

The Mathematica package for calculation of one-loop penguin diagrams for FCNC processes

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In this work, we present a Mathematica package Peng4BSM@LO which calculates the contributions to the Wilson Coefficients of certain effective operators originating from the one-loop penguin Feynman diagrams. The key feature of our package is the ability to find the corresponding expressions in almost any New Physics model which extends the SM and has no flavour changing neutral current (FCNC) transitions at tree level.

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1. Introduction

The flavor changing neutral current (FCNC) processes attract a lot of interest from both the theoretical and experimental side. Such transitions are absent in the Standard Model (SM) at the tree level and, thus, are suppressed in comparison to the charged current processes. Due to this FCNC can be used as an excellent probe of New Physics, which can considerably alter the predictions of the SM. At the moment, no significant deviations are found. Consequently, these rare processes impose very important constraints on Beyond-the-SM (BSM) physics. Typical examples are the $b \rightarrow s\gamma$ [1],[2],[3] and $B_s \rightarrow \mu\mu$ decays [4],[5],[6] which are used in different studies of various supersymmetric extensions of the SM. Since the new particles predicted by the BSM models are usually much heavier than the SM ones, the corresponding (short-distance) contribution to FCNC amplitudes can be absorbed into Wilson coefficients of the operators which enter the weak effective Hamiltonian.

In Sec.2, we present the generic diagrams and operators from the effective Hamiltonian. In Sec.3, we introduce the programming instruments and our method to define and calculate the Wilson coefficients and describe and explain the structural diagram of the package Peng4BSM@LO. In Sec.4, we test the Peng4BSM@LO package comparing its results for Z and γ exchange diagrams in the SM with the ones given in [7].

2. Generic diagrams and operators

In Peng4BSM@LO, we consider the following generic effective local operators and their form factors, $\left(E_{a,b}^{0,c}\right)_{L,R}$, $\left(E_{a,b}^{2,c}\right)_{L,R}$, $\left(M_{a,b}^{1,c}\right)_{L,R}$. The monopole operators which conserve chirality are of the form

$$\left(\bar{F}'_a \gamma^\mu P_{L,R} F_b V_\mu^c\right) \left(E_{a,b}^{0,c}\right)_{L,R}, \quad \left(\bar{F}'_a \gamma^\mu P_{L,R} F_b\right) \left(g^{\mu\nu} q^2 - q^\mu q^\nu\right) V_\nu^c \left(E_{a,b}^{2,c}\right)_{L,R}, \quad (2.1)$$

and the dipole operators which flip the chirality are

$$\left(\bar{F}'_a \sigma^{\mu\nu} P_{L,R} F_b q_\mu V_\nu^c\right) \left(M_{a,b}^{1,c}\right)_{L,R}, \quad (2.2)$$

where q_μ is the outgoing momentum of a neutral vector boson V entering the operator, F , F' are fermions of different families, a , b , c are some, e.g., color indices and $P_{L,R} = (1/2)(1 \mp \gamma^5)$ are the projection operators. From the form factors one can easily extract the corresponding Wilson coefficients. This kind of operators originate from expansion of the penguin amplitude over external momenta. The corresponding generic Feynman diagrams are presented in Fig. 1. In these diagrams, external particles correspond to quantum fields entering the considered operators and should be specified for each particular operator. Internal particles are automatically distributed by the package according to the chosen BSM model.

3. Structure of Peng4BSM@LO

In order to calculate the Wilson coefficients at low energy level for a particular FCNC process in supersymmetric or any two-higgs doublet models, one can use different codes available on the

market, e.g., SuperIso [8] (see also, Ref. [9]). We created a Mathematica package Peng4BSM@LO which can be used together with FeynArts¹ [10],[11],[12] and FeynCalc² [13]. Contrary to the above-mentioned packages our code gives an opportunity to obtain an expression for the Wilson coefficients in almost any model, which can be implemented with the help of FeynRules' [14] packages.

Our paradigm is based on the fact that the Lorentz structure of Feynman vertices is fixed only by the type of participating particles so one can define and calculate generic Wilson coefficients originating from generic diagrams. The corresponding amplitudes involve generic couplings which can be substituted later by actual expressions.

