Then the double differential rate is given by

$$\frac{\partial^2\Gamma}{\partial y\,\partial z} \sim [z^2\,|A+B|^2 + \left(y^2 - \frac{\lambda(1,r_\pi^2,z)}{4}\right)^2 \,|B|^2]\,, \tag{2.4}$$

where $\lambda(a,b,c)$ is the usual kinematical function and $r_\pi = m_\pi/m_K$. Thus in the region of small z (collinear photons) the B amplitude is dominant and can be determined separately from the A amplitude. This feature is crucial in order to disentangle the CP-conserving contribution $K_L \to \pi^0 e^+e^-$. In fact the lepton pair produced by photons in a S-wave, like an $A(z)$-amplitude, are suppressed by the lepton mass while the photons in $B(z,y)$ are in a D-wave and so the resulting $K_L \to \pi^0 e^+e^-$ amplitude, $A(K_L \to \pi^0 e^+e^-)_{CPC}$, does not suffer necesseraly from the electron mass suppression [8]. The important message is that the $K_L \to \pi^0\gamma\gamma$ $z-$spectrum study has been able to limit $\mathscr{B}(K_L \to \pi^0 e^+e^-) < 5\cdot 10^{-13}$ at 90% CL [22, 23].

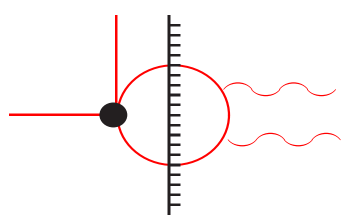

Figure 2: Unitarity contributions to $K \to \pi\gamma\gamma$

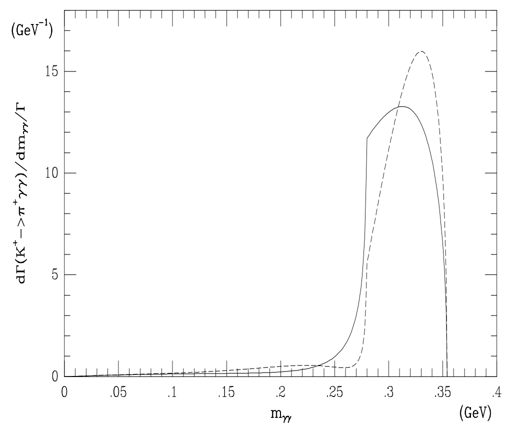

Figure 3: $K^+ \to \pi^+\gamma\gamma$: $\hat{c} = 0$, full line, $\hat{c} = -2.3$, dashed line, [25]

Recently a related channel, $K^+ \to \pi^+\gamma\gamma$, has attracted attention: new measurements of this decay have been performed using minimum bias data sets collected during a 3-day special NA48/2 run in 2004 with 60 GeV $K^\pm$ beams, and a 3-month NA62 run in 2007 with 74 GeV/c $K^\pm$ beams [28].

This channel starts at $\mathscr{O}(p^4)$, with pion (and kaon) loops and a local term $\hat{c}$. Due to the presence of the pion pole, there is a new helicity amplitude, C [24]; the unitarity contributions at $\mathscr{O}(p^6)$ in Fig.1 enhance the amplitude A by 30%-40% , along with the generation of a B-type amplitude [25]; as a result the differential decay rate is

$$\frac{d^2\Gamma}{dydz} \sim \left[z^2(|A+B|^2+|C|^2)+\left(y^2-\left(\frac{(1+r_\pi^2-z)^2}{4}-r_\pi^2\right)\right)^2|B|^2\right] \tag{2.5}$$

