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Gravity effects on the spectrum of scalar states on a thick brane

Oleg O. Novikov*
V.A. Fock Department of Theoretical Physics, Saint-Petersburg State University, ul. Ulianovskaya, 198504 St. Petersburg, Russia
oonovikov@gmail.com

Alexander A. Andrianov[a,b] **and Vladimir A. Andrianov**[a]
[a]*V.A. Fock Department of Theoretical Physics, Saint-Petersburg State University, ul. Ulianovskaya, 198504 St. Petersburg, Russia*
[b]*Institut de Ciències del Cosmos, Universitat de Barcelona, Martí Franquès, 1, E08028 Barcelona, Spain*
E-mail: andrianov@icc.ub.edu,v.andriano@rambler.ru

The model of a domain wall ("thick brane") in noncompact five-dimensional space-time is considered with geometries of AdS type generated by self-interacting scalar matter composed of two fields with minimal coupling to gravity. The mixing of scalar and gravitational degrees of freedom leads to contributions to the invariant scalar fluctuations potential that remain nontrivial in the limit of turned off gravity. This leads to the disappearance of the Goldstone mode associated with translational symmetry breaking and to the different mass of the Higgs-like scalar state.

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*Speaker.

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1. Introduction

The models of the BSM physics based on the hypothesis that our universe is a four-dimensional space-time hypersurface (3-brane) embedded in a fundamental multi-dimensional space have attracted recently much interest, see, for example, [1] and references therein. The gravity happens to play an important role in a (de) localization of matter fields on the brane [2] - [8], [9]. The purpose of this work is to study how the introduction of gravity influences the spectrum of scalar fluctuations.

In our talk we consider a model of the domain wall formation with finite thickness ("thick" branes)in five-dimensional noncompact space-time by the scalar matter minimally coupled to gravity [10], [11]. The scalar matter is taken to consist of two fields with $O(2)$ symmetric self interaction and with manifest $O(2)$ symmetry breaking by terms quadratic in fields. The limit of turned off gravity happens to be smooth for background fields and the solutions can be obtained using the perturbation theory [12]. On the other hand it was argued in [5], [8], [9] that for one scalar field, gravity induces singular repulsion towards the remote AdS horizon so that localized modes on a brane may be absent and a massless Goldstone-type mode of translational symmetry breaking disappears. The question of phenomenological importance that arises is whether this repulsion influences the mass of the light scalar produced by fluctuations of the second scalar field which can be identified with Higgs-like boson observed at LHC. The mass can be obtained by the perturbation theory. While the leading order is the same as in the model without gravity the next to leading order happens to be nontrivial.

2. Formulation of the model

Consider the five-dimensional space supplied with a pseudo Riemann metric g_{AB}, which is reduced to η_{AB} in flat space and for the rectangular coordinate system,

$$X^A = (x^\mu, y), \quad x^\mu = (x^0, x^1, x^2, x^3), \tag{2.1}$$

It is assumed that the size of extra dimension y is large or infinite.

We define the dynamics of two scalar fields $\Phi(X)$ and $H(X)$ with a minimal interaction to gravity by the following action functional,

$$S[g,\Phi,H] = \int d^5X \sqrt{|g|} \left(-\frac{1}{2} M_*^3 R + \mathscr{L}_{mat}(g,\Phi,H) \right), \tag{2.2}$$

$$\mathscr{L}_{mat} = Z \left(\frac{1}{2} (\partial_A \Phi \partial^A \Phi + \partial_A H \partial^A H) - V(\Phi, H) \right) \tag{2.3}$$

where R stands for a scalar curvature, $|g|$ is the determinant of the metric tensor. M_* denotes a five-dimensional gravitational Planck scale. In the scalar matter action the normalization coefficient Z has dimension of mass and is introduced to simplify the equations of motion.