With the help of FeynArts we generate all generic diagrams which are shown in Fig.1. Then we reformulate the generic amplitudes by means of FeynCalc in terms of the fundamental integrals of Passarino-Veltman's [15], which we expand in the limit of vanishing external momenta in order to obtain the generic Wilson coefficients. These generic Wilson coefficients serve as templates and can be used not only in a particular model but also in any model. Given the corresponding expressions, the substitution rules for particular diagrams and amplitudes can be applied and the Wilson coefficients for a particular model can be obtained. As we see from Fig.2, in order to obtain a contribution to $E^{0,2}$ or M^1 , one should define the model and the corresponding operator with external particles F_a, F_b, V_c from Eq.2.1 and Eq.2.2. Given this input, the package can be used in the following way. First of all, one should construct the relevant Feynman diagrams by calling "InsertFields[**PenguinTopologies**, $\{F_b\} \rightarrow \{F_a, V_c\}$, *Model* \rightarrow MOD]"³. Here "**PenguinTopologies**"⁴ is a list of topologies for Penguin-type diagrams in FeynArts notation. The model is defined by the option "Model \rightarrow "MOD" of "InsertFields". The corresponding Feynman rules in FeynArts notation are taken from the file "MOD.mod". Then, the penguin amplitudes are produced by "**PengCreateFeynAmp**"⁵ from the diagrams created with "InsertFields". After that, one should apply "**ExtractPenguinSubsRules**" to produce a list of substitution rules for each diagram (amplitude) generated by "**PengCreateFeynAmp**". The rules specify the actual couplings for each generic diagram (amplitude). They can be used later together with "**SubstituteMassesAndFeynmanRules**"⁶ to obtain final analytic expressions in the considered model.

4. Test of Peng4BSM@LO

We compared the results of application Peng4BSM@LO to induced $d\bar{s}Z$ vertex in the SM with those given in paper [7] and got the perfect agreement. For this particular case, the monopole operator is $\bar{s}_L \gamma^\mu d_L Z_\mu$ as in Eq.2.6 of [7]. We also consider the γ exchange diagrams for induced $d\bar{s}\gamma$ vertex for which the corresponding generic diagrams are the same as in Fig.1 with replacement of

¹A Mathematica package for the generation and visualization of Feynman diagrams and amplitudes.

²A Mathematica package for algebraic calculations in elementary particle physics.

³We state the elements of PHI, FeynArts and FeynCalc just in quotes.

⁴We state the elements of Peng4BSM@LO in bold case and in quotes.

⁵Similar to builtin CreateFeynAmp[*dia*] in FeynArts, where *dia* is list of diagrams.

⁶SubstituteMassesAndFeynmanRules[OP_][F_a, F_b, V_c][*rules*_...] gives a contribution to the coefficient function of the operator specified by OP, given the diagram substitution rules extracted by "**ExtractPenguinSubsRules**".

the outgoing Z-boson by γ quanta. The corresponding operators for the induced $d\bar{s}\gamma$ vertex are the monopole operators, $A_\mu \bar{s} (q^2 \gamma_\mu - q_\mu \not{q}) P_L d$ and the dipole operators, $\bar{s} \sigma_{\mu\nu} i q^\nu (m_s P_L + m_d P_R) d A_\mu$ as in Eq.B.1 of [7]. The consistency checks of Peng4BSM@LO are as follows. There are no UV-divergencies in the considered form-factor. For the case of Z boson there is no RR transition. For the case of γ quanta both $E_{L,R}^0 = 0$. As an example, we present the expression for E_L^0 corresponding to the monopole operator with external Z-boson:

