PROCEEDINGS OF SCIENCE

On description of the correlation between multiplicities in windows separated in azimuth and rapidity

Vladimir Vechernin[∗†]
Saint-Petersburg State University
E-mail: vechernin@pobox.spbu.ru

The forward-backward (FB) multiplicity correlations in two windows separated in rapidity and azimuth are analyzed in the framework of the model with independent identical emitters (strings). Along with the short-range contribution, originating from the correlation between particles produced by a single string, the long-range contribution, originating from the fluctuation in the number of strings, is taken into account. The connection of the FB multiplicity correlation coefficient with the two-particle correlation function and the di-hadron correlation analysis is traced. It's also shown that the direct azimuthal flow leads to the forward ridge structure in the resulting two-particle correlation function.

The XXI International Workshop High Energy Physics and Quantum Field Theory,
June 23 – June 30, 2013
Saint Petersburg Area, Russia

[∗]Speaker.
[†]The work was supported by the RFFI grant 12-02-00356-a.

PoS(QFTHEP 2013)055

1. Connection of the FB correlation coefficient with two-particle correlation function

Usually under the forward-backward (FB) correlation one implies the correlation between the multiplicities of charged particles n_F and n_B in two separated rapidity windows $\delta\eta_F$ and $\delta\eta_B$ in high energy pp, pA or AA interactions. In present report we consider a more general case - the FB correlation in windows separated both in rapidity and in azimuth, when two azimuthal sectors $\delta\phi_F$ and $\delta\phi_B$ are selected within these FB windows $\delta\eta_F$ and $\delta\eta_B$.

Traditionally one uses the following definition of the FB correlation coefficient [1]:

$$b_{abs} \equiv \frac{\langle n_F n_B\rangle - \langle n_F\rangle\langle n_B\rangle}{\langle {n_F}^2\rangle - \langle n_F\rangle^2} \qquad \text{or} \qquad b_{rel} \equiv \frac{\langle n_F\rangle}{\langle n_B\rangle} b_{abs} \tag{1.1}$$

The last one is using, when the analysis is performed in so-called relative or scaled variables [2], i.e. for the correlation between the normalized values $n_F/\langle n_F\rangle$ and $n_B/\langle n_B\rangle$.

The two-particle correlation function C_2 is defined through the inclusive ρ_1 and double inclusive ρ_2 distributions [3]. If we consider the distributions, integrated over the absolute value of transverse momenta, we have

$$C_2(\eta_F,\phi_F;\eta_B,\phi_B) \equiv \frac{\rho_2(\eta_F,\phi_F;\eta_B,\phi_B)}{\rho_1(\eta_F,\phi_F)\rho_1(\eta_B,\phi_B)} - 1 \tag{1.2}$$

$$\rho_1(\eta,\phi) = \frac{d^2N}{d\eta\, d\phi}\ , \qquad \rho_2(\eta_F,\phi_F;\eta_B,\phi_B) = \frac{d^4N}{d\eta_F\, d\phi_F\, d\eta_B\, d\phi_B} \tag{1.3}$$

In experiment one measures the ρ_1 taking a small window $\delta\eta\,\delta\phi$ around η, ϕ, then

$$\rho_1(\eta,\phi) = \frac{\langle n\rangle}{\delta\eta\,\delta\phi}\ , \tag{1.4}$$

here $\langle n\rangle$ is the mean multiplicity in the acceptance $\delta\eta\,\delta\phi$. Similarly, by definition (1.3) to measure the ρ_2 one has to take two small windows: $\delta\eta_F\,\delta\phi_F$ around η_F, ϕ_F and $\delta\eta_B\,\delta\phi_B$ around η_B, ϕ_B, then

$$\rho_2(\eta_F,\phi_F;\eta_B,\phi_B) = \frac{\langle n_F n_B\rangle}{\delta\eta_F\,\delta\phi_F\,\delta\eta_B\,\delta\phi_B}\ . \tag{1.5}$$

The formulae (1.4) and (1.5) are the base for the experimental measurement of the one- and two-particle densities of charge particles. By (1.4) and (1.5) the definition (1.2) leads to the following experimental procedure of the determination of the correlation function C_2:

$$C_2(\eta_F,\phi_F;\eta_B,\phi_B) = \frac{\langle n_F n_B\rangle - \langle n_F\rangle\langle n_B\rangle}{\langle n_F\rangle\langle n_B\rangle}\ , \tag{1.6}$$

where n_F and n_B are the event multiplicities in two small windows: $\delta\eta_F\,\delta\phi_F$ and $\delta\eta_B\,\delta\phi_B$.

