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Phenomenology of $SU(3)_L \times SU(3)_R \times SU(3)_C$ and the Higgs Boson

Berthold Stech*
Institut für Theoretische Physik, Philosophenweg 16, D-69120 Heidelberg, Germany
E-mail: `B.Stech@ThPhys.Uni-Heidelberg.DE`

The extension of the Standard Model to $SU(3)_L \times SU(3)_R \times SU(3)_C$ (the trinification group) augmented by the $SO(3)_G$ flavor group is considered. In our phenomenological treatment vacuum expectation values of the scalar Higgs fields play a dominant role. For the construction of the tree-level potential and to give all scalar fields masses apart from the would-be-Goldstone bosons, the combination of different matrix fields are necessary. In one scenario the main Higgs field is combined with a field that does not couple to fermions. As a consequence almost all scalar states have fermiophobic components. In a second scenario the fermiophobic field is replaced by the Higgs field which controls the generation (flavor) mixings. In this case most scalar states have components which can give rise to flavor-changing transitions. A few illustrating examples are given. Two flavor coupling matrices are needed, a diagonal one for the mass hierarchy and an antisymmetric one for the mixings of all fermions. The mixing with heavy fermions provides for an understanding of the difference between the up quark and the down quark mass spectrum. The two flavor matrices are also responsible for the neutrino properties. An inverted hierarchy is predicted. A quantitative description of the spectrum and mixings of all fermions is achieved using relatively few parameters.

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*Speaker.

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1. Introduction

According to present experimental results at the Large Hadron Collider supersymmetry has not been observed [1]. The hierarchy problem still persists. This is a serious problem for the vacuum expectation values (vevs) of scalar fields, but only for them. These important momentum independent quantities are presently not understood. Fortunately however, the particle masses are only indirectly affected by this hierarchy problem, only via their couplings to these vacuum expectation values. Also the well-known quadratic divergence of the Higgs self-energy caused by fermion loops is related to the vev of the Higgs field, a fact which can be demonstrated in the Standard Model.

In view of this situation we treat in this article all vacuum expectation values of scalar fields as fixed basic parameters. In addition, only dimensionless coupling constants are used. The idea is to start with a massless Lagrangian. By introducing vevs for the scalar fields, linear field components show up which have to be canceled. In a ϕ^4 model this cancellation is performed by adding a term of dimension 2 multiplied by the square of the corresponding vev:

$$\lambda\ \phi^4 \to \lambda\ \phi^4 - 2\lambda\ \langle\phi\rangle^2\ \phi^2. \tag{1.1}$$

Now the shift $\phi \to \langle\phi\rangle + \phi'$ can be applied and no linear term in ϕ' appears anymore. If log terms can be used more possibilities are open. For instance

$$\lambda\ \phi^4 \to \frac{\lambda}{1+\log[\frac{\phi^4}{\langle\phi\rangle^4}]}\ \phi^4 \tag{1.2}$$

has no linear term in ϕ' either.

The embedding of the Standard Model into a larger group allows us to connect the properties of quarks and charged leptons and their mixings with the properties of neutrinos and their different mixings. It also implies a Higgs sector with more scalar bosons. Here we extend the Standard Model symmetry group $SU(2)_L \times U(1) \times SU(3)_C$ to $SU(3)_L \times SU(3)_R \times SU(3)_C$, the trinification group [2–4], which is a subgroup of E_6 [5–8]. In this report we follow (with modifications) articles in reference [9–11], but restrict ourselves to the trinification group. As proposed in these articles, we also use in addition the flavor (generation) group $SO(3)_G$ and require all fermions to be 3-vectors in generation space. The coupling matrices for fermions are vevs of flavon fields. In our phenomenological treatment these couplings are parameters of the model, without regarding the flavon potential from which they originate. We take all Higgs fields to be singlets with respect to the flavor symmetry and all flavon fields to be singlets with respect to the trinification group.

The group $SU(3)_L \times SU(3)_R \times SU(3)_C$ can be unbroken only at and above the scale where the two electroweak gauge couplings g_1 and g_2 combine. According to the scale dependence of the Standard Model couplings this happens at a scale of about 10^{13} to 10^{14} GeV. Interestingly, this is just the scale relevant for the small values of the neutrino masses by applying the seesaw mechanism. It is also the place where the self coupling of the Higgs field approaches zero [12]. In this report we do not consider the possible complete unification of g_1, g_2 and g_3 at a still higher scale.