In order to build a thick $3+1$-dimensional brane we study such classical vacuum configurations which do not violate spontaneously 4-dimensional Poincare invariance. A background solution for the metric is searched for in the gaussian frame,

$$ds^2 = e^{-2\rho(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2. \tag{2.4}$$

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This kind of background metric suits well for interpretation of scalar fluctuation spectrum and corresponding resonance effects (i.e. scattering states) [10].

$$\rho'' = \frac{Z}{3M_*^3}(\Phi'^2 + H'^2), \quad \frac{2Z}{3M_*^3}V(\Phi, H) = \rho'' - 4(\rho')^2, \tag{2.5}$$

$$\Phi'' - 4\rho'\Phi' = \frac{\partial V}{\partial \Phi}, \quad H'' - 4\rho' H' = \frac{\partial V}{\partial H}. \tag{2.6}$$

One can prove [10], that only three of these equations are independent. In the limit of zero gravity the equations on classical backgrounds smoothly reproduce the corresponding equations in the model without gravity.

2.1 Minimal realization in ϕ^4 theory

Now let us restrict ourselves with studying the formation of a brane in the theory with a minimal potential bounded from below and admitting kink solutions which connect two potential minima. The effective action of scalar matter looks as follows,

$$\mathscr{L}_{mat} = \frac{3\kappa}{2M^2}\Big(\partial_A\Phi\partial^A\Phi + \partial_A H\partial^A H + 2M^2\Phi^2 + 2\Delta_H H^2 - (\Phi^2 + H^2)^2 - V_0\Big), \tag{2.7}$$

where the normalization of the scalar field lagrangian is chosen as $Z = 3\kappa M_*^3/M^2$ for simplification of Eqs. of motion and of the gravitational perturbation expansion. To associate it to the weak gravity limit we specify that $\kappa \sim M^3/M_*^3$ is a small parameter $\kappa \ll 1$ which characterizes the interaction of gravity and matter fields. Let us fix $M^2 > \Delta_H$ then the absolute minima correspond to $\Phi_{min} = \pm M,\ H_{min} = 0$ and a constant shift of the potential energy must be set $V_0 = M^4$ in order to determine properly the 5-dim cosmological constant Λ_c.

Depending on the relation between quadratic couplings M^2 and Δ_H there are two types of solutions of eqs. (2.6) inhomogeneous in y [11]. In the zero gravity limit the first solution dominates when $\Delta_H \le M^2/2$,

$$\Phi = \pm M\tanh(My) + O(\kappa M), \quad H(y) = 0. \tag{2.8}$$

To the leading order in κ it generates the following conformal factor ,

$$\rho_1(y) = \frac{2\kappa}{3}\left\{\ln\cosh(My) + \frac{1}{4}\tanh^2(My)\right\} + O\left(\kappa^2\right), \tag{2.9}$$

which is chosen to be an even function of y in order to preserve the so-called τ symmetry. The spontaneous breaking of this symmetry is associated with fermion mass generation [11, 12]. The very symmetry involves the intrinsic parity reflection of both fields and the reflection of fifth coordinate,

$$\Phi(y) \to -\Phi(-y), \quad H(y) \to -H(-y).$$

Evidently the parity reflection leaves the bosonic action (2.7) invariant and holds as a symmetry for the kink (2.8). In the presence of gravity induced by a background matter the τ symmetry survives for even conformal factors.

The second kink profile arises only when $M^2/2 \le \Delta_H \le M^2$ (in the zero gravity limit), i.e. $2\Delta_H = M^2 + \mu^2,\ \mu^2 < M^2$,

$$\Phi_0(y) = \pm M\tanh(\beta My), \quad H_0(y) = \pm\frac{\mu}{\cosh(\beta My)}, \quad \beta = \sqrt{1 - \frac{\mu^2}{M^2}}, \tag{2.10}$$

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and it breaks the τ symmetry. Therefrom one can find the conformal factor to the leading order in κ in the following form,

$$\rho_1(y) = \frac{\kappa}{3}\left\{\left(3-\beta^2\right)\ln\cosh(\beta M y) + \frac{1}{2}\beta^2\tanh^2(\beta M y)\right\} + O\left(\kappa^2\right), \tag{2.11}$$

which is as well symmetric against $y \to -y$.

These two solutions correspond to different phases with critical point at $\Delta_H = M^2/2 + O\Big(\kappa\Big)$. We are interested in the phase with broken τ-symmetry because the v.e.v. of the second field H can be used for the fermion mass generation. Let us choose further on the positive signs of $\Phi(y), H(y)$ at $y \to +\infty$.