Comparing (1.1) and (1.6) we see that for small FB windows we have

$$b_{abs} = \frac{\langle n_F\rangle\langle n_B\rangle}{D_{n_F}} C_2(\eta_F,\phi_F;\eta_B,\phi_B)\ , \qquad b_{rel} = \frac{\langle n_F\rangle^2}{D_{n_F}} C_2(\eta_F,\phi_F;\eta_B,\phi_B), \tag{1.7}$$

where $D_{n_F} = \langle {n_F}^2\rangle - \langle n_F\rangle^2$. Note that for small forward window: $D_{n_F} \to \langle n_F\rangle$ [4]. So we see that the traditional definition (1.1) of the FB correlation coefficient in the case of two small observation windows, separated in azimuth and rapidity, coincides with the standard definition of two-particle correlation function C_2 upto some common factor $\langle n_B \rangle$ or $\langle n_F \rangle$, which depends on the width of windows.

In practice, in di-hadron correlation analysis, the following alternative definition of the two-particle correlation function C is in use [5, 6]:

$$C = \frac{S}{B} - 1 \, , \qquad S = \frac{d^2N}{d\Delta\eta \, d\Delta\phi} \, , \tag{1.8}$$

where $\Delta\eta = \eta_1 - \eta_2$ and $\Delta\phi = \phi_1 - \phi_2$ are the distances between two particles in rapidity and in azimuth, and one takes into account all possible pair combinations of particles produced in given event in some one large rapidity interval $\Delta\eta \in (Y_1, Y_2)$. The B is the same, but in the case of uncorrelated particle production, obtained by the event mixing procedure.

At such definition, in contrast with (1.2), one implies from the very beginning that the translation invariance in rapidity takes place and the result depends only on $\Delta\eta = \eta_1 - \eta_2$ for any $\eta_1, \eta_2 \in (Y_1, Y_2)$. (All the pairs with the same value of difference $\eta_1 - \eta_2$ contribute to the same bin of the multiplicity distribution, irrespective of the value of $(\eta_1 + \eta_2)/2$.) This assumption is reasonable only in the central rapidity region at high energies. It means that we suppose that in the interval (Y_1, Y_2):

$$\rho_1(\eta) = \rho_0 \, , \qquad \rho_2(\eta_1, \eta_2; \Delta\phi) = \rho_2(\eta_1 - \eta_2, \Delta\phi) \, . \tag{1.9}$$

In this case we have for the enumerator of (1.8):

$$S(\Delta\eta, \Delta\phi) = \int_{Y_1}^{Y_2} dy_1 \, dy_2 \, \rho_2(y_1 - y_2, \Delta\phi) \, \delta(y_1 - y_2 - \Delta\eta) \, , \tag{1.10}$$

or in the case of commonly used symmetric interval $(-Y/2, Y/2)$:

$$S(\Delta\eta, \Delta\phi) = \rho_2(\Delta\eta, \Delta\phi) \, t_Y(\Delta\eta) \tag{1.11}$$

where the $t_Y(\Delta\eta)$ is the "triangular" weight function:

$$t_Y(\Delta\eta) = [\theta(-\Delta\eta)(Y + \Delta\eta) + \theta(\Delta\eta)(Y - \Delta\eta)] \, \theta(Y - |\Delta\eta|) \, . \tag{1.12}$$

In the denominator of (1.8) for mixed events we should replace the $\rho_2(\eta_1, \eta_2, \Delta\phi)$ by the product $\rho_1(\eta_1)\rho_1(\eta_2)$, which due to the translation invariance in rapidity reduces simply to ρ_0^2. Then

$$B(\Delta\eta, \Delta\phi) = \rho_0^2 \, t_Y(\Delta\eta) \, . \tag{1.13}$$

Substituting into (1.8) we get

$$C(\Delta\eta, \Delta\phi) = \frac{\rho_2(\Delta\eta, \Delta\phi)}{\rho_0^2} - 1 = C_2(\Delta\eta, \Delta\phi) \, , \tag{1.14}$$

We see if the translation invariance in rapidity takes place within the interval (Y_1, Y_2), then the definition (1.8) is equivalent to the standard one (1.2) (see meanwhile the remark in the end of the next section).

