PROCEEDINGS OF SCIENCE

The Influence Functional approach to the quantum systems dynamics

Alexander Biryukov
Samara State University - 1, Akademik Pavlov St. Samara, Russia
E-mail: biryukov@samsu.ru

Mark Shleenkov[*†]
Samara State University - 1, Akademik Pavlov St. Samara, Russia
E-mail: shleenkov@list.ru

We study the dynamics of multi-level quantum systems under the influence of electromagnetic field. Basing on the path integral approach to the quantum density matrix and transitions probability we represent functional averaging electromagnetic field influence. We demonstrate that the transition probability can be written as a path integral of real functional (the product of exponent and cosine). The numerical integration algorithm of the transitions probability in energy representation was developed. The numerical simulations of the quantum system dynamics under electromagnetic field action describe the well-known Rabi oscillations, their fluctuations and multi-photon quantum transitions.

The XXI International Workshop High Energy Physics and Quantum Field Theory,
June 23 – June 30, 2013
Saint Petersburg Area, Russia

[*]Speaker.

[†]The work was supported by the grant of the Russian Ministry of Science and Education 2.2459.2012. Numerical calculations were performed at Samara State Aerospace University computer system «Sergey Korolev».

PoS(QFTHEP 2013)076

1. Introduction

At the present time the investigations of non-linear processes in Physics are very intensive. There are atoms [1] and molecules [2] ionization and excitation by ultrashort strong laser pulses. In this case the multi-photon interaction between matter and light has the main role. To describe this interaction we need to find such equations, that describe the quantum systems dynamics in non-perturbation approaches. The Path Integral approach [3] is one of them. Within this approach we consider the Influence Functional method, which received the active development in many scientific papers [4, 5, 6, 7, 8, 9].

There are two fundamental problems. The first problem cover the influence function calculation for investigating model of matter and light interaction. The second is the path integral calculation for the description of investigating system density matrix or observation probabilities evolution. In the present article we present the path integral transformation from complex to real functionals over investigating system trajectories space. This trick allows us to calculate path integrals basing on modern numerical simulations methods (Monte-Carlo's methods).

2. The Influence Functional Approach

We consider the quantum systems (atom, molecule), which interact with electromagnetic field. The state of this model we describe in terms of statistical operator $\hat{\rho}(t)$. Its evolution can be presented as

$$\hat{\rho}(t) = \hat{U}(t)\hat{\rho}(0)\hat{U}^*(t), \tag{1}$$

where the evolution operator $\hat{U}(t)$

$$\hat{U}(t) = \hat{T}\exp[-\frac{\imath}{\hbar}\int_0^t \hat{H}_{full}(\tau)d\tau]. \tag{2}$$

The «matter+light» Hamiltonian $\hat{H}_{full}(\tau)$

$$\hat{H}_{full} = \hat{H}_{sys} + \hat{H}_{field} + \hat{H}_{int}, \tag{3}$$

where $\hat{H}_{sys}$ is the Hamiltonian of multi-level quantum system, which defines the stationary states spectrum of the free system

$$\hat{H}_{sys}|n\rangle = E_n|n\rangle; \tag{4}$$

$\hat{H}_{field}$ is the Hamiltonian of free electromagnetic field

$$\hat{H}_{field} = \int\sum_{\lambda}\frac{d^3k}{(2\pi)^3}\,\hbar\Omega_{\mathbf{k}}(\hat{a}^+_{\mathbf{k}\lambda}\hat{a}_{\mathbf{k},\lambda} + 1/2); \tag{5}$$

$\hat{H}_{int}$ is interaction Hamiltonian [10]:

$$\hat{H}_{int} = \int\frac{d^3k}{(2\pi)^3}\sum_{\lambda} e\hat{j}^{\mu}(\varepsilon^{\lambda}_{\mu}(\mathbf{k})\hat{a}^+_{\mathbf{k},\lambda} + \varepsilon^{\lambda *}_{\mu}(\mathbf{k})\hat{a}_{\mathbf{k},\lambda}), \tag{6}$$

where $\hat{j}^{\mu}$ is μ-component of the current operator, $\varepsilon^{\lambda}_{\mu}(\mathbf{k})$ is the polarization vector, $\hat{a}^+_{\mathbf{k},\lambda}, \hat{a}_{\mathbf{k},\lambda}$ are operators of photons creation and annihilation with wave vector $\mathbf{k}$ and polarization λ.