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All fermions are described by two-component (left-handed) Weyl fields. They occur in singlet and triplet $SU(3)$ representations of the trinification group. For each generation one has

$$\begin{aligned} \text{Quarks}: \quad & q(x) = (3,1,\bar{3}) \,, \\ \text{Leptons}: \quad & L(x) = (\bar{3},3,1) \,, \\ \text{Antiquarks}: \quad & \hat{q}(x) = (1,\bar{3},3) \end{aligned} \tag{1.3}$$

as in applications of E_6 [9, 10].

We introduce the 4 matrix fields (flavor singlet Higgs fields) $H, \tilde{H}, H_A$ and H_{Al}. The first 3 are 3×3 matrices, the last one a $(\bar{3}, \bar{6})$ matrix. The flavor (generation) coupling matrices are taken to be hermitian 3×3 matrices. Generation indices are denoted by $\alpha, \beta = 1,2,3$. The coupling of H to fermions is described by the symmetric matrix $G_{\alpha,\beta}$ which can be taken to be diagonal. For the flavor coupling of H_A and H_{Al} the antisymmetric matrix $A_{\alpha,\beta}$ is used. According to its origin from E_6 [9] H_A couples to quarks and H_{Al} to leptons only. The fields in $\tilde{H}$ are not coupled to fermions, only to gauge bosons.

With these matrices the effective Yukawa interaction is proposed to be [10, 11]

$$\begin{aligned} \mathscr{L}_Y^{\text{eff}} \;=\; & g_t G_{\alpha\beta} \left(\psi^{\alpha T} H \psi^{\beta}\right) + A_{\alpha\beta} \left(\psi^{\alpha T} H_A \psi^{\beta}\right) + A_{\alpha\beta} \left(\psi^{\alpha T} H_{Al} \psi^{\beta}\right) + \\ \frac{1}{M_N} \left(G^2\right)_{\alpha\beta} & \left((\psi^{\alpha T} H^{\dagger})_1 (\tilde{H}^{\dagger} \psi^{\beta})_1\right) \text{ or } \frac{1}{M_N} \left(G^2\right)_{\alpha\beta} \left((\psi^{\alpha T} H^{\dagger})_1 (H_A \psi^{\beta})_1\right) \quad + \; h.c. \end{aligned} \tag{1.4}$$

Taking vacuum expectation values for the scalar fields the first term in (1.4) gives the up quarks their masses. It also describes parts of the down quark and lepton mass matrices (with Dirac masses for the neutrinos). The second term performs the mixings of the Standard Model quarks as well as their mixings with the new heavier states occurring in the model. Together, the first and the second term are responsible for the masses of up quarks, down quarks, and the Cabibbo-Kobayashi-Maskawa matrix including its CP-violating phase. Similarly, the first term together with the third term gives the mass matrix for the leptons.

At this stage, however, the neutrinos are still Dirac neutrinos with masses comparable to the quark masses. An important assumption of our model is therefore the inclusion of the fourth or the fifth term. The fourth term is taken if the fermiophobic fields $\tilde{H}$ together with the fields H built the Higgs potential, or at least the most important part of it. We denote this case the fermiophobe model for the potential. As indicated fermions are coupled with $H^{\dagger}$ and $\tilde{H}^{\dagger}$ to form trinification singlets. This term could originate from the exchange of a very heavy trinification singlet with mass $M_N \approx M_I$. Obviously, it can only provide masses for neutral leptons. The fifth term in (1.4) is applied if no fermiophobic fields are used, i.e. $\tilde{H} = 0$. In this case the fields H_A together with the fields H built the important part of the Higgs potential. We denote this case the flavophile model. Also here neutrinos that are Standard Model singlets obtain large masses from vevs to be discussed below. The flavor matrix for the fourth and the fifth term term in (1.4) must be symmetric. Because of the second order form of these terms we choose the matrix G^2. As a consequence, the mass hierarchy of the heavy neutrinos ${}^3L_2$ and ${}^3L_3$ is a very strong one, proportional to the square of the up quark hierarchy [10, 11].