The constant $\hat{c}$ can be fixed by a precise determination of the rate and the spectrum as shown in Fig.2 [25]; this constant is predicted in terms of strong and weak counterterms, generated from the axial spin-1 contributions

$$\hat{c} = \frac{128\pi^2}{3}\left[3(L_9 + L_{10}) + N_{14} - N_{15} - 2N_{18}\right] \overset{\text{FM}}{=} 2.3\,(1 - 2\,k_f)\,,$$

with k_f is the factorization factor in the factorization model (FM) model [26]. BNL 787 got 31 events leading to $B(K^+ \to \pi^+\gamma\gamma) \sim (6 \pm 1.6) \cdot 10^{-7}$ [27] and a value of $\hat{c} = 1.8 \pm 0.6$. Recently NA48 has presented preliminary results normalizing $K^+ \to \pi^+\gamma\gamma$ with the channel $K^+ \to \pi^+\pi^0$: $\mathscr{B}(K^+ \to \pi^+\gamma\gamma) = (1.01 \pm 0.04 \pm 0.06) \cdot 10^{-6}$ and $\hat{c} = 2.00 \pm 0.24_{stat} \pm 0.09_{syst}$ [28].

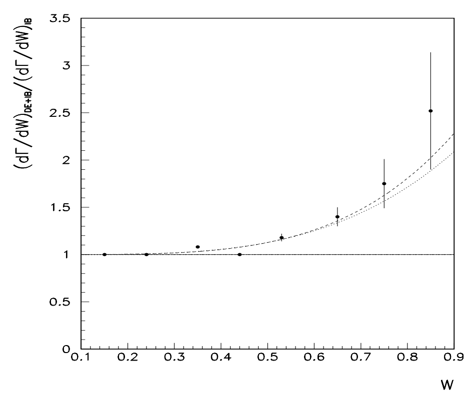

Figure 4: $K^+ \to \pi^+\pi^0\gamma$: deviations from IB

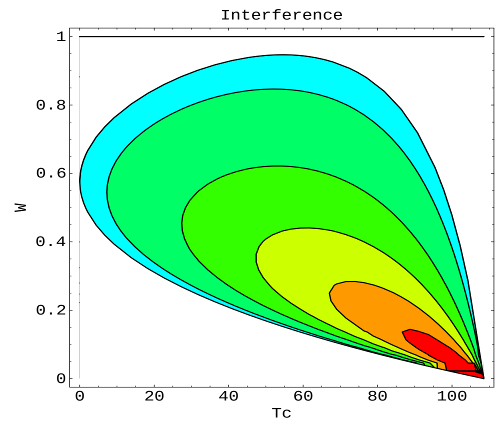

Figure 5: $K^+ \to \pi^+\pi^0\gamma$: $T_c^* - W$-Dalitz plot. In this contour plot of the interference Branching the red area corresponds to more dense and thus larger contribution

3. $K \to \pi\pi\gamma$ and $K \to \pi\pi ee$-decays

CP violation has been also studied in the $K \to \pi\pi\gamma$ and $K \to \pi\pi ee$ decays. According to gauge and Lorentz invariance we decompose $K(p) \to \pi(p_1)\pi(p_2)\gamma(q)$ decays, in electric (E) and magnetic (M) amplitudes [29]. In electric transitions one generally extracts the bremsstrahlung amplitude E_B. This is predicted by Low theorem in terms of the non-radiative amplitude and it is enhanced by the $1/E_\gamma$ pole. Summing over photon helicities: $\mathrm{d}^2\Gamma/(\mathrm{d}z_1\mathrm{d}z_2) \sim |E(z_i)|^2 + |M(z_i)|^2$. At the lowest order, (p^2), one obtains only E_B. Magnetic and electric direct emission amplitudes can be decomposed in a multipole expansion. In Table 2 we show the present experimental status of the DE amplitudes and the leading multipoles.

