PROCEEDINGS OF SCIENCE

Non-equilibrium photon production arising from the chiral mass shift

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We investigate the photon emission arising from the shift of the quark/antiquark masses during the chiral phase transition in the early stage of ultrarelativistic heavy-ion collisions. As this mass shift leads to spontaneous creation of quark-antiquark pairs and thus contributes to the formation of the quark-gluon plasma, our investigations are relevant in the context of finite-lifetime effects on the photon emission from the latter. Earlier investigations on this topic by Boyanovsky et al. were plagued by a UV divergent contribution from the vacuum polarization and the non-integrability of the remaining contributions in the ultraviolet domain. In contrast to these investigations, we do not consider the photon number-density at finite times but for free asymptotic states obtained by an adiabatic switching of the electromagnetic interaction according to the Gell-Mann and Low theorem. This approach eliminates possible unphysical contributions from the vacuum polarization and renders the resulting photon spectra integrable in the ultraviolet domain. It is emphasized that the consideration of free asymptotic states is indeed crucial to obtain such physically reasonable results.

International Winter Meeting on Nuclear Physics,
21-25 January 2013
Bormio, Italy

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PoS(Bormio 2013)055

1. Introduction and Motivation

Ultrarelativistic heavy-ion collision experiments allow for studying strongly interacting matter under extreme conditions. One main objective of these experiments is the creation and exploration of the so-called quark-gluon plasma (QGP), a state of matter of deconfined quarks and gluons. The two basic features of the strong interaction, namely confinement [1] and asymptotic freedom [2, 3], predict that this state is created at high densities and temperatures [4, 5, 6, 7, 8], which occur during ultrarelativistic heavy-ion collisions.

The lifetime of the QGP created during a heavy-ion collision is in the order of up to 10 fm$/c$. After that, it transforms into a gas of hadrons. Thus, experiments cannot access the QGP directly, and the investigations of its properties have to rely on experimental signatures. One important category of these signatures are direct photons as electromagnetic probes. As they only interact electromagnetically, their mean free path is much larger than the spatial extension of the QGP. Therefore, they leave the QGP almost undisturbed once they have been produced and thus provide direct insight into the early stage of the collision.

One important aspect in this context is that the QGP, as it occurs in a heavy-ion collision, is not a static medium. It first thermalizes over a finite interval of time, then keeps expanding and cooling down before it hadronizes finally. This non-equilibrium dynamics has always been a major motivation for investigations on non-equilibrium quantum field theory [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. Besides the role of possible memory effects during the time evolution [21, 22, 23, 24, 25, 26, 27, 28, 29], it is of particular interest how the finite lifetime QGP itself affects the resulting photon spectra.

The first investigations on this topic by Boyanovsky et al. [30, 31, 32] have shown that the finite lifetime of the QGP gives rise to the contribution to the photon yield from first-order QED processes, i.e., processes of linear order in the electromagnetic coupling constant, α_e, which are kinematically forbidden in thermal equilibrium. Furthermore, the results from [30, 31, 32] suggested that such non-equilibrium contributions dominate over leading-order thermal contributions in the ultraviolet (UV) domain. The latter contributions are linear both in the electromagnetic coupling constant, α_e, and the strong coupling constant, α_s, and hence of overall second order.

On the other hand, however, the investigations in [30, 31, 32] had been accompanied by two artifacts. In the first place, the vacuum polarization was found to lead to a divergent contribution to the photon number-density for given photon energy. Furthermore, the remaining contributions to this quantity did not decay fast enough with increasing photon energy for being integrable in the UV domain. In particular, the total number density and the total energy density of the emitted photons (after subtracting the contribution from the vacuum polarization) were logarithmically and linearly divergent, respectively.

Later on, the topic was also picked up by Fraga et al. [33, 34] where the ansatz used in [30, 31, 32] was considered as doubtful, as it came along with the mentioned problems. In particular, the concerns raised in [33, 34] were the following:

- In [30, 31, 32] the time at which the photons are observed has been kept finite. Either this corresponds to measuring photons that are not free asymptotic states or it corresponds to suddenly turning off the electromagnetic interaction at this point in time. Both cases are questionable.