$$\begin{aligned}
(E_{a,b}^{0,c})_L &= \frac{1}{256\pi^2 \cos \theta_W \sin^3 \theta_W (x(1)^2 - 1)^2 (x(2)^2 - 1)^2} e^3 CKM(2,1) CKM(2,2)^* \\
&\times \left\{ 4x(1)^2 [x(2)^2 - 1]^2 \times [6(2\sin^2 \theta_W + 2(1 - \sin^2 \theta_W) - 1)x(1)^2 + 4] \log x(1) \right. \\
&- [x(1)^2 - 1] \left[(x(1)^2 - x(2)^2) (x(2)^2 - 1) (\sin^2 \theta_W (3(x(2)^2 - 1)x(1)^2 - 3x(2)^2 - 23) \right. \\
&+ (1 - \sin^2 \theta_W) (3(x(2)^2 - 1)x(1)^2 - 3x(2)^2 - 23) + 7x(1)^2 - 7x(1)^2 x(2)^2 + 7x(2)^2 - 1) \\
&+ 4(x(1)^2 - 1)x(2)^2 (6(2\sin^2 \theta_W + 2(1 - \sin^2 \theta_W) - 1)x(2)^2 + 4) \log x(2) \left. \right] \Big\} \text{IndexDelta}(c1,c2) \\
&+ \frac{1}{256\pi^2 \cos \theta_W \sin^3 \theta_W (x(1)^2 - 1)^2 (x(3)^2 - 1)^2} e^3 CKM(3,1) CKM(3,2)^* \\
&\times \left\{ 4x(1)^2 [x(3)^2 - 1]^2 \times [6(2\sin^2 \theta_W + 2(1 - \sin^2 \theta_W) - 1)x(1)^2 + 4] \log x(1) \right. \\
&- [x(1)^2 - 1] \left[(x(1)^2 - x(3)^2) (x(3)^2 - 1) (\sin^2 \theta_W (3(x(3)^2 - 1)x(1)^2 - 3x(3)^2 - 23) \right. \\
&+ (1 - \sin^2 \theta_W) (3(x(3)^2 - 1)x(1)^2 - 3x(3)^2 - 23) + 7x(1)^2 - 7x(1)^2 x(3)^2 + 7x(3)^2 - 1) \\
&+ 4(x(1)^2 - 1)x(3)^2 (6(2\sin^2 \theta_W + 2(1 - \sin^2 \theta_W) - 1)x(3)^2 + 4) \log x(3) \left. \right] \Big\} \text{IndexDelta}(c1,c2).
\end{aligned} \tag{4.1}$$

Here $x(i) = m_i/M_W$, $CKM(i, j)$ is the CKM matrix with $i, j = 1, 2, 3$ are up-type quark family indices, $\text{IndexDelta}(c1, c2)$ is the Kronocker-delta in colour space with $c1$ and $c2$ being colour indices. The W-boson mass is given by M_W . The electric charge and sine function of the Weinberg angle are denoted by e and $\sin \theta_W$, respectively. The form factor for the second order of monopole operator for massless outgoing boson, in our case it is a photon, is:

$$\begin{aligned}
(E_{a,b}^{2,c})_L &= \frac{1}{1152\pi^2 M_W^2 \sin \theta_W (x(1)^2 - 1)^4 (x(2)^2 - 1)^4} e^3 CKM(2,1) CKM(2,2)^* \\
&\times \left\{ 4(3x(1)^8 - 30x(1)^6 + 54x(1)^4 - 32x(1)^2 + 8) (x(2)^2 - 1)^4 \log x(1) \right. \\
&- (x(1)^2 - 1) \left[(x(2)^2 - 1) \left((32x(2)^4 - 57x(2)^2 + 19) x(1)^6 \right. \right. \\
&+ \left. \left. (-32x(2)^6 + 75x(2)^2 - 25) x(1)^4 + (57x(2)^6 - 75x(2)^4) x(1)^2 - 19x(2)^6 + 25x(2)^4 \right) \right]
\end{aligned}$$