The drawback of this approach is that it supposes from the very beginning the translation invariance and hence can't be applied for an investigation of the multiplicity correlation at large rapidity distances, where the translation invariance in rapidity (1.9) is not valid. At that by (1.6) and (1.7) we see that the approaches based on the analysis of the standard (1.1) FB correlation coefficient with two remote windows of small acceptance in rapidity and azimuth enable in any case to measure the correlation function $C_2(\eta_1,\eta_2;\phi_1-\phi_2)$ without using of the event mixing procedure.

2. Model with strings as independent identical emitters

We now calculate the FB correlations in windows separated in rapidity and azimuth using the simple two stage model [7, 8, 9], inspired by a string picture of hadronic interactions. In this model one suggests that at the initial stage of interaction some number N of strings are formed, which fluctuates event-by-event with some variance $D_N = \langle N^2\rangle - \langle N\rangle^2$ or scaled variance

$$\omega_N = D_N/\langle N\rangle \, . \tag{2.1}$$

Note that the fluctuation in the number of strings in pp and especially in AA collisions is not poissonian [10] and hence $\omega_N \neq 1$. Its value depends on the collision energy. At next stage one considers these strings as identical independent emitters of observed charge particles.

In the present paper, along with the so-called long-range (LR) part of the correlation [11], originating from the fluctuation in the number of strings, we take into account also the short-range (SR) contribution, originating from the correlation between particles produced by a single string.

To characterize the last property of the string we introduce, similarly to the consideration in the section 1, the $\lambda_1(\eta,\phi)$ and $\lambda_2(\eta_1,\phi_1;\eta_2,\phi_2)$ - the one- and two-particle densities of charge particles produced by one string. In this section we'll suppose that the particle emission from one string is isotropic in ϕ, then at fixed number of strings (N) in the framework of the model we have:

$$\rho_1^N(\eta) = N\lambda_1(\eta) \, , \tag{2.2}$$

$$\rho_2^N(\eta_F,\eta_B;\Delta\phi) = N\lambda_2(\eta_F,\eta_B;\Delta\phi) + N(N-1)\lambda_1(\eta_F)\lambda_1(\eta_B) \, . \tag{2.3}$$

After averaging over N the one- and two-particle densities of charge particles are given by

$$\rho_1(\eta) = \langle N\rangle\lambda_1(\eta) \, , \tag{2.4}$$

$$\rho_2(\eta_F,\eta_B;\Delta\phi) = \langle N\rangle[\lambda_2(\eta_F,\eta_B;\Delta\phi) - \lambda_1(\eta_F)\lambda_1(\eta_B)] + \langle N^2\rangle\lambda_1(\eta_F)\lambda_1(\eta_B) \, . \tag{2.5}$$

Introducing similarly (1.2) the two-particle correlation function for charged particles produced from a decay of a single string:

$$\Lambda(\eta_1,\eta_2;\Delta\phi) = \frac{\lambda_2(\eta_1,\eta_2;\Delta\phi)}{\lambda_1(\eta_1)\lambda_1(\eta_2)} - 1 \, , \tag{2.6}$$

we find for the two-particle correlation function C_2 the following expression:

$$C_2(\eta_F,\eta_B;\Delta\phi) = \frac{\Lambda(\eta_F,\eta_B;\Delta\phi) + \omega_N}{\langle N\rangle} \, , \tag{2.7}$$