Table 2 DE_{exp}

$K_S \to \pi^+\pi^-\gamma$	$< 9 \cdot 10^{-5}$	$E1$
$K^+ \to \pi^+\pi^0\gamma$	$(0.44 \pm 0.07)10^{-5}$	$M1, E1$
$K_L \to \pi^+\pi^-\gamma$	$(2.92 \pm 0.07)10^{-5}$	$M1$,VMD

Particularly interesting are the recent interesting NA48/2 data regarding $K^+ \to \pi^+\pi^0\gamma$ decays [30]. Due to the $\Delta I = 3/2$ suppression of the bremsstrahlung, interference between E_B and the electric dipole ($E1$) and magnetic transitions can be measured. Defining $z_i = p_i \cdot q/m_K^2 \quad z_3 = p_K \cdot q/m_K^2$

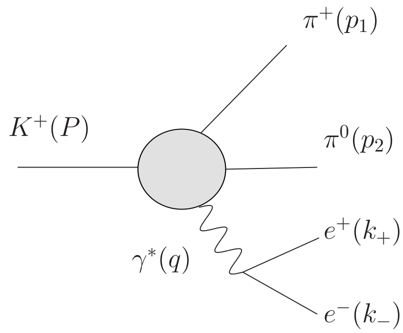

Figure 6: *Photon-mediated $K^+ \to \pi^+\pi^0 e^+e^-$ decay with our kinematical conventions. The blob represents the hadronic tensor H_μ.*

and $z_3 z_+ = \frac{m_{\pi^+}^2}{m_K^2} W^2$ we can study the deviation from bremsstrahlung (see Fig.3)

$$\frac{\partial^2\Gamma}{\partial T_c^* \partial W^2} = \frac{\partial^2\Gamma_{IB}}{\partial T_c^* \partial W^2}\left[1+\frac{m_{\pi^+}^2}{m_K}2Re\left(\frac{E_{DE}}{eA}\right)W^2+\frac{m_{\pi^+}^4}{m_K^2}\left(\left|\frac{E_{DE}}{eA}\right|^2+\left|\frac{M_{DE}}{eA}\right|^2\right)W^4\right],$$

where $A = A(K^+ \to \pi^+\pi^0)$. The Dalitz plot distribution of the interference term is shown in Fig. 4. Study of the Dalitz plot has lead NA48 to these results [30]

Table 3

NA48/2		$T_c^* \in [0,80]$ MeV
Frac(DE)	=	$(3.32\pm0.15\pm0.14)\times10^{-2}$
Frac(INT)	=	$(-2.35\pm0.35\pm0.39)\times10^{-2}$

Also the interesting CP bound was obtained [30]:

$$\frac{\Gamma(K^+\to\pi^+\pi^0\gamma)-\Gamma(K^-\to\pi^-\pi^0\gamma)}{\Gamma(K^+\to\pi^+\pi^0\gamma)+\Gamma(K^-\to\pi^-\pi^0\gamma)} < 1.5\cdot10^{-3} \quad \text{at} \quad 90\% \quad \text{CL.} \tag{3.1}$$

With more statistics the Dalitz plot analysis in Fig. 4 will be more efficient.

We have studied also the decay $K^\pm \to \pi^\pm\pi^0 e^+e^-$ in Fig. 5 [31]. Historically kaon four body semileptonic decays, K_{e4} have been studied as a tool to tackle final state rescattering effects in $K \to \pi\pi$-decays: crucial to this goal has been finding an appropriate set of kinematical variables which would allow i) to treat the system as two body decay in dipion mass $M_{\pi\pi}$ and dilepton mass $M_{l^+l^-}$ [32] and ii) to identify appropriate kinematical asymmetries to extract observables crucially dependent on final state interaction. In Fig. 7 we show the traditional kinematical variables for the four body kaon semileptonic decay which allow to write the four body phase space Φ in terms of the two two-body phase space Φ_π Φ_ℓ from [32]

$$d\Phi = \frac{1}{4m_K^2}(2\pi)^5\int ds_\pi \int ds_\ell \lambda^{1/2}(m_K^2, p_\pi^2, q^2)\Phi_\pi\Phi_\ell. \tag{3.2}$$

Then defining $q^2 = M_{e\nu}^2$ and the he $\pi\pi$ invariant mass $p_\pi^2 = M_{\pi\pi}^2$, we can write

$$d^5\Phi = \frac{1}{2^{14}\pi^6 m_K^2}\frac{1}{s_\pi}\sqrt{1-\frac{4m_\ell^2}{q^2}}\lambda^{1/2}(m_K^2, p_\pi^2, q^2)\lambda^{1/2}(p_\pi^2, m_{\pi^+}^2, m_{\pi^0}^2)dp_\pi^2 dq^2 d\cos\theta_\pi d\cos\theta_\ell d\phi, \tag{3.3}$$