- The initial state in [30, 31, 32] has been specified for a system of thermalized quarks, antiquarks and gluons not containing any photons. On the other hand, however, a system of quarks and gluons, which undergoes electromagnetic interactions, necessarily contains photons. Hence, taking an initial state without any photons and without the Hamiltonian for electromagnetism corresponds to switching on the electromagnetic interaction at the initial time, which is questionable as well. It was shown in [35] that the ansatz used in [30, 31, 32] is indeed equivalent to such a scenario.

- The divergent contribution from the vacuum polarization is unphysical and thus needs to be renormalized. Nevertheless, the renormalization procedure presented in [32] is not coherent since no derivation of the photon yield with rescaled field operators has been presented in [32].

The authors of [33, 34] did, however, not provide an alternative approach for how to handle the mentioned problems in a consistent manner. Solely in [35] it was indicated that the question of finite-lifetime effects could be addressed within the 2PI (two-particle irreducible) approach even though the conservation of gauge invariance remains challenging. Later on Boyanovsky et al. insisted on their approach [36] and objected to the arguments by [33, 34] as follows:

- Non-equilibrium quantum field theory is an initial-value problem. This means that the density matrix of the considered system is first specified at some initial time and then propagated to a later time by the time-evolution operator. For that reason, the Hamiltonian is not modified by introducing a time-dependent artificial coupling as it would be the case for a 'switching on' and a later 'switching off' of the electromagnetic interaction.

- The quark-gluon plasma, as it occurs in a heavy-ion collision, has a lifetime of only a few fm/c. Therefore, taking the time to infinity is unphysical as this limit requires the inclusion of non-perturbative phase-transition effects on the photon production.

- The renormalization technique of [36] provides a rescaling of the photon field operators such that the photon number operator actually counts asymptotic photon states with amplitude one.

During this debate, however, the original problems encountered in [30, 31, 32] remained open. In order to resolve them in a satisfactory manner, we have followed two different approaches.

In the first approach [37], we have modeled the finite lifetime of the (thermalized) quark-gluon plasma during a heavy-ion collision by time-dependent occupation numbers in the photon self-energy. By means of this procedure, we have been able to renormalize the divergent contribution from the vacuum polarization in a consistent manner. However, it has not been possible to get the problem with the UV behavior fully under control.

We have expected in the first place that this remaining shortcoming results from a violation of the Ward-Takahashi identities. For that reason, we have also considered a conceptionally different scenario [38] where the production of quark-antiquark pairs together with photons results from a change in the quark-antiquark mass. Such a change occurs during the chiral phase transition in the very early stage of a heavy-ion collision. It has been shown in [39, 40] that it leads to the spontaneous and non-perturbative pair production of quarks and antiquarks, which effectively contributes to the formation of the QGP. We have investigated the photon production arising from this pair-creation process. As this photon production is effectively induced by the change of the quark/antiquark mass and thus by the chiral phase transition, it is from now on referred to as chiral photon production.

In contrast to [37], such a scenario has the crucial advantage that it allows for a first-principle description by introducing a Yukawa-like source term in the QED Lagrangian. This source term couples the quarks and antiquarks to a time-dependent, scalar background field. In this way, they obtain a time-dependent mass, which conserves the Ward-Takahashi identities. As in [37], we have restricted ourselves to first-order QED processes. They are kinematically possible since the quarks and antiquarks obtain additional energy by the coupling to the background field. To the contrary, the coupling to the background field is resummed to all orders as to properly take into account the non-perturbative nature of the pair-creation process.

In this context, there is another crucial difference to the approaches in [30, 31, 32, 37]: There the photon number-density has been considered at finite times. In the course of our investigations [38], however, we have shown that the photon number-density has to be extracted in the limit $t \to \infty$ for free asymptotic states, which are the only observable ones since they reach the detectors. Such states have been obtained by introducing an adiabatic switching of the electromagnetic interaction according to the Gell-Mann and Low theorem. The photon number-density is then considered in the asymptotic time limit and the switching parameter is taken to zero at the very end of our calculation.

We shall demonstrate that this procedure eliminates a possible unphysical vacuum contribution and, furthermore, leads to photon spectra being integrable in the ultraviolet domain if the time evolution of the quark/antiquark mass is modeled in a physically reasonable manner. It is also emphasized that the consideration of the photon number-density for free asymptotic states is indeed essential to obtain such physically reasonable results.