The vev of H can be taken to be a diagonal matrix while the vevs of $\tilde{H}$ and H_A will then in general have 5 nonzero elements.

$$\langle H\rangle = \begin{pmatrix} v_1 & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & M_I \end{pmatrix}, \quad \langle \tilde{H}\rangle = \begin{pmatrix} v_2 & 0 & 0 \\ 0 & b_2 & b_3 \\ 0 & M_R & M_3 \end{pmatrix}, \quad \langle H_A\rangle = \begin{pmatrix} {}^1f_1 & 0 & 0 \\ 0 & {}^2f_2 & {}^2f_3 \\ 0 & {}^3f_2 & {}^3f_3 \end{pmatrix}. \tag{1.5}$$

M_I and M_R or ${}^3f_2$ have to be large compared to the weak scale. M_I is presumably close to the meeting point of g_1 and g_2 mentioned above. These vevs break the trinification group down to the Glashow-Weinberg-Salam group. Finally, because of the small vevs in the first and second lines only the electromagnetic $U_e(1)$ symmetry remains. The vevs for H_{Al} are denoted by ${}^if^{\{jk\}}$. As a consequence of our scheme it is seen that v_1 and b are related to the top mass and the (unmixed) bottom quark mass:

$$m_t = g_t\, v_1, \;\; m_b^0 = g_t\, b \quad i.e.\;\; \frac{b}{v_1} = \frac{m_b^0}{m_t}. \tag{1.6}$$

All mixings are supposed to be induced by the mixings to high mass fermions present in the representation (1.3). Since the top has no higher partner we take ${}^1f_1 = {}^1f^{\{jk\}} = 0$.

The known value $v = 174$ GeV ($= \frac{246}{\sqrt{2}}$ GeV) for the vev of the Higgs field of lowest mass is related to vevs of H, $\tilde{H}$ and H_A :

$$v^2 = v_1^2 + v_2^2 + b^2 + b_2^2 + b_3^2 + ({}^2f_2)^2 + ({}^2f_3)^2 + ({}^2f^{\{13\}})^2 + ({}^2f^{\{12\}})^2. \tag{1.7}$$

Thus, if the b_j and f_j are similarly small as b, ($b \approx 2.8$ GeV at the scale m_Z) the vevs v_1 and v_2 are restricted according to $v_1^2 + v_2^2 \simeq v^2$

The effective Yukawa interaction with G, A in (1.4) and the vev configurations of H, H_A, H_{Al} and $\tilde{H}$ contain all the necessary information about the generation structure and the fermion spectrum.

2. The scalar sector and the Higgs Boson.

The embedding of the Standard Model into a larger group implies an extended Higgs structure formed by numerous scalar fields. This is difficult to deal with since only the information about the just discovered Higgs boson [13] can be incorporated. Our aim is to construct in a phenomenological way examples of tree-level potentials for the scalar fields and to calculate the corresponding boson mass spectrum. The tree potential has to be formed from $SU(3)_L \times SU(3)_R$ invariants. Colored fields are not used. Because of the large vevs of color singlet fields the cyclic symmetry between these and colored fields is strongly broken giving the latter a large mass. As mentioned in section 1 our input consists of vacuum expectation values only. They determine the spontaneous symmetry breaking pattern and fix the position of the minimum of the potential.

The presence of Higgs fields with different properties and the necessity of combining gauge group invariants in order to get non-zero masses leads to interesting properties of the bosons obtained. Some will drastically differ from the Standard Model Higgs-like states. As already indicated in the introduction we will discuss here two scenarios of interest.

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2.1 The fermiophobe model.

The potential responsible for the scalar particle spectrum is constructed from invariants of the fields in H and $\tilde{H}$. From these 36 real fields 21 of them should become massive while leaving 15 would-be-Goldstone particles massless. The remaining fields H_A and H_{Al} are supposed to have little influence on the scalar particle spectrum, at least below the TeV region. Starting from a massless Lagrangian the individual invariants for H and $\tilde{H}$ are

$$J_1 = (Tr[H^\dagger \cdot H])^2,\ J_2 = Tr[H^\dagger \cdot H \cdot H^\dagger \cdot H],$$
$$J_3 = (Tr[\tilde{H}^\dagger \cdot \tilde{H}])^2,\ J_4 = Tr[\tilde{H}^\dagger \cdot \tilde{H} \cdot \tilde{H}^\dagger \cdot \tilde{H}]. \tag{2.1}$$

However, only J_1 and J_3 can be modified as in (1.1) in order to have no linear fields after the appropriate shift $H \to \langle H \rangle + H$ and a similar shift for $\tilde{H}$. Moreover, the masses obtained from these two modified invariants are of order M_I, M_R. None of them are of order v_1 or v_2 like the Higgs boson observed at the LHC. On the other hand, phenomenologically satisfactory results are obtained from a naive ansatz analogue to (1.2) [14–16]

$$V_0 = \frac{\kappa}{t}\,(c_1 J_1 + c_2 J_2 + c_3 J_3 + c_4 J_4) \tag{2.2}$$
$$\text{with}$$
$$t = 1 + \kappa \log[\frac{J_1 J_2 J_3 J_4}{\langle J_1\rangle\langle J_2\rangle\langle J_3\rangle\langle J_4\rangle}].$$