We write the evolution equation (1) in the mixed representation. The basis vectors of this representation are

$$|x, a_{\mathbf{k},\lambda}\rangle = |x\rangle \otimes |a_{\mathbf{k},\lambda}\rangle,$$

where definition of vectors $|a_{\mathbf{k},\lambda}\rangle$ is the following: $\hat{a}_{\mathbf{k},\lambda}|a_{\mathbf{k},\lambda}\rangle = a_{\mathbf{k},\lambda}|a_{\mathbf{k},\lambda}\rangle$,

$$\langle a'_{\mathbf{k},\lambda}|a_{\mathbf{k},\lambda}\rangle = \exp\{-\frac{1}{2}(|a'_{\mathbf{k},\lambda}|^2 + |a_{\mathbf{k},\lambda}|^2 - 2a'^*_{\mathbf{k},\lambda}a_{\mathbf{k},\lambda})\}, \qquad \int |a_{\mathbf{k},\lambda}\rangle\langle a_{\mathbf{k},\lambda}| \frac{|a_{\mathbf{k},\lambda}|d|a_{\mathbf{k},\lambda}|d\phi_{\mathbf{k},\lambda}}{\pi} = 1;$$

the vectors $|x\rangle$ are to define:

$$\hat{x}|x\rangle = x|x\rangle, \qquad \langle x'|x\rangle = \delta(x'-x), \qquad \int |x\rangle\langle x|dx = 1.$$

The equation of density matrix evolution in this representation is the following

$$\rho(a^*_f, x_f, a'_f, x'_f; t) = \int \frac{da^*_{in}a_{in}}{2\pi\imath}\frac{da'^*_{in}a'_{in}}{2\pi\imath}dx_{in}dx'_{in}\times$$

$$\times U(a^*_f, x_f, a_{in}, x_{in}; t)\rho(a^*_{in}, x_{in}, a'_{in}, x'_{in}; 0)U^*(a'_f, x'_f, a^*_{in}, x'_{in}; t), \tag{7}$$

where

$$\rho(a^*_{in}, x_{in}, a'_{in}, x'_{in}; 0) = < a_{in}, x_{in}|\hat{\rho}(0)|a'_{in}, x'_{in} > \qquad \rho(a^*_f, x_f, a'_f, x'_f; t) = < a_f, x_f|\hat{\rho}(t)|a'_f, x'_f > .$$

The evolution operator (2) kernel can be presented as path integral [11]

$$U(a^*_f, x_f, a_{in}, x_{in}; t) = \int (\prod_{k,\lambda} \mathfrak{D}a^{\lambda *}_{\mathbf{k}}(\tau)\mathfrak{D}a^{\lambda}_{\mathbf{k}}(\tau)\mathfrak{D}p(\tau)\mathfrak{D}x(\tau)) \exp[-\frac{\imath}{\hbar}S_{full}[p(\tau), x(\tau), a^{\lambda *}_{\mathbf{k}}(\tau), a^{\lambda}_{\mathbf{k}}(\tau)]], \tag{8}$$

where action in (8)

$$S_{full}[p(\tau), x(\tau), a^{\lambda *}_{\mathbf{k}}(\tau), a^{\lambda}_{\mathbf{k}}(\tau)] = \int_0^{\tau} [p(\tau)\dot{x}(\tau) - H_{sys}(p(\tau), x(\tau)) +$$

$$+ \int \sum_{\lambda} (\frac{\dot{a}^{\lambda *}_{\mathbf{k}}(\tau)a^{\lambda}_{\mathbf{k}}(\tau) - a^{\lambda *}_{\mathbf{k}}(\tau)\dot{a}^{\lambda}_{\mathbf{k}}(\tau)}{2\imath} - H_{field}(a^{\lambda *}_{\mathbf{k}}(\tau), a^{\lambda}_{\mathbf{k}}(\tau)))\frac{d^3k}{(2\pi)^3} -$$

$$- \int \frac{d^3k}{(2\pi)^3} \sum_{\lambda} j^{\mu}(x(\tau))(\varepsilon^{\lambda}_{\mu}(\mathbf{k})a^{\lambda *}_{\mathbf{k}}(\tau) + \varepsilon^{\lambda *}_{\mu}(\mathbf{k})a^{\lambda}_{\mathbf{k}}(\tau))]d\tau;$$

obviously, that the kernel $U^*(a'_f, x'_f, a^*_{in}, x'_{in}; t)$ has the same structure. We consider such systems, in which the interaction turns on at initial moment $t = 0$, so that

$$\rho(a^*_{in}, x_{in}, a'_{in}, x'_{in}; 0) = \rho_{sys}(x_{in}, x'_{in}; 0) \cdot \rho_{field}(a^*_{in}, a'_{in}; 0). \tag{9}$$