2. Model description

Before we turn to our first-principle description on chiral photon production, we first provide a short insertion on our model description [37], where we have made an ansatz on the two-time dependence of the photon self-energy. For this purpose, we have taken into account that the vacuum contribution to the photon self-energy is always persistent, whereas the medium contribution only occurs as long as the QGP is actually present. These two aspects had been disregarded in [30, 31, 32]. This time dependence has been in implemented by introducing time-dependent quark/antiquark occupation numbers

$$n_{\mathrm{F}}(E) \to n_{\mathrm{F}}(E,t) = f(t) n_{\mathrm{F}}(E) , \tag{2.1}$$

in the photon self-energy. Here $n_{\mathrm{F}}(E)$ denotes the Fermi-Dirac distribution at the given temperature, T, and $f(t)$ is the function modeling its time evolution. This function changes monotonously from zero for $t \to -\infty$ (vacuum) to one for $t \to \infty$ (QGP fully persistent) over an assumed formation time interval, τ. As in [30, 31, 32], we have considered the photon number-density at finite times.

By means of this procedure, we have been able to renormalize the divergent contribution from the vacuum polarization in a consistent manner. For our numerical investigations on the remaining medium contributions, we have compared the resulting photon spectra for different switching functions, $f_i(t)$, which are depicted in Figure 1. As one can see, $f_1(t)$ describes an instantaneous formation of the QGP at $t = 0$, whereas $f_2(t)$ describes a formation over a finite interval of time, τ, for which we have assumed $\tau = 1.0$ fm$/c$.

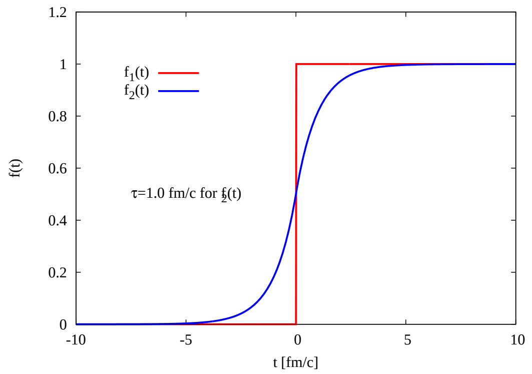

Figure 1: The time evolution of the QGP is modeled by different switching functions, $f_i(t)$.

It has been our initial hope that in addition to the renormalization of the vacuum contribution our model description would also lead to UV integrable photon spectra if the the formation of the QGP is assumed to occur over a finite interval of time. Such an assumption is reasonable from the phenomenological point of view. A comparison of the photon spectra for the different switching functions, which is done in Figure 2, does, however, show that this is only partly the case.

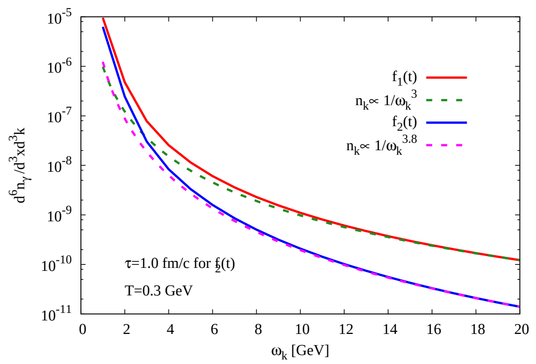

Figure 2: Comparison of the photon spectra for different switching functions. For $f_1(t)$ describing an instantaneous formation, the photon spectrum decays as $1/\omega_{\vec{k}}^3$ and exhibits only a slightly steeper decay $\propto 1/\omega_{\vec{k}}^{3.8}$ for $f_2(t)$. $\omega_{\vec{k}} = |\vec{k}|$ denotes the photon energy for given photon three-momentum, $\vec{k}$.