$$\begin{aligned}
& + 4 \left(3x(2)^8 - 30x(2)^6 + 54x(2)^4 - 32x(2)^2 + 8 \right) \left(x(1)^2 - 1 \right)^3 \log x(2) \Bigg] \Bigg\} \text{IndexDelta}(c1, c2) \\
& + \frac{1}{1152\pi^2 M_W^2 \sin \theta_W} \frac{1}{(x(1)^2 - 1)^4 (x(3)^2 - 1)^4} e^3 \text{CKM}(3, 1) \text{CKM}(3, 2)^* \\
& \times \left\{ 4 \left(3x(1)^8 - 30x(1)^6 + 54x(1)^4 - 32x(1)^2 + 8 \right) \left(x(3)^2 - 1 \right)^4 \log x(1) \right. \\
& - \left(x(1)^2 - 1 \right) \left[\left(x(3)^2 - 1 \right) \left(\left(32x(3)^4 - 57x(3)^2 + 19 \right) x(1)^6 \right. \right. \\
& + \left. \left. \left(-32x(3)^6 + 75x(3)^2 - 25 \right) x(1)^4 + \left(57x(3)^6 - 75x(3)^4 \right) x(1)^2 - 19x(3)^6 + 25x(3)^4 \right) \right. \\
& \left. \left. + 4 \left(3x(3)^8 - 30x(3)^6 + 54x(3)^4 - 32x(3)^2 + 8 \right) \left(x(1)^2 - 1 \right)^3 \log x(3) \right] \right\} \text{IndexDelta}(c1, c2)
\end{aligned} \tag{4.2}$$

Finally, the nontrivial contribution to the dipole operator for massless photon is given by

$$\begin{aligned}
& \left(M_{a,b}^{1,c} \right)_{L,R} = - \frac{1}{384\pi^2 M_W^2 \sin^2 \theta_W} \frac{1}{(x(1)^2 - 1)^4 (x(2)^2 - 1)^4} i e^3 m_{s,d} \text{CKM}(2, 1) \text{CKM}(2, 2)^* \\
& \times \left\{ \left[x(1)^2 - 1 \right] \left[\left(x(2)^2 - 1 \right) \left(\left(-29x(2)^4 + 31x(2)^2 - 8 \right) x(1)^6 \right. \right. \right. \\
& + \left. \left. \left(29x(2)^6 - 6x(2)^2 - 5 \right) x(1)^4 + \left(-31x(2)^6 + 6x(2)^4 + 7 \right) x(1)^2 \right. \right. \\
& \left. \left. + x(2)^2 \left(8x(2)^4 + 5x(2)^2 - 7 \right) \right) + 12 \left(x(1)^2 - 1 \right)^3 \left(3x(2)^2 - 2 \right) x(2)^4 \log x(2) \right] \\
& - 12x(1)^4 \left(3x(1)^2 - 2 \right) \left(x(2)^2 - 1 \right)^4 \log x(1) \Bigg\} \text{IndexDelta}(c1, c2) \\
& - \frac{1}{384\pi^2 M_W^2 \sin^2 \theta_W} \frac{1}{(x(1)^2 - 1)^4 (x(3)^2 - 1)^4} i e^3 m_{s,d} \text{CKM}(3, 1) \text{CKM}(3, 2)^* \\
& \times \left\{ \left[x(1)^2 - 1 \right] \left[\left(x(3)^2 - 1 \right) \left(\left(-29x(3)^4 + 31x(3)^2 - 8 \right) x(1)^6 \right. \right. \right. \\
& + \left. \left. \left(29x(3)^6 - 6x(3)^2 - 5 \right) x(1)^4 + \left(-31x(3)^6 + 6x(3)^4 + 7 \right) x(1)^2 \right. \right. \\
& \left. \left. + x(3)^2 \left(8x(3)^4 + 5x(3)^2 - 7 \right) \right) + 12 \left(x(1)^2 - 1 \right)^3 \left(3x(3)^2 - 2 \right) x(3)^4 \log x(3) \right] \\
& - 12x(1)^4 \left(3x(1)^2 - 2 \right) \left(x(3)^2 - 1 \right)^4 \log x(1) \Bigg\} \text{IndexDelta}(c1, c2).
\end{aligned} \tag{4.3}$$

Also, we remind that the Eq.4.3 can be rewritten in terms of Fermi constant $G_F = e^2 / (8M_W^2 \sin^2 \theta_W)$ in order to factorize the result from the Wilson Coefficient together with the CKM matrix elements. Here e is the charge of an electron, θ_W is the electroweak mixing angle of which value varies as a

function of the momentum transfer, and M_W is the mass of the W -boson which mediates the decay in question.

5. Conclusion

We present the new package which we called Peng4BSM@LO. Peng4BSM@LO is written in Mathematica as an additional package which works with FeynArts and FeynCalc. Peng4BSM@LO defines and calculates contributions to the Wilson coefficients of particular operators for the one loop penguin diagrams in FCNC processes.