The next approximation to the solutions in the second phase can be obtained with the help of the perturbation theory in the parameters κ expressing the strength of gravity and μ/M parameterizing the deviation from the critical point. To reduce the complexity of the analytic calculations it is useful to introduce new dimensionless coordinate for the extra dimension $\tau = M\beta y$ and use the following perturbation expansions,

$$\Phi(\tau) = M\textstyle\sum_{m,n=0}^{\infty}\kappa^m\left(\frac{\mu}{M}\right)^{2n}\Phi_{m,n}(\tau), \quad \Phi_{n,0}\equiv\Phi_n, \tag{2.12}$$

$$H(\tau) = M\textstyle\sum_{m,n=0}^{\infty}\kappa^m\left(\frac{\mu}{M}\right)^{2n+1}H_{m,n}(\tau), \quad H_{n,0}\equiv H_n, \tag{2.13}$$

$$\rho(\tau) = \kappa\textstyle\sum_{n,m=0}^{\infty}\kappa^n\left(\frac{\mu}{M}\right)^{2m}\rho_{n+1,m}(\tau), \quad \rho_{n,0}\equiv\rho_n, \tag{2.14}$$

$$\Delta_H = \Delta_{H,c}(\kappa) + \tfrac{1}{2}\mu^2, \quad \Delta_{H,c}(\kappa) = \tfrac{1}{2}M^2\textstyle\sum_{n=0}^{\infty}\kappa^m\Delta_H^n, \tag{2.15}$$

$$\tfrac{1}{\beta^2} = \textstyle\sum_{m,n=0}^{\infty}\kappa^m\left(\frac{\mu}{M}\right)^{2n}\left(\frac{1}{\beta^2}\right)_{m,n}; \tag{2.16}$$

The use of the gaussian coordinates happen to be important for the well-behaved perturbation expansions. The computation that we are not presenting here because of its complexity gives the following leading order corrections,

$$\Delta_{H,c} = \frac{1}{2} - \frac{22}{27}\kappa + O\Big(\kappa^2\Big), \quad \frac{1}{\beta^2}\Big|_{\mu=0} = 1 + \frac{4}{3}\kappa + O\Big(\kappa^2\Big) \tag{2.17}$$

3. Fluctuation equations

Let us consider small localized deviations of the fields from the background values and find the action squared in these fluctuations. The fluctuations of the metric $h_{AB}(X)$ and of the scalar fields $\phi(X)$ and $\chi(X)$ against the background solutions of EoM are introduced in the following way,

$$g_{AB}(X)\,dx^A dx^B = e^{-2\rho(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2 + e^{-2\rho(y)}h_{AB}(X)\,dx^A dx^B; \tag{3.1}$$

$$\Phi(X) = \Phi(y) + \phi(X); H(X) = H(y) + \chi(X). \tag{3.2}$$

Since 4dim Poincare symmetry is not broken, we select the corresponding 4dim part of the metric $h_{\mu\nu}$ and employ the notation for gravi-vectors $h_{5\mu} \equiv v_\mu$ and gravi-scalars $e^{-2\rho}h_{55} \equiv S$. The major simplification can be achieved by separation of different spin components of the field $h_{\mu\nu}$

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and v_μ. It can be accomplished by description of ten components of 4-dim metric in terms of the traceless-transverse tensor, vector and scalar components [5, 13],

$$h_{\mu\nu} = b_{\mu\nu} + F_{\mu,\nu} + F_{\nu,\mu} + E_{,\mu\nu} + \eta_{\mu\nu}\psi, \quad v_\mu = v^\perp_\mu + \partial_\mu \eta, \tag{3.3}$$

$$b^{;\mu}_{\mu\nu} = b = 0 = F^{;\mu}_\mu = v^{\perp,\mu}_\mu. \tag{3.4}$$

After this separation in the action is performed the scalar sector that we are interested in decouples from the fields with higher spins up to quadratic orders in fluctuations.