The next steps are substitution the density matrix (9) in equation (7) and electromagnetic field variables exclusion. After that we have the equation for quantum system density matrix $\rho(x_f, x'_f; t)$ evolution in the following form

$$\rho(x_f, x'_f; t) = Sp_{a_f = a'_f}\rho(a^*_f, x_f, a'_f, x'_f; t) = \int (\mathfrak{D}p(\tau)\mathfrak{D}x(\tau))dx_f dx_{in}(\mathfrak{D}p'(\tau)\mathfrak{D}x'(\tau))dx'_f dx'_{in}\times$$

$$\times \exp[-\frac{\imath}{\hbar}(S_{syst}[p(\tau),x(\tau)] - S_{syst}[p'(\tau),x'(\tau)])]F[x(\tau),x'(\tau)]\rho_{sys}(x_{in},x'_{in};0), \quad (10)$$

where $F[x(\tau),x'(\tau)]$ the influence functional of electromagnetic field on the quantum system is under study:

$$F[x(\tau),x'(\tau)] = \int (\prod_{k,\lambda} \mathfrak{D}a^*_{\mathbf{k},\lambda}(\tau)\mathfrak{D}a_{\mathbf{k},\lambda}(\tau))da_f da_{in}(\prod_{k',\lambda'} \mathfrak{D}a'^*_{\mathbf{k'},\lambda'}(\tau)\mathfrak{D}a'_{\mathbf{k'},\lambda'}(\tau))da'_f da'_{in}\times$$

$$\times \exp[-\frac{\imath}{\hbar}(S_{infl}[a^*_{\mathbf{k},\lambda}(\tau),a_{\mathbf{k},\lambda}(\tau),x(\tau)] - S_{infl}[a'^*_{\mathbf{k'},\lambda'}(\tau),a'_{\mathbf{k'},\lambda'}(\tau),\tau])]\rho_{field}(a^*_{in},a'_{in};0). \quad (11)$$

$S_{sys}[p(\tau),x(\tau)]$ is free quantum system action:

$$S_{sys}[p(\tau),x(\tau)] = \int_0^t [p(\tau)\dot{x}(\tau) - H_{sys}(p(\tau),x(\tau))]d\tau;$$

$S_{infl}[a^*_{\mathbf{k},\lambda}(\tau),a_{\mathbf{k},\lambda}(\tau),x(\tau)]$ is the action, which describes the electromagnetic field influence on the system under study:

$$S_{infl}[a^*_{\mathbf{k},\lambda}(\tau),a_{\mathbf{k},\lambda}(\tau),x(\tau)] = -\int_0^t [\int \frac{d^3k}{(2\pi)^3} \sum_\lambda j^\mu(x(\tau))(\varepsilon^\lambda_\mu(\mathbf{k})a^{\lambda *}_{\mathbf{k}}(\tau) + \varepsilon^{\lambda *}_\mu(\mathbf{k})a^\lambda_{\mathbf{k}}(\tau)) -$$

$$- \int \sum_\lambda (\frac{\dot{a}^{\lambda *}_{\mathbf{k}}(\tau)a^\lambda_{\mathbf{k}}(\tau) - a^{\lambda *}_{\mathbf{k}}(\tau)\dot{a}^\lambda_{\mathbf{k}}(\tau)}{2\imath} - H_{field}(a^{\lambda *}_{\mathbf{k}}(\tau),a^\lambda_{\mathbf{k}}(\tau)))\frac{d^3k}{(2\pi)^3}]d\tau. \quad (12)$$

Particularly, the influence functional (11) can describe the thermal photon bath influence.

We present the equation (7) for the density matrix evolution in energy representation. We consider such model, in which the quantum system state $|n\rangle$ state is determined by wave function $\phi_n(x)$; the electromagnetic field state with wave vector k is determined by function in holomorphic functions space $\psi_k(a)$. The product of them creates full system orthonormal basis:

$$\int\int \phi^*_{n'}(x)\psi^*_{k'}(a)\phi_n(x)\psi_k(a)dxda = \delta_{n'n}\delta_{k'k}.$$