PoS(QFTHEP 2013)055

In the central rapidity region, where one has the translation invariance in rapidity and each string contributes to the particle production in the whole rapidity region, we have

$$\lambda_1(\eta) = \mu_0 = const\ , \qquad \Lambda(\eta_F,\eta_B;\Delta\phi) = \Lambda(\eta_F-\eta_B,\Delta\phi)\ , \tag{2.8}$$

then

$$\rho_1(\eta) = \langle N\rangle\mu_0 = const\ , \tag{2.9}$$

$$C_2(\Delta\eta,\Delta\phi) = \frac{\Lambda(\Delta\eta,\Delta\phi)+\omega_N}{\langle N\rangle}\ . \tag{2.10}$$

Important that by (2.10) we see that the value of the common constant (pedestal) in $C_2(\Delta\eta,\Delta\phi)$ is physically important. The height of the pedestal ($\omega_N/\langle N\rangle$=$D_N/\langle N\rangle^2$) contains the important physical information on the magnitude of the fluctuation of the number of emitters N at different energies and centrality fixation [4, 11].

In a conclusion of the section we note that if one uses the so-called di-hadron correlation approach, described above, for the experimental determination of the two-particle correlation function $C(\Delta\eta,\Delta\phi)$ (1.8) the result can depend on the details of track and/or event mixing used in that approach for the determination of B through the imitation of the uncorrelated particle production and also on arbitrary using of unjustified normalization procedure in S and B.

One can illustrate this in the framework of the model with strings as independent identical emitters. By (1.11) and (2.5) we have for the enumerator and the denominator of (1.8):

$$S(\Delta\eta,\Delta\phi) = \langle\rho_2^N(\Delta\eta;\Delta\phi)\rangle\, t_Y(\Delta\eta) = [\langle N\rangle\Lambda(\Delta\eta,\Delta\phi)+\langle N^2\rangle]\mu_0^2\, t_Y(\Delta\eta)\ , \tag{2.11}$$

$$B(\Delta\eta,\Delta\phi) = \int_{-Y/2}^{Y/2} dy_1\, dy_2\, \langle\rho_1^N(y_1)\rangle\langle\rho_1^N(y_2)\rangle\, \delta(y_1-y_2-\Delta\eta) = \langle N\rangle^2\mu_0^2\, t_Y(\Delta\eta)\ . \tag{2.12}$$

Then by $C = S/B-1$ we get again that $C(\Delta\eta,\Delta\phi) = C_2(\Delta\eta,\Delta\phi)$, which is given by (2.10).

But if instead of (2.12) one will use another event mixing procedure, for example, the mixing only between events with the same multiplicity (i.e. the same N), then instead of (2.12) we'll have

$$B(\Delta\eta,\Delta\phi) = \int_{-Y/2}^{Y/2} dy_1\, dy_2\, \langle\rho_1^N(y_1)\rho_1^N(y_2)\rangle\, \delta(y_1-y_2-\Delta\eta) = \langle N^2\rangle\mu_0^2\, t_Y(\Delta\eta)\ , \tag{2.13}$$

which leads instead of (2.10) to

$$C(\Delta\eta,\Delta\phi) = \frac{\langle N\rangle}{\langle N^2\rangle}\Lambda(\Delta\eta,\Delta\phi)\ . \tag{2.14}$$

The last result does not coincide with the standard two-particle correlation function C_2, defined by (1.2). Compare (2.14) with (2.10) we see that in this case the resulting $C(\Delta\eta,\Delta\phi)$ does not have an additional constant contribution reflecting the event-by-event fluctuation in the number of emitters. It depends only on the pair correlation function of a single string $\Lambda(\Delta\eta,\Delta\phi)$ and, therefore, is equal to zero in the absence of the pair correlation from one string.

The same effect can take place if one use unjustified artificial normalization procedure in S and B.

3. Connection between the ridge and the azimuthal flows

In this section we consider a simple model, in which we will not take into account the two-particle correlation between particles originating from the decay of a same string ($\Lambda(\Delta\eta,\Delta\phi)=0$), but try to understand the influence of the event-by-event fluctuation of azimuthal distribution on the resulting two-particle correlation function. The physical reason which leads to the event anisotropy of the azimuthal distribution, for example in the framework of the string fusion approach [12], is the final state interaction (FSI) of produced particles with the fused string medium.