For the case of an instantaneous formation the photon number-density scales as $1/\omega_{\vec{k}}^3$ (with $\omega_{\vec{k}} = |\vec{k}|$ denoting the photon energy for given photon three-momentum, $\vec{k}$) in the UV domain. This implies that the total number density and the total energy density of the radiated photons are logarithmically and linearly divergent, respectively. This artifact is only partly resolved if we turn to a formation over a finite interval of time describing a physically more realistic scenario. In this case, the UV scaling behavior of the photon number-density is only suppressed to $1/\omega_{\vec{k}}^{3.8}$ such that only the total number density is rendered UV finite whereas the total energy density remains UV divergent. For our numerical investigations, we have chosen $f_2(t)$ to be continuously differentiable once but one can show that this artifact persists for arbitrarily smooth switching functions.

We have assumed in the first place that the still problematic UV behavior arises from a violation of the Ward-Takahashi identities for the photon self-energy within our model description [37]. Therefore, we aimed to find a new approach in which these identities are conserved. For that reason, we have eventually studied chiral photon production as this scenario allows for an accordant first-principle description.

3. Chiral photon production

We model the change of the quark/antiquark mass during the chiral phase transition by introducing a Yukawa-like source term in the QED Lagrangian

$$\hat{\mathscr{L}}(x) = \hat{\mathscr{L}}_{\mathrm{QED}}(x) - g\phi(t)\hat{\bar{\psi}}(x)\hat{\psi}(x) \ . \tag{3.1}$$

The source field, $\phi(t)$, is assumed to be classical and time-dependent only, which effectively assigns the quarks and antiquark a time-dependent mass

$$m(t) = m_c + g\phi(t) \ . \tag{3.2}$$

Since such an ansatz keeps the Lagrangian gauge invariant it conserves the Ward-Takahashi identities of QED, which we will demonstrate explicitly below for the photon self-energy.

The change of the quark/antiquark mass from its constituent value, m_c, to its bare value, m_b leads to the spontaneous and non-perturbative pair creation of quarks and antiquarks. We investigate the photon emission resulting from this pair-creation process. Thereby, we assume that our system does not contain any quarks/antiquarks or photons initially. The initial density matrix is hence given by

$$\hat{\rho}(t_0) = \left|0_{q\bar{q}}\right\rangle \otimes \left|0_\gamma\right\rangle \ . \tag{3.3}$$

Here $\left|0_{q\bar{q}}\right\rangle$ and $\left|0_\gamma\right\rangle$ denote the vacuum states of the fermionic and the photonic sector, respectively. As the quark/antiquark mass changes with time, it is important to point out that the former is defined with respect to the initial constituent mass, m_c. The initial time, t_0, is chosen from the domain where the quark/antiquark mass is still at this value, i.e., $t_0 < t_0'$ with t_0' denoting the time at which the change of the quark/antiquark mass begins. In the case of parameterizations, $m(t)$, where the time derivative, $\dot{m}(t)$, has a non-compact support, it is sufficient to ensure that one has $|g\phi(t)| \ll m_c$ for $t \le t_0'$.

Since the quark mass is time-dependent only and our system is initially given by the vacuum state (3.3), it is spatially homogeneous. For such a system, the photon number-density is formally given by

$$2\omega_{\vec{k}} \frac{\mathrm{d}^6 n_\gamma(t)}{\mathrm{d}^3x\mathrm{d}^3k} = \frac{1}{(2\pi)^3 V} \sum_{\lambda=\perp} \left\langle \hat{a}^\dagger(\vec{k},\lambda,t)\hat{a}(\vec{k},\lambda,t) \right\rangle \ . \tag{3.4}$$

The sum runs over all physical (transverse) polarizations and the average is taken with respect to the initial density matrix (3.3). Moreover, $\hat{a}(\vec{k},\lambda,t)$ together with its Hermitian conjugate corresponds to the expansion coefficients in the plane-wave decomposition of the photon-field operator, $\hat{A}_\mu(\vec{x},t)$, i.e.,

$$\hat{A}_\mu(x) = \sum_\lambda \int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{\vec{k}}}}\left[\varepsilon_\mu(\vec{k},\lambda)\hat{a}(\vec{k},\lambda,t)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} + \varepsilon^*_\mu(\vec{k},\lambda)\hat{a}^\dagger(\vec{k},\lambda,t)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}}\right] . \tag{3.5}$$