We choose the basis as a $\phi_n(x)\psi_k(a)$ and consider the density matrix (7) representation if the formulas (8), (9) are true:

$$\rho(m,k_f,m',k'_f;t) = \int (\mathfrak{D}p(\tau)\mathfrak{D}x(\tau))dx_f dx_{in}(\mathfrak{D}p(\tau)\mathfrak{D}x(\tau))dx'_f dx'_{in}\times$$

$$\times \phi_m(x_f)\phi_{m'}(x'_f)\exp[-\frac{\imath}{\hbar}(S_{syst}[p(\tau),x(\tau)] - S_{syst}[p'(\tau),x'(\tau)])]$$

$$F_{k_f,k'_f;k_{in},k'_{in}}[x(\tau),x'(\tau)]\sum_{n,n'} \phi^*_{n'}(x'_{in})\phi_n(x_{in})\rho_{sys}(n,n';0), \quad (13)$$

where

$$\rho(m,k_f,m',k'_f;t) = \int \phi_m(x_f)\psi_{k_f}(a_f)\rho(x_f,a_f,x'_f,a'_f;t)\phi_{m'}(x'_f)\psi_{k'_f}(a'_f) \quad (14)$$

- the density matrix of quantum system and electromagnetic field at the time moment t in energy representation;

$$\rho_{sys}(n,n';0) = \int\int \phi^*_n(x_{in})\rho_{sys}(x_{in},x'_{in};0)\phi_{n'}(x'_{in})dx_{in}dx'_{in} \quad (15)$$

is the density matrix of quantum system at the time moment t in energy representation;

$$F_{k_f,k'_f;k_{in},k'_{in}}[x(\tau),x'(\tau)] = \int (\prod_{k,\lambda} \mathfrak{D}a^*_{\mathbf{k},\lambda}(\tau)\mathfrak{D}a_{\mathbf{k},\lambda}(\tau))(\prod_{k',\lambda'} \mathfrak{D}a'^*_{\mathbf{k}',\lambda'}(\tau)\mathfrak{D}a'_{\mathbf{k}',\lambda'}(\tau))\times$$

$$\times \psi_{k_f}(a_f)\psi_{k'_f}(a'_f)\exp[-\frac{\imath}{\hbar}(S_{infl}[a^*_{\mathbf{k},\lambda}(\tau),a_{\mathbf{k},\lambda}(\tau),x(\tau)] - S_{infl}[a'^*_{\mathbf{k}',\lambda'}(\tau),a'_{\mathbf{k}',\lambda'}(\tau),\tau])]\times$$

$$\times \sum_{k_{in},k'_{in}} \psi_{k_{in}}(a_{in})\rho_{field}(k_{in},k'_{in};0)\psi^*_{k'_{in}}(a'_{in})da_f da_{in} da'_f da'_{in} \quad (16)$$

is the influence functional of electromagnetic field in the case that its states in initial and final time moments are defined.

$$\rho_{field}(k_{in},k'_{in};0) = \int\int \psi_{k_f}(a_{in})\rho(a_{in},a'_{in};0))\psi_{k'_i n}(a'_{in}) \quad (17)$$

is the density matrix of electromagnetic field at the initial time moment $t = 0$ in energy representation; here indexes in, f are define the value status at the initial ($t = 0$) and final ($t > 0$) time moments.

The equations (13) - (17) are defined concretely for specific models of quantum system and electromagnetic field.

We consider the model, in which the initial and final states of quantum system are pure quantum states, are defined with wave functions $\phi_{n_0}(x'_{in}), \phi_m(x_f)$, while in the equations (13), (15) $m = m'$, $\rho_{sys}(n,n';0) = \delta_{n_0-n}\delta_{n-n'}$. The electromagnetic field state is also pure and they are defined functions $\psi_{k_f}(a_f), \psi_{k_{in}0}(a_{in})$, in equations (13), (16) $k_f = k'_f$, $\rho_{field}(k_{in},k'_{in};0) = \delta_{k_{in}0-k_{in}}\delta_{k_{in}-k'_{in}}$.

In this model the equation (13) describe probabilities $P(m,t|n,0)$ of investigated quantum system transition under influence of electromagnetic field with wave vector k from state $\phi_n(x'_{in}$ to state $\phi_m(x_f)$

$$P(m,t|n,0) = \int (\mathfrak{D}p(\tau)\mathfrak{D}x(\tau))dx_f dx_{in}(\mathfrak{D}p(\tau)\mathfrak{D}x(\tau))dx'_f dx'_{in}\times$$

$$\times \phi_m(x_f)\phi_m(x'_f)\exp[-\frac{\imath}{\hbar}(S_{syst}[p(\tau),x(\tau)] - S_{syst}[p'(\tau),x'(\tau)])]F_k[x(\tau),x'(\tau)]\phi_n(x_{in})\phi^*_n(x'_{in}), \quad (18)$$

where the influence functional $F_k[x(\tau),x'(\tau)]$ is defined by (16) and model parameters