In papers [13, 14] in the framework of this approach the azimuthal flows v_n for ultrarelativistic heavy ion collisions were found by Monte Carlo (MC) simulations. In that papers the flows were calculated by Fourier decomposition of the azimuthal inclusive distribution of charged particles $\rho_1^i(\phi)$, produced by the given string configuration i, obtaining by MC simulations (it was supposed that in the central region this configurations are homogeneous in rapidity):

$$\rho_1^i(\phi)=\bar\rho^i[1+2\sum_{n=1}^{\infty}(a_n^i\cos n\phi+b_n^i\sin n\phi)]=\bar\rho^i[1+2\sum_{n=1}^{\infty}v_n^i\cos n(\phi-\psi_n^i)]\ , \tag{3.1}$$

where

$$a_n^i=\frac{1}{2\pi\bar\rho^i}\int\rho_1^i(\phi)\cos n\phi\,d\phi\ ,\qquad b_n^i=\frac{1}{2\pi\bar\rho^i}\int\rho_1^i(\phi)\sin n\phi\,d\phi\ , \tag{3.2}$$

$$\bar\rho^i=\frac{1}{2\pi}\int\rho_1^i(\phi)d\phi\ ,\qquad v_n^i=\sqrt{a_n^{i2}+b_n^{i2}}\ ,\qquad \mathrm{tg}\,n\psi_n^i=b_n^i/a_n^i\ , \tag{3.3}$$

Here $\bar\rho^i$ is the mean multiplicity for given string configuration. The flows were founded by averaging over string configurations $i=1,...,K$:

$$v_n=\frac{1}{K}\sum_{i=1}^{K}v_n^i=\frac{1}{K}\sum_{i=1}^{K}\sqrt{a_n^{i2}+b_n^{i2}}\ . \tag{3.4}$$

Since there are no correlations between particles produced by different strings and we don't take into account the correlations between particles originating from the decay of a same string, then for a given string configuration i, we have

$$\rho_2^i(\phi_1,\phi_2)=\rho_1^i(\phi_1)\rho_1^i(\phi_2)\ . \tag{3.5}$$

We'll show now that nevertheless one has in this model the so-called ridge structure in the two-particle correlation function C_2, which can be expressed through the same Fourier harmonics a_n^i and b_n^i, as the azimuthal flows v_n.

As discussed, in the central rapidity region di-hadron correlation function is given by the expression (1.14). In the framework of this model we have for the di-hadron correlation function:

$$C(\Delta\phi)=C_2(\phi_1-\phi_2)=\frac{\rho_2(\phi_1-\phi_2)}{\rho_1^2}-1\ , \tag{3.6}$$

where ρ_1 is the mean multiplicity density:

$$\rho_1=\frac{1}{K}\sum_{i=1}^{K}\frac{1}{2\pi}\int_0^{2\pi}\rho_1^i(\phi+\widetilde\phi^i)\,d\widetilde\phi^i=\frac{1}{K}\sum_{i=1}^{K}\bar\rho^i\equiv\langle\bar\rho^i\rangle\ , \tag{3.7}$$

and

$$\rho_2(\phi_1-\phi_2)=\frac{1}{K}\sum_{i=1}^{K}\frac{1}{2\pi}\int_0^{2\pi}\rho_1^i(\phi_1+\widetilde{\phi}^i)\rho_1^i(\phi_2+\widetilde{\phi}^i)\,d\widetilde{\phi}^i\ . \tag{3.8}$$

Here $\rho_1^i(\phi)$ is given by (3.1) and $\bar{\rho}^i$ and v_n^i are given by (3.3). The $\widetilde{\phi}^i$ is an additional common random phase, which arises due to the event-by-event fluctuation of the reaction plane. Note that we also add an additional averaging over this phase for each string configuration, which corresponds to the azimuthal rotation of a given string configuration, and $\langle...\rangle$ means averaging over string configurations. Substituting now (3.1) in (3.8) we get

$$\rho_2(\Delta\phi)=\langle(\bar{\rho}^i)^2\rangle+2\sum_{n=1}^{\infty}\langle(\bar{\rho}^i v_n^i)^2\rangle\cos(n\Delta\phi)\ . \tag{3.9}$$