The condition for the vanishing of all first derivatives of V_0 at the point $H = \tilde{H} = 0$ for the shifted fields has a straightforward solution. It determines the parameter κ and the coefficients c_2, c_3, c_4 in terms of c_1 [16].

$$\kappa = \frac{1}{4}, \qquad c_2 = \frac{c_1\left(b^2 + M_I^2 + v_1^2\right)^2}{b^4 + M_I^4 + v_1^4}, \tag{2.3}$$
$$c_3 = \frac{c_1\left(b^2 + M_I^2 + v_1^2\right)^2}{\left(b_2^2 + b_3^2 + v_2^2 + M_R^2 + M_3^2\right)^2},$$
$$c_4 = \frac{c_1\left(b^2 + M_I^2 + v_1^2\right)^2}{\left(b_2^2 + b_3^2\right)^2 + v_2^4 + 2(b_2 M_R + b_3 M_3)^2 + (M_R^2 + M_3^2)^2}.$$

From the second derivatives of $\frac{1}{2}\,V_0$ with respect to all 36 fields at the point $H = \tilde{H} = 0$ of the shifted fields one gets the 36×36 mass matrix whose eigenvalues - shown here for large M_I, M_R and $M_3 \ll M_R$ - are

$$m_1^2 = c_1\,(v_1^2 + b^2), \qquad m_2^2 = c_1 (v_2^2 + b_3^2)\frac{M_I^4}{M_R^4},$$
$$m_3^2 = 4\,c_1\,M_I^2, \qquad m_4^2 = 4\,c_1\,\frac{M_I^4}{M_R^2}$$
$$m_i^2 = 0 \quad i = 5.............36 \tag{2.4}$$

For an easy presentation, we took in these equations $b_2 = 0$.

In (2.4) we have 2 scalars with low masses [15, 16]. The first one can be identified with the Higgs boson found at the LHC. It is coupled to fermions and gauge bosons. Its mass is $m^2_{Higgs} = c_1(v_1^2 + b^2)$. The second boson is fermiophobic since it is not directly coupled to fermions, but only to gauge bosons. Its mass depends decisively on the vevs v_2, M_I, M_R and M_3.

The 32 massless states can be divided into 15 massless Goldstone states and 17 additional states of mass zero. They are due to our provisional neglecting of invariants that connect the fields in H with the fields in $\tilde{H}$.

Taking as in (1.7) b_j, f_j to be small ($\approx m_b$), an interesting case would be the equality $v_1 = v_2 \simeq \frac{v}{\sqrt{2}} = 123$ GeV not very different from the Higgs mass found at the LHC [13]. This could happen if vevs of the matrix field H appear also in $\tilde{H}$. It would imply $m_{Higgs} = \sqrt{c_1}\frac{v}{\sqrt{2}}$ with $c_1 \simeq 1.04$ [16]. But it would also imply the top quark coupling g_t of the field ${}^1H_1$ to be a factor $\sqrt{2}$ larger than its Standard Model value. Thus, $v_1 \approx v$, $v_2 \ll v$, $c_1 \approx \frac{1}{2}$ is more likely the case.

A close connection between the other vevs of H and $\tilde{H}$ would be the correspondence

$$M_R = M_I, \;\; M_3 = 0, \;\; b_2, b_3 \text{ of order } b \tag{2.5}$$

($b_2 = 0$ and $b_3 = -b$ corresponds to a $\frac{\pi}{2}$ U_R-spin rotation of the vev of $\tilde{H}$ with respect to the vev of H). If (2.5) holds near degeneracies are expected. Even the Higgs at $\simeq 125$ GeV could be a state of twins with about equal masses [16]. However, the so far neglected invariants connecting the fields in H with the fields in $\tilde{H}$ will in general change this picture. We need these invariants in order to give masses to the 17 massless bosons obtained above in order to get an acceptable scalar boson spectrum.

There are quite a number of different invariants containing the fields of both multiplets H and $\tilde{H}$ [4]. Restrictions have to guarantee that all new masses are positive and not in conflict with the data. We cannot perform this task in general but will show examples. We take $v_1 = v\cos\tilde{\beta}$, $v_2 = v\sin\tilde{\beta}$ and restrict ourselves to the perhaps unlikely but interesting scheme where (2.5) holds. Moreover, we will use $b_2 = 0$ and $|b_3| = b$. In order to avoid abrupt discontinuities in the mass matrices constructed below we also require $v_2 > |b_3|$.