$$F_{k_f;k_{in}}[x(\tau),x'(\tau)] = \int (\prod_{k,\lambda} \mathfrak{D}a^*_{\mathbf{k},\lambda}(\tau)\mathfrak{D}a_{\mathbf{k},\lambda}(\tau))da_f da_{in}(\prod_{k',\lambda'} \mathfrak{D}a'^*_{\mathbf{k}',\lambda'}(\tau)\mathfrak{D}a'_{\mathbf{k}',\lambda'}(\tau))da'_f da'_{in}\times$$

$$\times \psi_{k_f}(a_f)\psi^*_{k_f}(a'_f)\exp[-\frac{\imath}{\hbar}(S_{infl}[a^*_{\mathbf{k},\lambda}(\tau),a_{\mathbf{k},\lambda}(\tau),x(\tau)] - S_{infl}[a'^*_{\mathbf{k}',\lambda'}(\tau),a'_{\mathbf{k}',\lambda'}(\tau),\tau])]\times$$

$$\times \psi_{k_{in}}(a_{in})\psi^*_{k_{in}}(a'_{in}). \quad (19)$$

In the case the final state of electromagnetic field is not specified (can be any), then we sum over all possible finite states of electromagnetic field $\psi_{k_f}(a_f)$ and take into account

$$\sum_{k_f} \psi_{k_f}(a_{k_f})\psi^*_{k_f}(a'_{k_f}) = \delta(a_{k_f} - a'_{k_f}),$$

then the influence functional (19) we convert to the following form

$$F_{k_{in}}[x(\tau),x'(\tau)] = \int\limits_{a_f=a'_f} (\prod_{k,\lambda} \mathfrak{D}a^*_{\mathbf{k},\lambda}(\tau)\mathfrak{D}a_{\mathbf{k},\lambda}(\tau))(\prod_{k',\lambda'} \mathfrak{D}a'^*_{\mathbf{k'},\lambda'}(\tau)\mathfrak{D}a'_{\mathbf{k'},\lambda'}(\tau))\times$$

$$\times \exp[-\frac{\imath}{\hbar}(S_{infl}[a^*_{\mathbf{k},\lambda}(\tau),a_{\mathbf{k},\lambda}(\tau)] - S_{infl}[a'^*_{\mathbf{k'},\lambda'}(\tau),a'_{\mathbf{k'},\lambda'}(\tau)])]\psi_{k_{in}}(a_{in})\psi^*_{k_{in}}(a'_{in}). \quad (20)$$

The influence functionals (19),(20) were calculated for vacuum and pure coherent state of electromagnetic field [**?**]. If the initial and finite states of electromagnetic field are one mode vacuum

$$\psi_{k_{in,vac}}(a_{k,in}) = \exp[-\frac{|a_{k,in}|^2}{2}], \quad \psi_{k,fvac}(a_{k,f}) = \exp[-\frac{|a_{k,f}|^2}{2}],$$

then the influence functional (12) is calculated in the following form:

$$F_k^{vac\to vac}[x(\tau),x'(\tau)] =$$

$$= \exp\{-\int_0^\tau \int_0^{\tau'} [\gamma_k(\tau',\tau'')j(x(\tau'))j(x(\tau'')) + \gamma^*_k(\tau',\tau'')j(x'(\tau'))j(x'(\tau''))]\}d\tau''d\tau'\}. \quad (21)$$

If the initial state of electromagnetic field is one mode vacuum $\psi_{k_{in,vac}}(a_{k,in}) = \exp[-\frac{|a_{k,in}|^2}{2}]$ and we have no interest in the final state, then the influence functional (20) was calculated in the following form:

$$F_k^{vac}[x(\tau),x'(\tau)] = F_k^{vac\to vac}[x(\tau),x'(\tau)]\cdot$$

$$\cdot\exp\{\int_0^\tau \int_0^{\tau'} [\gamma^*_k(\tau',\tau'')j(x(\tau'))j(x'(\tau'')) + \gamma_k(\tau',\tau'')j(x'(\tau'))j(x(\tau''))]d\tau''d\tau'\}. \quad (22)$$

In the equations (21), (22) the functions $\gamma(\tau',\tau'')$ depend on time interval and fundamental constant

$$\gamma_k(\tau',\tau'') = \frac{e^2}{\hbar^2}\frac{\hbar\Omega_k}{2\varepsilon_0 V}e^{-\imath\Omega_k(\tau'-\tau'')}.$$

The analytical calculation of the spontaneous emission probability was done in [12, 13]. The result agree with other theoretical models and experimental data.