Then

$$C_2(\Delta\phi)=\frac{2}{\langle\bar{\rho}^i\rangle^2}\sum_{n=1}^{\infty}\langle(\bar{\rho}^i v_n^i)^2\rangle\cos(n\Delta\phi)+C_0=2\sum_{n=1}^{\infty}\langle(\frac{\bar{\rho}^i}{\langle\bar{\rho}^i\rangle}v_n^i)^2\rangle\cos(n\Delta\phi)+C_0\ , \tag{3.10}$$

where

$$C_0=\frac{\langle(\bar{\rho}^i)^2\rangle-\langle\bar{\rho}^i\rangle^2}{\langle\bar{\rho}^i\rangle^2}\ . \tag{3.11}$$

In section 2 we have emphasized the importance of the observation of the common “pedestal” value in C_2, which in that model was equal to the variance of the number of emitters divided by the square of their mean number (2.10). In the present model by (3.11) we see again that the value of this constant C_0 is equal to the variance of the mean (for given string configuration i) multiplicity $\bar{\rho}^i$ (3.3) from one string configuration to another divided by the square of the averaged multiplicity.

We see also that the ridge like structure in (3.10) is expressed through the same Fourier harmonics a_n^i and b_n^i (3.2), as the azimuthal flows v_n (3.4), and the mean multiplicity $\bar{\rho}^i$ for given string configuration i (3.3) .

Further rough evaluation of (3.10) is possible only if we will consider that the mean multiplicity $\bar{\rho}^i$ weakly depends on string configuration i: $\bar{\rho}^i\approx\langle\bar{\rho}^i\rangle=const$, which is poorly justified assumption. Under this assumption $C_0=0$ and

$$C_2(\Delta\phi)=2\sum_{n=1}^{\infty}\langle(v_n^i)^2\rangle\cos(n\Delta\phi)=2\sum_{n=1}^{\infty}(v_n^{ms})^2\cos(n\Delta\phi)\ . \tag{3.12}$$

Note that even in this very rude approximation the $C_2(\Delta\phi)$ is expressed not directly through the flows (3.4), but through the "mean squared flows" v_n^{ms}:

$$v_n^{ms}\equiv\sqrt{\langle(v_n^i)^2\rangle}=\sqrt{\frac{1}{K}\sum_{i=1}^{K}(v_n^i)^2}=\sqrt{\frac{1}{K}\sum_{i=1}^{K}(a_n^{i2}+b_n^{i2})} \tag{3.13}$$

By (3.10) and (3.12) we see that in any model the nonzero direct flow v_1 leads to the forward ridge structure in the resulting two-particle correlation function.

References

[1] ALICE collaboration, *J. Phys. G* **32** (2006) 1295 [Section 6.5.15 - Long-range correlations].

[2] G.A. Feofilov, et al. (for ALICE collaboration), PoS(Baldin ISHEPP XXI)075.

[3] C. Pruneau, S. Gavin, S. Voloshin, *Phys. Rev. C* **66** (2002) 044904.

[4] V.V. Vechernin, arXiv:1210.7588; 1305.0857 [hep-ph].

[5] B.I. Abelev et al. (STAR Collaboration), *Phys. Rev. C* **80** (2009) 064912.

[6] CMS Collaboration, *JHEP* **09** (2010) 091.

[7] M.A. Braun, C. Pajares, V.V. Vechernin, *Phys. Lett. B* **493** (2000) 54.

[8] M.A. Braun, R.S. Kolevatov, C. Pajares, V.V. Vechernin, *Eur. Phys. J. C* **32** (2004) 535.

[9] V.V. Vechernin, R.S. Kolevatov, *Phys. Atom. Nucl.* **70** (2007) 1797.

[10] V.V. Vechernin, H.S. Nguyen, *Phys. Rev. C* **84** (2011) 054909.

[11] A. Capella and A. Krzywicki, *Phys. Rev. D* **18** (1978) 4120.

[12] M.A. Braun, C. Pajares, *Phys. Lett. B* **287** (1992) 154; *Nucl. Phys. B* **390** (1993) 542.

[13] M.A.Braun and C.Pajares, *Eur. Phys. J. C* **71** (2011) 1558.

[14] M.A. Braun, C. Pajares, V.V. Vechernin, *Nucl. Phys. A* **906** (2013) 14.