By comparison of the Wilson coefficients obtained by Peng4BSM@LO in the SM for induced $d\bar{s}Z$ and $d\bar{s}\gamma$ vertices with those from [7] and we have shown that they match with each other, which serves as validity check of our code.

The next steps are the calculation of the box diagrams and to provide the ability to calculate in the next order and to control more parameters at the input level of Peng4BSM@LO.

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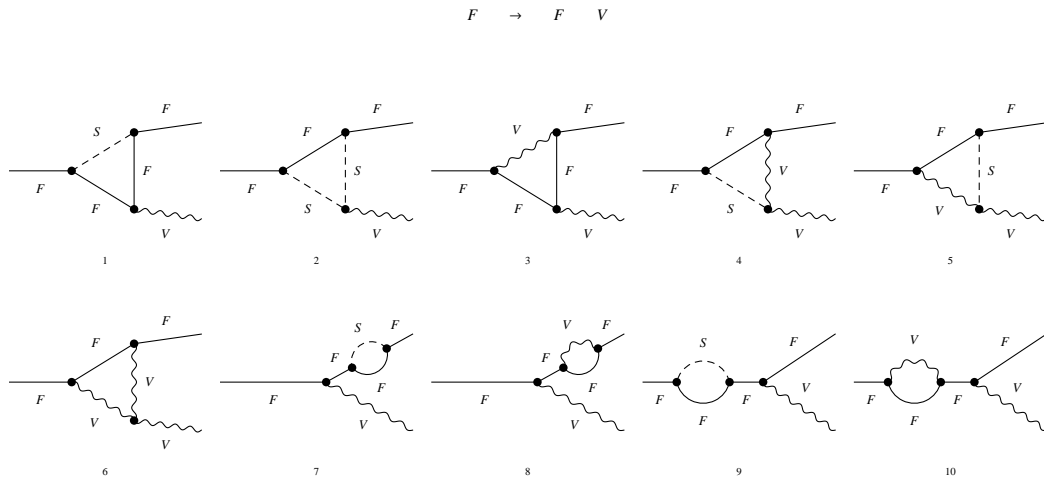


Figure 1: The one-loop generic diagrams, where F is a fermion, S is a scalar and V is a vector. We do not consider the self-energy insertions in the neutral vector boson leg, since we assume that there is no tree-level FCNC.

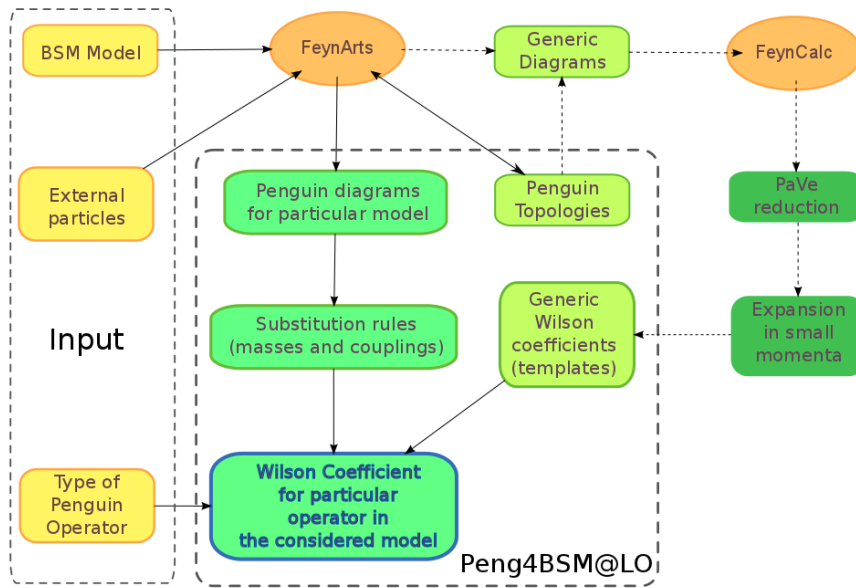


Figure 2: The new package to calculate and define Wilson coefficients for the one loop penguin diagrams in FCNC processes.