If the initial state of electromagnetic field is one mode pure coherent state

$$\psi_{a_0}(a_{in}) = \langle a_{in}|a_0\rangle = \exp[-\frac{|a_{in}|^2}{2} - \frac{|a_0|^2}{2} + a^*_{in}a_0]$$

and we have no interest in the final state, then the influence functional (20) is calculated in the following form:

$$F_k^{coh}[x(\tau),x'(\tau)] = F_k^{vac}[x(\tau),x'(\tau)]\exp\{-\frac{\imath}{\hbar}\sqrt{\frac{\hbar\Omega_k}{2\varepsilon_0 V}}e\int_0^\tau (j(x'(\tau')) - j(x(\tau')))\cos(\Omega_k t' - \phi_k)dt'\}. \quad (23)$$

The influence functionals of multi-mode electromagnetic field $F[x(\tau),x'(\tau)]$ can be calculated by product of the independent modes k influence functionals [3]:

$$F[x(\tau),x'(\tau)] = \prod_k F_k[x(\tau),x'(\tau)]. \quad (24)$$

3. The transitions probability as a path integral of real functional

The influence functionals for wide class of electromagnetic field models was investigated in the papers [12]. The analysis of resulting influence functionals allows us to write its general form as

$$F[x(\tau),x'(\tau)] = \exp[-\frac{1}{\hbar}S_{0F}[x(\tau),x'(\tau)]]\exp[-\frac{\imath}{\hbar}S_f[x(\tau),x'(\tau)]], \tag{25}$$

where $S_{0F}[x(\tau),x'(\tau)]$ and $S_F[x(\tau),x'(\tau)]$ are real functionals, which defined in trajectories space of investigated quantum system. We should note, that structure (25) of the electromagnetic field influence functional describes the time irreversible evolution of the quantum system density matrix (10), (13) and transition probabilities (18).

We present the equation (18) with the (25) in the following form

$$P(m,t|n,0) = \int \mathfrak{D}p(\tau)\mathfrak{D}x(\tau)\mathfrak{D}p'(\tau)\mathfrak{D}x'(\tau)dx_f dx'_f dx_{in}dx'_{in}\ \phi^*_m(x_f)\phi_m(x'_f)\phi^*_n(x_{in})\phi_n(x'_{in})\times$$

$$\times\exp[-\frac{1}{\hbar}S_{0F}[x(\tau),x'(\tau)]]\exp[-\frac{\imath}{\hbar}(S_{sys}[p(\tau),x(\tau)]-S_{sys}[p'(\tau),x'(\tau)]+S_F[x(\tau),x'(\tau)])]. \tag{26}$$

For numerical simulations we present the path integral (26) in such form, in which the integrand is the real functional.

We transform formula (26) using the Euler formula to the following form

$$P(m,t|n,0) = \int \mathfrak{D}p(\tau)\mathfrak{D}x(\tau)\mathfrak{D}p'(\tau)\mathfrak{D}x'(\tau)dx_f dx'_f dx_{in}dx'_{in}\ \phi^*_m(x_f)\phi_m(x'_f)\phi^*_n(x_{in})\phi_n(x'_{in})\times$$

$$\times\exp\{\frac{1}{\hbar}S_{0F}[x(\tau),x'(\tau)]\}\cos\{\frac{1}{\hbar}(S_{sys}[p(\tau),x(\tau)]-S_{sys}[p'(\tau),x'(\tau)]-S_F[x(\tau),x'(\tau)])\}-$$

$$-\imath\int \mathfrak{D}p(\tau)\mathfrak{D}x(\tau)\mathfrak{D}p'(\tau)\mathfrak{D}x'(\tau)dx_f dx'_f dx_{in}dx'_{in}\ \phi^*_m(x_f)\phi_m(x'_f)\phi^*_n(x_{in})\phi_n(x'_{in})\times$$

$$\times\exp\{\frac{1}{\hbar}S_{0F}[x(\tau),x'(\tau)]\}\sin\{\frac{1}{\hbar}(S_{sys}[p(\tau),x(\tau)]-S_{sys}[p'(\tau),x'(\tau)]-S_F[x(\tau),x'(\tau)])\}. \tag{27}$$