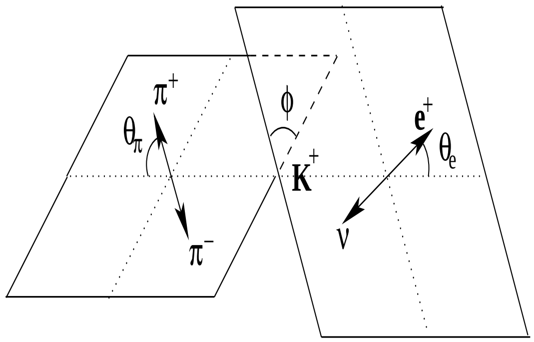

Figure 7: $K^+ \to \pi^+\pi^0 e^+e^-$ kinematical planes: N. Cabibbo and A. Maksymowicz defintion of the angles [32]

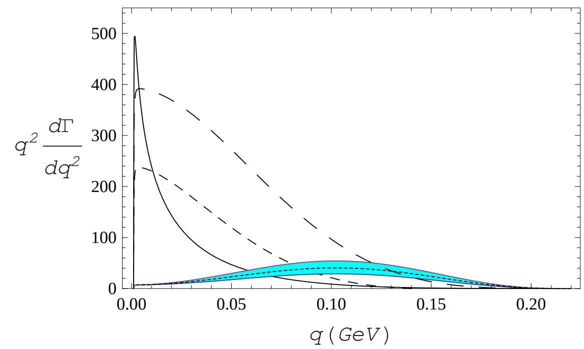

Figure 8: q dependence of the different contributions. The solid line represents the Bremsstrahlung. The dashed lines (from bigger dash to smaller dash) are 100×M, 100×BE and 300×E, respectively.

Then the K_{e4} amplitude is written as

$$\mathcal{M}_{l4} = \frac{G_F}{\sqrt{2}} V_{us} \left[\overline{u}(p_e)\gamma^\mu (1-\gamma^5) v(p_\nu)\right] H_\mu(p_1, p_2, q), \tag{3.4}$$

where H_μ is the hadronic vector, which can be written in terms of 3 form factors $F_{1,2,3}$:

$$H^\mu(p_1, p_2, q) = F_1 p_1^\mu + F_2 p_2^\mu + F_3 \varepsilon^{\mu\nu\alpha\beta} p_{1\nu} p_{2\alpha} q_\beta. \tag{3.5}$$

The goal was to obtain some asymmetry strongly dependent on the final state $\delta_j^i(s)$ in the form factors

$$F_i(s) = f_i(s) e^{i\delta_0^0(s)} + ..$$

Indeed

$$\begin{aligned} \frac{d^5\Gamma}{dE_\gamma^* dT_c^* dq^2 d\cos\theta_\ell d\phi} &= \mathcal{A}_1 + \mathcal{A}_2 \sin^2\theta_\ell + \mathcal{A}_3 \sin^2\theta_\ell \cos^2\phi \\ &+ \mathcal{A}_4 \sin 2\theta_\ell \cos\phi + \mathcal{A}_5 \sin\theta_\ell \cos\phi + \mathcal{A}_6 \cos\theta_\ell \\ &+ \mathcal{A}_7 \sin\theta_\ell \sin\phi + \mathcal{A}_8 \sin 2\theta_\ell \sin\phi + \mathcal{A}_9 \sin^2\theta_\ell \sin 2\phi, \end{aligned} \tag{3.6}$$

where θ_ℓ and ϕ are two variables for K_{l4} decays [32] and $\mathcal{A}_i$ are dynamical functions that can be parameterized in terms of three form factors. The amplitudes $\mathcal{A}_{8,9}$, odd in θ_ℓ, are also linearly dependent on the final state, establishing a clear way to determine them; while $\mathcal{A}_{5,6,7}$ are generated by interference with the axial leptonic current.