The potential to be added to V_0 needs 5 new invariants

$$V_S = (r_1J_1 + r_2J_2 + r_3J_3 + r_4J_4)\frac{v^2}{M_I^2} + r_5J_5 + r_6J_6 + r_7J_7 + r_8J_8 + r_9J_9. \tag{2.6}$$

The coefficients $r_1 ... r_4$ are multiplied by the small factor $\frac{v^2}{M_I^2}$ since the corresponding invariants appear already in V_0. The new invariants are

$$\begin{aligned} J_5 &= Tr[H^\dagger \cdot \tilde{H} \cdot \tilde{H}^\dagger \cdot H], \;\; J_6 = Tr[H^\dagger \cdot H \cdot \tilde{H}^\dagger \cdot \tilde{H}], \\ J_7 &= Tr[H^\dagger \cdot \tilde{H} \cdot H^\dagger \cdot \tilde{H}] + Tr[\tilde{H}^\dagger \cdot H \cdot \tilde{H}^\dagger \cdot H], \\ J_8 &= v\,(\det H + \det H^\dagger), \quad J_9 = v\,(\det \tilde{H} + \det \tilde{H}^\dagger). \end{aligned} \tag{2.7}$$

The first seven invariants J_1 to J_7 respect the symmetry $H \to -H$ and $\tilde{H} \to -\tilde{H}$, while J_8 and J_9 break this symmetry. The latter two also break the invariance under a common phase transformation of the fields. Their contribution to the potential will turn out to be very small. The total potential $V = V_0 + V_S$ should now provide non-zero masses for all fields except the Goldstone ones. After requiring the vanishing of all first derivatives of V at the proposed minimum, there remain 3 free parameters among the coefficients r_i. We choose r_1, r_2, r_7. For a range of values for these parameters the Higgs boson in (2.4) appears again together with an acceptable mass spectrum of the other 20 bosons. Taking $\tan\tilde{\beta} \approx 0$, $M_I = 10^{13}$ GeV, $b = 2.8$ GeV and $c_1 \simeq \frac{1}{2}$ the boson at 125 GeV is not degenerate. Its field content is ${}^1H_1$ with a fermiophobic admixture ${}^1\tilde{H}_1$ of about 2% only. Thus it is very close to the Standard Model Higgs. The masses of the first scalar states above 125 GeV depend in detail on the parameters r_i. Using for example $r_1 = 1, r_2 = 2, r_7 = -1$, there are 4 almost degenerate states at $\simeq 603$ GeV: two neutral states formed from ${}^2H_3$ and ${}^2\tilde{H}_2$ fields and a positively and a negatively charged state with the field components ${}^1H_3$ and ${}^1\tilde{H}_2$. Their usual and fermiophobic components are of about equal magnitudes.

For $\tan\tilde{\beta}$ values different from zero and an appropriate value for c_1 the single state at 125 GeV is a $\tilde{\beta}$ dependent superposition of a usual Higgs and a fermiophobic scalar boson. Because the production and decay properties are now different from the Standard Model Higgs, limits on $\tilde{\beta}$ may soon be available [17] .

Taking a value of $\tan\tilde{\beta}$ close to 1, and $c_1 \simeq 1$ it is possible to have a twin state at $\simeq 125$ GeV that is mass degenerate within the experimental resolution. Both members are compositions of fields from H and of fermiophobic fields from $\tilde{H}$. In other words, the twin state mentioned earlier may survive the addition of the new invariants [16]. Its existence requires $r_1 \simeq 0.004$ with the parameters r_2 and r_7 remaining unchanged. To exclude or verify a mass degeneracy the method presented in ref [18] could be applied. Our model allows this degeneracy but does not necessarily favor it. A different parameter for r_1 lift this degeneracy.

2.2 The flavophile model

Here we set $\tilde{H} = 0$ and thus have no fermiophobic components in the scalar particle spectrum. Instead, the potential is constructed from the fields occurring in H and H_A. This is another way to obtain an acceptable boson spectrum with no zero-mass members except the 15 would-be-Goldstone bosons. Unavoidably, it will lead to flavor-changing contributions. The fields from H_{Al} are supposed either not to be relevant for low-lying states and/or to have their own additional potential not involving the other fields. This is required because the existence of a state coupled to leptons having simultaneously flavor conserving and flavor changing components would be inconsistent with the known strong limits [19] on the decay $\mu \to e\,\gamma$.