Still there are redundant degrees of freedom because the action (2.2) is invariant under diffeomorphisms. Infinitesimal diffeomorphisms correspond to the Lie derivative along an arbitrary vector field $\tilde{\zeta}^A(X)$, defining the coordinate transformation $X \to X = X + \tilde{\zeta}(X)$. The further analysis of the scalar spectrum is conveniently performed in the following gauge invariant variables:

$$\check{\eta} = E' - 2\eta + \frac{e^{2\rho}}{\rho'}\psi, \quad \check{\phi} = \phi + \frac{\Phi'}{2\rho'}\psi, \quad \check{\chi} = \chi + \frac{H'}{2\rho'}\psi, \quad \check{S} = S - \frac{1}{\rho'}\psi' + \frac{\rho''}{(\rho')^2}\psi. \tag{3.5}$$

The scalar part of the lagrangian quadratic in fluctuations takes the form:

$$\begin{aligned}\sqrt{|g|}\mathscr{L}_{(2),scal} = \frac{1}{2} Z e^{-2\rho} \Bigg\{ & \check{\phi}_{,\mu}\check{\phi}^{,\mu} + \check{\chi}_{,\mu}\check{\chi}^{,\mu} - e^{-2\rho}\Bigg[(\check{\phi}')^2 + (\check{\chi}')^2 \\ & + \begin{pmatrix}\check{\phi}\\ \check{\chi}\end{pmatrix}^T \partial^2 V \begin{pmatrix}\check{\phi}\\ \check{\chi}\end{pmatrix} + \frac{1}{2}V(\Phi,H)\check{S}^2 + \check{S}\Big(\Phi'\check{\phi}' + H'\check{\chi}' - \frac{\partial V}{\partial \Phi}\check{\phi} - \frac{\partial V}{\partial H}\check{\chi}\Big)\Bigg]\Bigg\} \\ & + \frac{3}{4}M_*^3 e^{-4\rho}\Box\check{\eta}\Big(-\rho'\check{S} + \frac{2Z}{3M_*^3}(\Phi'\check{\phi} + H'\check{\chi})\Big),\end{aligned} \tag{3.6}$$

where $\partial^2 V$ is a matrix of second derivatives of the background solutions.

From the last line it follows that the scalar field $\check{\eta}$ is a gauge invariant Lagrange multiplier and therefore it generates the gauge invariant constraint,

$$\rho'\check{S} = \frac{2Z}{3M_*^3}(\Phi'\check{\phi} + H'\check{\chi}). \tag{3.7}$$

Thus after resolving this constraint only two independent scalar fields remain. To normalize kinetic terms the fields should be redefined $\check{\psi} = \Omega\hat{\psi}$, $\check{\chi} = \Omega\hat{\chi}$, where $\Omega = Z^{-1/2}e^\rho$. Then the scalar action is reduced to the following form,

$$\sqrt{|g|}\mathscr{L}_{(2),scal} = \frac{1}{2}\Big(\partial_\mu\hat{\phi}\partial^\mu\hat{\phi} + \partial_\mu\hat{\chi}\partial^\mu\hat{\chi} - e^{-2\rho}\begin{pmatrix}\hat{\phi}\\ \hat{\chi}\end{pmatrix}^T\Big(-\partial_y^2 + 2\rho'\partial_y + \hat{\mathscr{M}}\Big)\begin{pmatrix}\hat{\phi}\\ \hat{\chi}\end{pmatrix}\Big), \tag{3.8}$$

$$\hat{\mathscr{M}} = \partial^2 V + \hat{\mathscr{M}}_{NP} - \rho'' + 3(\rho')^2, \quad \hat{\mathscr{M}}_{NP} = \frac{2Z}{3M_*^3}(-\partial_y + 4\rho')\left[\frac{1}{\rho'}\begin{pmatrix}(\Phi')^2 & \Phi'H'\\ \Phi'H' & (H')^2\end{pmatrix}\right], \tag{3.9}$$

$\hat{\mathscr{M}}_{NP}$ is a correction to the mass operator that generally speaking can change the spectrum of the scalar fluctuations non-perturbatively.

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Let us perform the mass spectrum expansion,

$$\begin{pmatrix} \hat{\phi}(X) \\ \hat{\chi}(X) \end{pmatrix} = e^{\rho} \sum_m \Psi^{(m)}(x) \begin{pmatrix} \phi^{(m)}(y) \\ \chi^{(m)}(y) \end{pmatrix}, \quad \partial_\mu \partial^\mu \Psi^{(m)} = -m^2 \Psi^{(m)}, \tag{3.10}$$

where the factor $\exp(\rho)$ is introduced to eliminate first derivatives in the equations. We obtain the following equations,

$$\left(-\partial_y^2 + \hat{\mathscr{M}} - \rho'' + (\rho')^2 \right) \begin{pmatrix} \phi^{(m)} \\ \chi^{(m)} \end{pmatrix} = e^{2\rho} m^2 \begin{pmatrix} \phi^{(m)} \\ \chi^{(m)} \end{pmatrix}, \tag{3.11}$$