One can easily show that the Bremsstrahlung, direct emission and electric interference terms contribute to $\mathcal{A}_{1-4}$. In contrast, $\mathcal{A}_{8,9}$ receive contributions from the electric-magnetic interference terms (BM and EM) and therefore capture long-distance induced P-violating terms. $\mathcal{A}_{5,6,7}$ are also P-violating terms but generated through the interference of Q_{7A} with long distances.

Essentially two groups [33] studied the decay $K_L \to \pi^+\pi^- e^+e^-$. Here the targets are mainly short distance physics, *i.e.* $\mathcal{A}_{5,6,7}$ and the diplane angular asymmetry proportional to $\mathcal{A}_{8,9}$. This last observable is large and has been measured by KTeV and NA48 [34, 2]. However this observable is proportional to electric (bremsstrahlung) and magnetic interference, both contributions known already from $K_L \to \pi^+\pi^-\gamma$. In fact it was known that these contributions were large and they may obscure smaller but more interesting short distance physics effects.

We have performed a similar analysis for the decay $K^+ \to \pi^+\pi^0 e^+e^-$ trying to focus on i) short distance physics and ii) all possible Dalitz plot analyses to disentangle all possible interesting long and short distance effects [31]. This decay has not been observed yet, and the interesting physics is hidden by bremsstrahlung [31, 35]

$$\begin{aligned} \mathscr{B}(K^+ \to \pi^+\pi^0 e^+e^-)_B &\sim (330 \pm 15) \cdot 10^{-8} \\ \mathscr{B}(K^+ \to \pi^+\pi^0 e^+e^-)_M &\sim (6.14 \pm 1.30) \cdot 10^{-8}, \end{aligned} \tag{3.7}$$

and so Dalitz plot analysis is necessary in order to capture the more interesting direct emission contributions. The $K^+ \to \pi^+\pi^0 e^+e^-$-amplitude is written as

$$\mathscr{M}_{LD} = \frac{e}{q^2}\left[\overline{u}(k_-)\gamma^\mu v(k_+)\right] H_\mu(p_1, p_2, q), \tag{3.8}$$

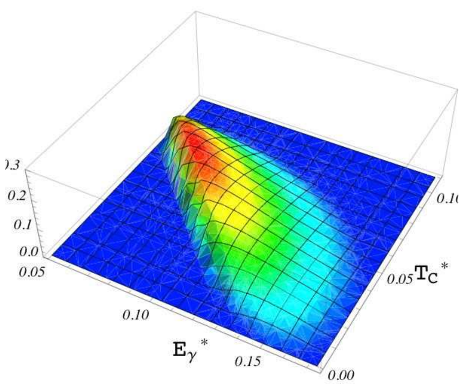

Figure 9: Dalitz plot in the (E^*_γ, T^*_c) plane at $q^2 = (50\text{ MeV})^2$ for the P-violating BM contribution

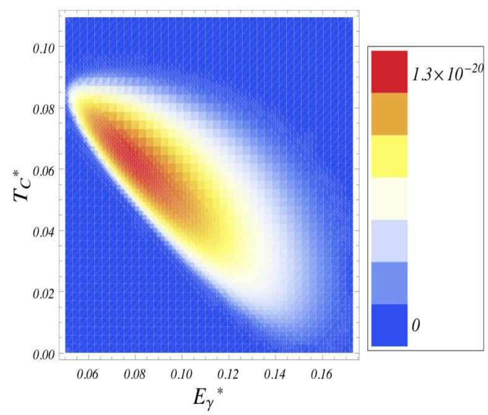

Figure 10: Dalitz plot BM contribution: two-dimensional density projection

We may wonder also what it is the advantage to study this four body decay, $K^+ \to \pi^+\pi^0 e^+e^-$, versus $K^+ \to \pi^+\pi^0\gamma$; infact there are two reasons to investigate this channel, i) first trivially there are more short distance operators and also more long distance observables (for instance interfering electric and magnetic amplitudes) and ii) going to large dilepton invariant mass there is an extra tool compared to $K^+ \to \pi^+\pi^0\gamma$ to separate the bremsstrahlung component [31]. For instance at large dilepton invariant mass the bremsstrahlung can be even 100 time smaller than the magnetic contribution. In our paper we give practically all the distributions in eq. (3.6), here as example we show in Figs. 8 and 9 the Dalitz plot distribution for the novel electric magnetic interference. This decay has been analyzed by NA48/2-NA62.