The path integral (27) is equal to path integral with real and sing varied integrand functional, so we present the transition probability in the form:

$$P(m,t|n,0) = \int \mathfrak{D}p(\tau)\mathfrak{D}x(\tau)\mathfrak{D}p'(\tau)\mathfrak{D}x'(\tau)dx_f dx'_f dx_{in}dx'_{in}\phi^*_m(x_f)\phi_m(x'_f)\phi_n(x_{in})\phi^*_n(x'_{in})\times$$

$$\times\exp\{\frac{1}{\hbar}S_{0F}[x(\tau),x'(\tau)]\}\cos\{\frac{1}{\hbar}(S_{sys}[p(\tau),x(\tau)]-S_{sys}[p'(\tau),x'(\tau)]-S_F[x(\tau),x'(\tau)])\}. \tag{28}$$

The formula (28) can be proved by integration independence over trajectories with prime and without one in formula (27). The asymmetry of the functional $\sin\{\frac{1}{\hbar}(S_{sys}[p(\tau),x(\tau)]-S_{sys}[p'(\tau),x'(\tau)]-S_F[x(\tau),x'(\tau)])\}$ relatively the replacement these trajectories and if following formula are true

$$S_{0F}[x(\tau),x'(\tau)] = S_{0F}[x'(\tau),x(\tau),] \qquad S_F[x(\tau),x'(\tau)] = -S_F[x'(\tau),x(\tau)],$$

$$\phi^*_m(x_f)\phi_m(x'_f) = \phi^*_m(x'_f)\phi_m(x_f), \qquad \phi_n(x_{in})\phi^*_n(x'_{in}) = \phi_n(x'_{in})\phi^*_n(x_{in}).$$

We should remind, that first reference on possibility of probability transition presentation as path integral of real and sign variable functional was developed by G. V. Ryazanov [14].

Basing on (28) we consider the non-relativistic multi-level quantum system transition under electromagnetic field influence in the dipole approximation. The intensity of electromagnetic field in our model is so large, that the vacuum influence is neglected. In this case we assume $S_{OF}[x(\tau),x'(\tau)]=0$ in the formula (28), that makes the calculations simpler and leads to unitary dynamics.

The formula (28) for transition probability for numerical simulations is more convenient in the energy representation.

$$P(n_f,t_f|n_{in},t_{in})=\lim_{K\to\infty}\sum_{n_1,..,n_K=1}^{N}\sum_{m_1,..,m_K=1}^{N}\int_0^1..\int_0^1 d\xi_0..d\xi_K d\zeta_0..d\zeta_K\times$$

$$\times\cos[S[n_f,n_K,\xi_K;..;n_k,n_{k-1},\xi_{k-1};..;n_1,n_{in},\xi_0]-S[n_f,m_K,\zeta_K;..;m_k,m_{k-1},\zeta_{k-1};..;m_1,n_{in},\zeta_0]], \tag{29}$$

where action $S[n_f,n_K,\xi_K;..;n_k,n_{k-1},\xi_{k-1};..;n_1,n_{in},\xi_0]=\sum_{k=1}^{K+1}S[n_k,n_{k-1},\xi_{k-1}]$ is functional in trajectories space, which defined in space of discrete n_k and continuous ξ_k variables. The size of discrete space is one of the quantum system parameter and the continuous space is limited on the interval $[0,1]$. The boundary conditions are $t_{K+1}=t$, $n_{K+1}=n_f$, $t_0=0$.

$$S[n_k,t_k;n_{k-1},t_{k-1};\xi_{k-1}]=2\pi(n_k-n_{k-1})\xi_{k-1}+\Omega^R_{n_kn_{k-1}}(\cos(2\pi(n_k-n_{k-1})\xi_{k-1}+$$

$$+(\Omega+\omega_{n_k,n_{k-1}})\frac{t_k+t_{k-1}}{2})+\cos(2\pi(n_k-n_{k-1})\xi_{k-1}-(\Omega-\omega_{n_k,n_{k-1}})\frac{t_k+t_{k-1}}{2}))(t_k-t_{k-1}). \tag{30}$$

The action (30) was calculated without rotating wave approximation (RWA). In this approximation, terms in a Hamiltonian which oscillate rapidly are neglected. Obviously, that within RWA the action is

$$S^{RWA}[n_k,t_k,n_{k-1},t_{k-1};\xi_{k-1}]=2\pi(n_k-n_{k-1})\xi_{k-1}+\Omega^R_{n_kn_{k-1}}(t_k-t_{k-1})\times$$

$$\times\cos(2\pi|n_k-n_{k-1}|\xi_{k-1}-\frac{\Omega-|\omega_{n_k,n_{k-1}}|}{2}(t_k+t_{k-1})). \tag{31}$$

Basing on (29-31) we have calculated the multi-level quantum system transition probabilities with rotating wave approximation and without it, using the computer cluster. The results predict fluctuations of the Rabi oscillation [15]. They are not contrary to the quantum system description by the perturbation theory [16, 17, 18, 19, 20] and the experimental data [21], [22] for two-photon Rabi oscillations. This facts confirm, that this approach (29)-(31) describe the multi-photon processes.