For the scalar potential we take the same approach as in the fermiophobe model and simply replace $\tilde{H}$ by H_A. Now we have to set $v_1 \simeq v$, $v_2 \to {}^1f_1 \approx 0$, $b_2 \to {}^2f_2$, $b_3 \to {}^2f_3$, $M_R \to {}^3f_2$ and $M_3 \to {}^3f_3$. As an example we use again ${}^3f_2 = M_I$, ${}^3f_3 = 0$, ${}^2f_2 = 0$, ${}^2f_3 = b = 2.8$ GeV and $c_1 \simeq \frac{1}{2}$. In section 2 we have set ${}^1f_1 = 0$ since flavor mixing should only occur where mixing with heavy states can take place. We now have to allow a small value for this quantity for stability reasons. As in the fermiophobe model v_2 (now ${}^1f_1$) had to be slightly larger than $|b_3|$. We choose ${}^1f_1 = 3$ GeV but could also take a somewhat larger value without any noticeable effect on the spectrum.

Apart from the very different interpretation one can copy the results from the fermiophobe model. Again there is a range of values for the parameters r_1, r_2, r_7 for which the state at $\simeq 125$ GeV coincides with the low mass Higgs boson in (2.4). It is barely affected by the heavier bosons. The additional field component ${}^1(H_A)_1$ that can lead to flavor-changing transitions via the matrix A is only $\simeq 2\%$ of the field ${}^1H_1$. This boson can hardly be distinguished from the Standard Model Higgs. Nevertheless, a careful analysis like the one performed in [20] would be highly desirable. In almost all higher mass states however, field components from H_A are present with the same strength as the fields from H. If such states exist one could look for their decays to two quarks of different flavors, for instance to a jet with a leading top quark and a jet with a leading charm quark. An intensive search for such decays as well as for low energy processes induced by virtual boson exchange (in analogy to penguin type processes) is suggested here.

3. Charged Fermion Masses and Mixings

In this report we will only shortly mention our way to describe the fermion properties. The flavor matrices G and A occurring in (1.4) are abstracted from the up quark masses and CKM properties. All masses used here and in the following are running masses taken at the scale m_Z. The experimental values can be found in [21].

$$G = \frac{1}{g_1 v_1} \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} = \begin{pmatrix} 1.4\,\sigma^4 & 0 & 0 \\ 0 & 1.4\,\sigma^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad A = i \begin{pmatrix} 0 & \sigma & -\sigma \\ -\sigma & 0 & \tau \\ \sigma & -\tau & 0 \end{pmatrix} \tag{3.1}$$
$$\sigma = 0.05 \quad \text{and} \quad \tau = 0.45\,.$$

The up quark masses are well described by m_t and the small parameter $\sigma = 0.05$. (The factor 1.4 can be interpreted as a renormalization factor implying $m_u/m_c = m_c/m_t$ at the scale M_I [10]). By writing the matrix A we choose the element $(A)_{1,2} = i\sigma$ and did absorb the remaining multiplication factor into the vevs of H_A. Solely the parameter $\tau = 0.45$ is then necessary for a good choice of the matrix A.

Besides the down quarks of the Standard Model there exists - according to (1.3) - also a state which is a singlet with respect to Standard Model gauge group transformations. Thus, the down quark mass matrix is a 6×6 matrix. This new quark D with $SU(3)_L$ index $i = 3$ is very heavy due to the vev M_I of $\langle H\rangle$. One can integrate out this heavy state if the contributions from H_A are taken to be small compared to M_I. This way one finds the wanted 3×3 mass matrix for the Standard Model particles. The mixings with the high mass state cannot be neglected. It is seen to be essential for our understanding of the CKM matrix and the deviations of the mass pattern of down quarks from the mass pattern of the up quarks. The light down quark mass matrix is therefore

$$m_d = m_b^0\, G + f^d A + f_0^d\, \sigma^3 A\, (G)^{-1} A\,. \tag{3.2}$$

Here $m_b^0 = g_t b$ is the value of m_b before mixing, f^d is proportional to the vev of ${}^2(H_A)_2$ and f_0^d a parameter resulting from integrating out the heavy D-quark masses. The factor σ^3 serves to cancel the negative powers of σ in $A(G)^{-1}A$ and thus allows a smooth formal limit $\sigma \to 0$. Taking $m_b^0 = 2.81$ GeV, $f^d = -0.236$ GeV and $f_0^d = 1.18$ GeV (together with the values for m_t, σ and τ) an almost perfect representation for all up and down quark masses, and the CP-violating CKM matrix is achieved. The masses at the scale m_Z agree within error limits with the experimental ones [21]. Also the calculated CKM elements describe the data correctly. Only the angle β of the unitary triangle is larger than its experimental value ($\beta \simeq 26^o$ instead of $\beta \simeq 21^o$).