Expression (3.4) for the photon number-density has also been used in [30, 31, 32] with the initial density matrix, $\hat{\rho}(t_0)$, being instead specified for a thermalized QGP not containing any photons. However, before one can start with any further evaluation and numerical analysis of (3.4), one has to clarify under which circumstances $\hat{a}(\vec{k},\lambda,t)$ and its Hermitian conjugate actually allow for an interpretation as a single-photon operator. Such an interpretation is not given in general since $\hat{A}_\mu(\vec{x},t)$ describes an interacting electromagnetic field. It is, however, possible in the asymptotic time limit $t\rightarrow\infty$ for free asymptotic fields. Such fields are obtained by introducing an adiabatic switching of the electromagnetic interaction according to the Gell-Mann and Low Theorem, i.e.,

$$\hat{H}_{\mathrm{EM}}(t) = e\int \mathrm{d}^3x\; \hat{\bar{\psi}}(x)\gamma_\mu\hat{\psi}(x)\hat{A}^\mu(x) \rightarrow \mathrm{e}^{-\varepsilon|t|}\hat{H}_{\mathrm{EM}}(t) , \quad \text{with} \quad \varepsilon > 0 . \tag{3.6}$$

In order to obtain physically well-defined results for the photon number-density, we have to specify our initial state for $t_0\rightarrow-\infty$ and consider (3.4) in the limit $t\rightarrow\infty$ for free asymptotic states. This procedure is illustrated in Figure 3. Since the introduction of an adiabatic switching of the electromagnetic interaction per se is an artificial procedure we have to take $\varepsilon\rightarrow 0$ at the very end of our calculation.

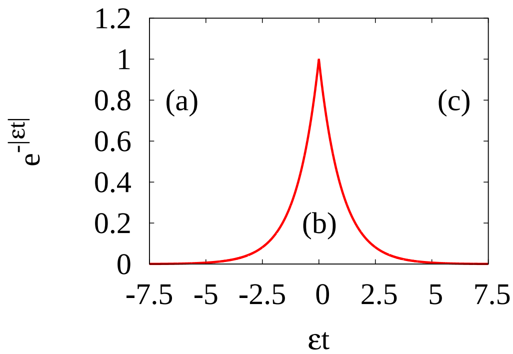

Figure 3: Schematic representation of our asymptotic description. The initial state is specified for $t_0\rightarrow-\infty$ (a). From the adiabatic switching of the electromagnetic interaction, one then has interacting fields around $t=0$ (b) which evolve into free fields in the asymptotic time limit $t\rightarrow\infty$ (c), where the photon number-density is considered. Only at the very end of our calculation, the switching parameter, ε, is taken to zero.

As the electromagnetic coupling is small, we evaluate (3.4) to first order in α_e, but keep all orders in g. The latter is achieved by constructing an interaction-picture representation that incorporates the source term [38]. Together with (3.6), this procedure leads to

$$2\omega_{\vec{k}}\frac{\mathrm{d}^6 n^\varepsilon_\gamma(t)}{\mathrm{d}^3x\mathrm{d}^3k} = \frac{1}{(2\pi)^3}\int_{-\infty}^{t}\underline{\mathrm{d}t_1}\int_{-\infty}^{t}\underline{\mathrm{d}t_2}\mathrm{i}\Pi^<_{\mathrm{T}}(\vec{k},t_1,t_2)\mathrm{e}^{\mathrm{i}\omega_k(t_1-t_2)} , \tag{3.7}$$

with the underlining denoting that

$$\int_{-\infty}^{t} \underline{\mathrm{d}u} = \int_{-\infty}^{t} \mathrm{d}u \, \mathrm{e}^{-\varepsilon|u|} \,.$$

Moreover, $\mathrm{i}\Pi_{\mathrm{T}}^{<}(\vec{k},t_1,t_2)$ describes the transverse part of the photon self-energy, i.e.,

$$\mathrm{i}\Pi_{\mathrm{T}}^{<}(\vec{k},t_1,t_2) = \gamma^{\mu\nu}(\vec{k})\mathrm{i}\Pi_{\nu\mu}^{<}(\vec{k},t_1,t_2) \,. \tag{3.8}$$