4. Conclusions

We are looking forward to the upcoming $K_L \to \pi^0\nu\overline{\nu}$ KOTO [36] and $K^+ \to \pi^+\nu\overline{\nu}$ [28] NA62 experiments probing deeply the flavour structure of the SM and we hope ORKA will join this enterprise [37]. We have also shown that there are other decay modes like $K_L \to \pi^0 e^+e^-$, $K^+ \to \pi^+\gamma\gamma$

and $K^+ \to \pi^+\pi^0 e^+e^-$ which are very useful, in particular these last two have been studied recently by NA62. I would like also to mention CPT tests in kaon decays [38] through Bell-Steinberger relations, recently updated in [2]; these leads to best CPT limit and an accurate determination of the CP violating parameter ε.

Acknowledgements

I thank the organizers of KAON 2013, in particular the scientific organization by Monica Tecchio and also partial support by MIUR under project SIMAMI

References

[1] R. Aaij *et al.* [LHCb Collaboration], JHEP **1301**, 090 (2013) [arXiv:1209.4029 [hep-ex]].

[2] J. Beringer *et al.* [Particle Data Group Collaboration], Phys. Rev. D **86**, 010001 (2012).

[3] L. M. Sehgal, Phys. Rev. **183**, 1511 (1969) [Erratum-ibid. D **4**, 1582 (1971)]; M. K. Gaillard and B. W. Lee, Phys. Rev. D **10**, 897 (1974).

[4] P. Herczeg, Phys. Rev. D **27**, 1512 (1983).

[5] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996) [hep-ph/9512380].

[6] G. Ecker and A. Pich, Nucl. Phys. B **366**, 189 (1991);

[7] G. D'Ambrosio and D. Espriu, *Phys. Lett.* **B 175** 237 (1986); J.L. Goity, *Z. Phys.* **C 34,** (1987) 341.

[8] G. Buchalla, G. D'Ambrosio and G. Isidori, Nucl. Phys. B **672**, 387 (2003) [hep-ph/0308008].

[9] G. Isidori and R. Unterdorfer, JHEP **0401**, 009 (2004) [hep-ph/0311084].

[10] L. Littenberg, hep-ex/0212005.

[11] G. D'Ambrosio, G. Isidori and J. Portolés, Phys. Lett. B **423**, 385 (1998).

[12] D. Greynat and E. de Rafael, hep-ph/0303096.

[13] D. Gomez Dumm and A. Pich, *Phys. Rev. Lett.* **80** (1998) 4633 [hep-ph/9801298].

[14] G. Valencia, *Nucl. Phys.* **B 517** (1998) 339 [hep-ph/9711377].

[15] G. Buchalla and A.J. Buras, *Nucl. Phys.*, B **412** 106 (1994); see also J.O. Eeg, K. Kumericki, I. Picek Eur. Phys. J. **C 1** 531 (1998).

[16] G. Ecker, A. Pich and E. de Rafael, Nucl. Phys. **B 291,** 692 (1987).

[17] G. D'Ambrosio, G. Ecker, G. Isidori and J. Portolés, JHEP **08,** 004 (1998).

[18] J. R. Batley *et al.* [NA48/1 Collaboration], Phys. Lett. B **576**, 43 (2003) [arXiv:hep-ex/0309075].

[19] J. R. Batley *et al.* [NA48/1 Collaboration], Phys. Lett. B **599**, 197 (2004) [arXiv:hep-ex/0409011].