Like the down quarks the charged leptons have heavy partners as well and mix with them via the flavor matrix A. Again, by integrating out these heavy states the 6×6 mass matrix is reduced to the 3×3 matrix for the usual leptons. Apart from a sign, its form is the same as for the down quarks.

$$m_e = -m_\tau^0\, G - f^e A - f_0^e\, \sigma^3 A\, (G)^{-1} A\,. \tag{3.3}$$

Here m_τ^0 is the value of m_τ before mixing. m_τ^0, f^e and f_0^e are now used to fit the masses of τ, μ and the electron. A good fit is obtained by setting $m_\tau^0 = 1.64$ GeV, $f^e = -0.186$ GeV, $f_0^e = 3.03$ GeV. These values are not simply related to the corresponding values for the down quarks since for leptons the matrix elements of $\langle H_{Al} \rangle$ have to be used [9]. The charged lepton mass matrix obtained allows to calculate the charged lepton mixings, which is a necessary ingredient for the discussion of the neutrino properties.

4. Neutrino Masses and Mixings.

According to the lepton assignments in (1.3) one has to deal with 5 neutral leptons in each generation. Thus, the matrix for neutral leptons is a 15×15 matrix. Again, leptons that obtain high masses because of the large vevs in the effective Yukawa interaction (1.4) can be integrated out giving rise to a generalized seesaw mechanism. According to our Yukawa interaction the Dirac matrix for the light neutrinos is $m_t\, G$, while the heavy neutrinos have masses proportional to G^2. Therefore, a unit matrix is part of the light neutrino mass matrix. Together with a further contribution due to the particle mixing matrix A one obtains the 3×3 matrix for the light neutrinos [10, 11] in terms of the two parameters κ and f_g

$$m_\nu \simeq \frac{m_t^2}{M_I} \kappa\, \mathbf{1} + f_g\, \sigma^3\, (A\, \frac{1}{G} - \frac{1}{G} A). \tag{4.1}$$

Taking the neutrino mass matrix m_ν only up to first order in the small parameter σ, one can write m_ν in a very simple form: setting $f_g = 1.40\, m_0$ and $\kappa \frac{m_t^2}{M_I} = m_0\, \rho$ one obtains

$$m_\nu \simeq m_0 \begin{pmatrix} \rho & -\mathrm{i} & \mathrm{i} \\ -\mathrm{i} & \rho & -\mathrm{i}\tau\sigma \\ \mathrm{i} & -\mathrm{i}\tau\sigma & \rho \end{pmatrix} \tag{4.2}$$

with $\tau = 0.45$ and $\sigma = 0.05$ as used for quarks and charged leptons. The eigenvalues of $m_\nu . m_\nu^\dagger$ to first order in σ are

$$\begin{aligned} (m_2)^2 &\simeq (\rho^2 + 2 + \sqrt{2}\, \tau\, \sigma)\, m_0^2\,, \\ (m_1)^2 &\simeq (\rho^2 + 2 - \sqrt{2}\, \tau\, \sigma)\, m_0^2\,, \\ (m_3)^2 &\simeq \rho^2 m_0^2\,. \end{aligned} \tag{4.3}$$

It is now easy to see the following properties of the light neutrinos:

- The neutrino mass spectrum has the form of an inverted hierarchy.

- The ratio between the solar and the atmospheric mass squared differences is independent of the two neutrino parameters m_0 and ρ. This ratio is $\sqrt{2}\,\tau\,\sigma = 0.031$ in good agreement with experiment.
- The experimentally observed atmospheric mass squared difference can be used to fix the mass parameter m_0 for the light neutrinos. Then ρ is fixed by the lightest neutrino mass m_3.

$$\begin{aligned} m_0 &\simeq \frac{1}{\sqrt{2}}\sqrt{\Delta m^2_{\rm atm}} \simeq 0.035 \text{ eV} , \\ m_3 &\simeq m_0\,\rho \simeq 0.035\,\rho \text{ eV}. \end{aligned} \tag{4.4}$$

- Without taking account of effects from diagonalizing the charged lepton mass matrix and renormalization, the neutrino mass matrix m_ν leads to almost strict bimaximal mixing.