References

[1] V. Florescu, O. Budriga, H. Bachau, *Two-photon ionization of hydrogen and hydrogenlike ions: Retardation effects on differential and total generalized cross sections*, *Phys. Rev. A* **86** (2012) 033413.

[2] R. Antoine, T. Doussineau, P. Dugourd, F. Calvo, *Multiphoton dissociation of macromolecular ions at the single-molecule level*, *Phys. Rev. A* **87** (2013) 013435.

[3] R. P. Feynman, A. R. Hibbs, *Quantum mechanics and Path Integrals*, McGraw-Hill, New York 1965.

[4] R. P. Feynman, F. L. Vernon Jr., *The Theory of a General Quantum System Interacting with a Linear Dissipative System*, *Annals of Physics* **24** (1963) 118-173.

[5] J. An, M. Feng, W. Zhang, *Non-Markovian decoherence dynamics of entangled coherent states* (2007) [quant-ph/07052472].

[6] S. Albeverio, L. Cattaneo, S. Mazzucchi, L. Persio, *A rigorous approach to the Feynman-Vernon influence functional and its applications I*, *J. Math. Phys.* **48** (2007) 102109.

[7] J. Jin, M. W. Tu, W. Zhang, Y. Yan, *A nonequilibrium theory for transient transport dynamics in nanostructures via the Feynman-Vernon influence functional approach* (2009) [cond-mat.mes-hall/09101675].

[8] F. Marquardt, J. Delft, R. A. Smith, V. Ambegaokar, *Decoherence in weak localization I: Pauli principle in influence functional* (2005) [cond-mat.dis-nn/0510556].

[9] F. Marquardt, J. Delft, R. A. Smith, V. Ambegaokar, *Decoherence in weak localization II: Bethe-Salpeter calculation of Cooperon* (2005) [cond-mat.dis-nn/0510557].

[10] A. I. Akhiezer, V. B. Berestetskii *Quantum Electrodynamics*, Intersci. Publ., New York 1965.

[11] L. D. Fadeev, A. A. Slavnov *Gauge Fields. Introduction to Quantum theory* Addison-Wesley P. C., 1991.

[12] A. A. Biryukov, M. A. Shleenkov, *The Influence Functional of quantum electromagmetic field*, *Theoretical Physics* **12** (2011) 28-49.

[13] A. A. Biryukov, M. A. Shleenkov, *The functional approach for the description of multilevel quantum systems dynamics in electromagnetic field*, *Bulletin of the Samara Scientific Center of the Russian Academy of Sciences* **15** (2013) 140-144.

[14] G. V. Ryazanov, *Quantum-mechanical probability as a sum over path*, *JETP* **35** (1958) 121-131.

[15] A. A. Biryukov, M. A. Shleenkov, *The transition probabilities calculation by paths integration of real functionals*, *Theoretical Physics* **13** (2012) 8-41.

[16] A. A. Biryukov, J. P. Philippov, *Amplitudes and probabilities of transitions pf many-level quantum system under the action of quantum electromagnetic field at the third order of perturbation theoty*, *Theoretical Physics* **9** (2008) 187-194.

[17] A. A. Biryukov, B. V. Danilyuk, *Transitions probabilities in the multi-level quantum system at the third order of perturbation theory*, *Proc. SPIE* **7024** (2008) 70240A.

[18] A. A. Biryukov, B. V. Danilyuk, *Rabi oscillations in many-level quantum system*, *Proc. SPIE* **7024** (2008) 702405.

[19] A. A. Biryukov, B. V. Danilyuk, A. N. Kosygin *Quantum transition probabilities of a particle in potencial wall under the effect of variable external field*, *Theoretical Physics* **7** (2006) 102-108.

[20] A. A. Biryukov, B. V. Danilyuk *Resonance interaction of oscillating electrical field with many-level quantum system*, *Theoretical Physics* **7** (2006) 130-140.

[21] M. Ornigotti, G. D. Valle at al *Visualization of two-photon Rabi oscillations in evanescently coupled optical waveguides*, *J. Phys. B: At. Mol. Opt. Phys.* **41** (2008) 085402.

[22] T. R. Gentile, B. J. Hughey at al *Experimental study of one- and two-photon Rabi oscillations Phys. Rev. A* **40** (1989) 5103-5115.