These coupled channel equations of second order in derivative contain the spectral parameter m^2 as a coupling constant of a part of potential (a non-derivative piece). For $m^2 > 0$ the mass term in the potential makes it unbounded below. Thus any eigenfunction of the spectral problem (3.11) is at best a resonance state though it could be quasi-localized in a finite volume around a local minimum of the potential. In [10] the probability for quantum tunneling of quasi-localized light resonances with masses $m \ll M$ was estimated as $\sim \exp\{-\frac{3}{\kappa} \ln \frac{2M}{m}\}$ which for phenomenologically acceptable values of $\kappa \sim 10^{-15}$ and $M/m \gtrsim 30$ means an enormous suppression.

4. Scalar fluctuation spectrum in the ϕ^4 model

In the phase with unbroken τ-symmetry when $\langle H \rangle = 0$ eqs. of motion (2.6) entail $\partial^2 V / \partial\Phi\partial H = 0$ and the two scalar sectors decouple. In this case the equation on $\phi^{(m)}$ (3.11) can be written using eqs. (2.5)-(2.6) in the following factorized form,

$$\left(-\partial_y + \frac{\rho''}{\rho'} - \frac{\Phi''}{\Phi'} + 2\rho' \right) \left(\partial_y + \frac{\rho''}{\rho'} - \frac{\Phi''}{\Phi'} + 2\rho' \right) \phi^{(m)} = e^{2\rho} m^2 \phi^{(m)}, \tag{4.1}$$

Because $\rho'|_{y=0} = 0$ the potential in the ϕ-channel contains singular barrier [9]. As a consequence in the presence of gravity there is no (normalizable) Goldstone zero-mode related to spontaneous breaking of translational symmetry. The cause is evident: the corresponding brane fluctuation represents, in fact, a gauge transformation and does not appear in the invariant part of the spectrum.

On the other hand the equation for the χ-channel,

$$\left[-\partial_\tau^2 + \frac{1}{\beta^2 M^2} e^{-2\rho} \left(-2\Delta_H + 2\Phi^2 \right) + 4(\rho')^2 - 2\rho'' \right] \chi_m = \frac{m^2}{M^2 \beta^2} e^{2\rho} \chi^{(m)}, \tag{4.2}$$

does not gain nonperturbative correction and in the limit $\kappa \to 0$ coincides with the corresponding equation in the model without gravity.

For the minimal model in the phase with unbroken τ-symmetry the χ-channel contains zero-mode that remains massless when next order in κ corrections are taken into account [12]. In the phase with broken τ-symmetry this state gains mass and nonzero ϕ component that can be computed using the perturbation theory in the parameter μ/M controlling the deviation from the critical point.

$$\chi^{(m)} = \sum_{n,k}^{\infty} \kappa^n \left(\frac{\mu}{M} \right)^k \chi_{n,k}, \quad \phi^{(m)} = \sum_{n,k}^{\infty} \kappa^n \left(\frac{\mu}{M} \right)^{k+1} \phi_{n,k}, \quad m^2 = M^2 \sum_{n,k}^{\infty} \kappa^n \left(\frac{\mu}{M} \right)^k (m^2)_{n,k}. \tag{4.3}$$

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The computation that we do not present here because of its complexity gives the same leading order of mass as in the model without gravity

$$(m^2)_{0,1} = 2, \tag{4.4}$$

However the next leading order of mass happen to be

$$(m^2)_{0,2} = -128\sqrt{3}\text{arctanh}\frac{\sqrt{3}}{3} + 146 + \frac{4}{3}\ln 2\cdot(1+\ln 2) - \frac{\pi^2}{9} \approx +0.4817, \tag{4.5}$$

whereas similar computation for the model without gravity gives,

$$(m^2)_{0,2}^{NG} = -\frac{130442}{121275} \approx -1.0756. \tag{4.6}$$

Thus we have revealed the unambiguous puzzle of discontinuity in the scalar fluctuation mass spectrum between a theory without gravity since the very beginning and a theory with minimal gravity interaction in the zero gravity limit.

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