Including the charged lepton mixings obtained from (3.3) a better, but still not satisfactory agreement with the experimentally determined neutrino mixing angles is achieved. Detailed renormalization group calculations would be necessary, but are not performed here. Instead we introduce a parameter which may in part simulate these effects. It should not change the successful mass pattern obtained so far and is therefore taken to be an orthogonal transformation in generation space. Mixing the first with the third generation by the angle ϕ one gets with $c = cos\ \phi$, $s = sin\ \phi$ the modified neutrino mass matrix:

$$m_\nu \Rightarrow m_0 \begin{pmatrix} \rho + 2\mathrm{i}cs & -\mathrm{i}(c+\tau\sigma s) & \mathrm{i}(c^2-s^2) \\ -\mathrm{i}(c+\tau\sigma s) & \rho & -\mathrm{i}(\tau\sigma c - s) \\ \mathrm{i}(c^2-s^2) & -\mathrm{i}(\tau\sigma c - s) & \rho - 2\mathrm{i}cs \end{pmatrix}. \tag{4.5}$$

Since m_0, τ and σ are fixed, this matrix depends, for a given mass of the lightest neutrino, only on the angle ϕ. Of course, the mixing matrix of charged leptons, obtainable from section 2, has yet to be included.

As an illustrative example we take $\rho = 1$ and choose ϕ to fit the third neutrino mixing angle $\theta^\nu_{13} \approx 9^o$ in accord with data analysis [22]. This leads to $\phi \simeq \pi/14$. One then finds for the neutrino masses and angles:

$$\begin{aligned} & m_2 = 0.06012 \text{ eV},\ m_1 = 0.05949 \text{ eV},\ m_3 = 0.03456 \text{ eV} \\ & \theta^\nu_{12} \simeq 36^o, \quad \theta^\nu_{23} \simeq 49^o, \quad \theta^\nu_{13} \simeq 9.4^o. \end{aligned} \tag{4.6}$$

The CP-violating phase δ, the Majorana angles and the mass parameter for the neutrinoless double β-decay in this example are

$$\begin{aligned} & \delta \simeq -13^o,\ \frac{\alpha_{21}}{2} \simeq -81^o,\ \frac{\alpha_{31}}{2} \simeq 89^o, \\ & |\langle m_{\beta\beta}\rangle| \simeq 0.032 \text{ eV}. \end{aligned} \tag{4.7}$$

The phase and angles are given according to the standard parametrization [23]. For different values for ρ and even for $\rho \to 0$ the mixing angles shown in (4.6, 4.7) are almost unaffected. In view of our simple approach these results for the mixing angles and the mass squared differences are satisfactory. Nevertheless, in case the inverted hierarchy predicted here turns out to be established by experiment, a more detailed study of the model will be necessary. By integrating out heavy states renormalization group effects and the violation of unitarity of the mixing matrices must certainly be incorporated before any final judgement is possible.

5. Summary

In this work we considered the generalization of the Standard Model to the trinification group $SU(3)_L \times SU(3)_R \times SU(3)_C$ augmented by the generation symmetry $SO(3)_G$ under which all fermions are 3-vectors in generation space. In our phenomenological approach essential use is made of the vacuum expectation values of scalar Higgs fields. They provide the spontaneous symmetry breaking down to the Standard Model and finally to $U(1)_e$. Phenomenological tree-level potentials have been constructed giving mass to all fields apart from the 15 would-be Goldstone particles. Due to the combination of different matrix fields required to obtain finite masses, the mass eigenstates have either fermiophobic components or parts that induce flavor changing processes. Within a suitable range of parameters a notable exception is the Higgs-like state at 125 GeV. It has only a very small fraction of those components and is thus barely different from the Standard Model Higgs. However, already the next higher states will strongly differ from usual Higgs-like bosons. These new bosons, if existing at all, would have interesting properties and would allow the study of exciting new processes.

As in [10, 11] an effective Yukawa interaction is proposed that, beside flavor singlet Higgs fields, contains two flavor (generation) matrices G and A. G determines the mass hierarchy of all fermions and A all mixings. The difference between the up quark spectrum and the spectrum of down quarks as well as the structure of the CKM matrix is related to the mixing of fermions with heavy states present in the group representation. Also the form of the neutrino mass matrix is determined by this mixing. Our model leads to an inverted neutrino hierarchy. With the measured atmospheric mass squared difference as input and having only one fit parameter, a satisfactory result for the experimentally observed solar neutrino mass difference and the neutrino mixing pattern is achieved.

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