[20] A. Alavi-Harati *et al.* [KTeV Collaboration], Phys. Rev. Lett. **93**, 021805 (2004) [hep-ex/0309072].

[21] A. Alavi-Harati *et al.* [KTEV Collaboration], Phys. Rev. Lett. **84**, 5279 (2000) [hep-ex/0001006].

[22] A. Lai *et al.* [NA48 Collaboration], Phys. Lett. B **536**, 229 (2002) [hep-ex/0205010].

[23] E. Abouzaid *et al.* [KTeV Collaboration], Phys. Rev. D **77**, 112004 (2008) [arXiv:0805.0031 [hep-ex]].

[24] G. Ecker, A. Pich and E. de Rafael, Nucl. Phys. B **303**, 665 (1988).

[25] G. D'Ambrosio and J. Portoles, Phys. Lett. B **386**, 403 (1996) [Phys. Lett. B **389**, 770 (1996 ERRAT,B395,390.1997)] [arXiv:hep-ph/9606213].

[26] G. Ecker, J. Kambor and D. Wyler, Nucl. Phys. B **394**, 101 (1993); G. D'Ambrosio and J. Portoles, *Nucl. Phys.* **B 533** (1998) 494 [hep-ph/9711211]; L. Cappiello, O. Cata and G. D'Ambrosio, Phys. Rev. D **85**, 015003 (2012) [arXiv:1106.0467 [hep-ph]].

[27] P. Kitching *et al.* [E787 Collaboration], Phys. Rev. Lett. **79**, 4079 (1997) [arXiv:hep-ex/9708011].

[28] E. Goudzovski, arXiv:1112.5365 [hep-ex]; M. S. Sozzi, arXiv:1102.0893 [hep-ex].

[29] L. Cappiello and G. D'Ambrosio, Phys. Rev. D **75**, 094014 (2007) [arXiv:hep-ph/0702292].

[30] J. R. Batley *et al.* [NA48/2 Collaboration], Eur. Phys. J. C **68**, 75 (2010) [arXiv:1004.0494 [hep-ex]].

[31] L. Cappiello, O. Cata, G. D'Ambrosio and D. -N. Gao, Eur. Phys. J. C **72**, 1872 (2012) [arXiv:1112.5184 [hep-ph]].

[32] N. Cabibbo and A. Maksymowicz, Phys. Rev. **137**, B438 (1965) [Erratum-ibid. **168**, 1926 (1968)]; A. Pais and S. B. Treiman, Phys. Rev. **168**, 1858 (1968); J. Bijnens, G. Colangelo, G. Ecker and J. Gasser, arXiv:hep-ph/9411311.

[33] L. M. Sehgal and M. Wanninger, Phys. Rev. D **46**, 1035 (1992) [Erratum-ibid. D **46**, 5209 (1992)]; P. Heiliger and L. M. Sehgal, Phys. Rev. D **48**, 4146 (1993) [Erratum-ibid. D **60**, 079902 (1999)]; J. K. Elwood, M. B. Wise, M. J. Savage, Phys. Rev. **D52**, 5095 (1995). [hep-ph/9504288]; J. K. Elwood, M. B. Wise, M. J. Savage, J. W. Walden, Phys. Rev. **D53**, 4078-4081 (1996). [hep-ph/9506287].

[34] E. Abouzaid *et al.* [KTeV Collaboration], Phys. Rev. Lett. **96**, 101801 (2006) [hep-ex/0508010]; A. Lai *et al.* [NA48 Collaboration], Eur. Phys. J. C **30**, 33 (2003).

[35] H. Pichl, Eur. Phys. J. C **20**, 371 (2001) [hep-ph/0010284].

[36] `http://koto.kek.jp/` T. K. Komatsubara, Prog. Part. Nucl. Phys. **67**, 995 (2012) [arXiv:1203.6437 [hep-ex]].

[37] http://www.fnal.gov/directorate/OPMO/Projectsns/ORKA/home.htm

[38] F. Ambrosino *et al.* [KLOE Collaboration], JHEP **0612**, 011 (2006) [hep-ex/0610034].