$\gamma^{\mu\nu}(\vec{k})$ is the photon polarization tensor reading

$$\gamma^{\mu\nu}(\vec{k}) = \sum_{\lambda=\perp} \varepsilon^{\mu,*}(\vec{k},\lambda)\varepsilon^{\nu}(\vec{k},\lambda) = \begin{cases} -\eta^{\mu\nu} - \frac{k^\mu k^\nu}{\omega_{\vec{k}}^2} & \text{for} \quad \mu,\nu \in \{1,2,3\} \\ 0 & \text{otherwise} \end{cases} \,, \tag{3.9}$$

with $\eta^{\mu\nu} = \mathrm{diag}\,\{1,-1,-1,-1\}$. The photon self-energy, $\mathrm{i}\Pi_{\mu\nu}^{<}(\vec{k},t_1,t_2)$, is in turn given by the one-loop approximation

$$\mathrm{i}\Pi_{\mu\nu}^{<}(\vec{k},t_1,t_2) = e^2 \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \mathrm{Tr}\left\{ \gamma_\mu S_{\mathrm{F}}^{<}(\vec{p}+\vec{k},t_1,t_2)\gamma_\nu S_{\mathrm{F}}^{>}(\vec{p},t_2,t_1) \right\} \,, \tag{3.10}$$

with the propagators fulfilling the equations of motion

$$\left[\mathrm{i}\gamma^0 \partial_{t_1} + \gamma^j p_i - m(t_1)\right] S_{\mathrm{F}}^{\lessgtr}(\vec{p},t_1,t_2) \;=\; 0 \,, \tag{3.11a}$$

$$\left[\mathrm{i}\gamma^0 \partial_{t_2} - \gamma^j p_i + m(t_2)\right] S_{\mathrm{F}}^{\lessgtr}(\vec{p},t_1,t_2) \;=\; 0 \,. \tag{3.11b}$$

It follows from these equations that the photon self-energy (3.10) fulfills the Ward-Takahashi identities

$$\partial_{t_1} \mathrm{i}\Pi_{0\mu}^{<}(\vec{k},t_1,t_2) - \mathrm{i}k^j \mathrm{i}\Pi_{j\mu}^{<}(\vec{k},t_1,t_2) = 0 \,, \tag{3.12a}$$

$$\partial_{t_2} \mathrm{i}\Pi_{\mu 0}^{<}(\vec{k},t_1,t_2) + \mathrm{i}k^j \mathrm{i}\Pi_{\mu j}^{<}(\vec{k},t_1,t_2) = 0 \,. \tag{3.12b}$$

The one-loop approximation for the photon self-energy includes the processes of first order in α_e, which is shown in Figure 4. In particular, these processes are (one-body) quark/antiquark Bremsstrahlung, quark-antiquark pair annihilation into a single photon and the spontaneous creation of a quark-antiquark pair together with a photon out of the vacuum. They are kinematically possible since the quarks and antiquarks obtain additional energy by the coupling to the background field, $\phi(t)$.

As the above diagrammatic arguments indicate, the photon number-density (3.7) can be written as the absolute square of a first-order QED transition amplitude and, as a consequence, is positive semidefinite. To demonstrate this explicitly, we first undo the contraction (3.8) in (3.7), which leads to

$$2\omega_{\vec{k}} \frac{\mathrm{d}^6 n_\gamma^\varepsilon(t)}{\mathrm{d}^3x \mathrm{d}^3k} = \frac{\gamma^{\mu\nu}(\vec{k})}{(2\pi)^3} \int_{-\infty}^{t} \underline{\mathrm{d}t_1} \int_{-\infty}^{t} \underline{\mathrm{d}t_2} \mathrm{i}\Pi_{\nu\mu}^{<}(\vec{k},t_1,t_2) \mathrm{e}^{\mathrm{i}\omega_k(t_1-t_2)} \,. \tag{3.13}$$

As the next step, we expand the fermion propagators in terms of positive- and negative-energy wavefunctions, $\psi_{\vec{p},s\uparrow\downarrow}(t)$ for given momentum, $\vec{p}$, and spin, s, i.e.,

$$S_{\mathrm{F}}^{>}(\vec{p},t_1,t_2) \;=\; -\mathrm{i}\sum_s \psi_{\vec{p},s,\uparrow}(t_1)\,\bar{\psi}_{\vec{p},s,\uparrow}(t_2) \,, \tag{3.14a}$$

$$S_{\mathrm{F}}^{<}(\vec{p},t_1,t_2) \;=\; \mathrm{i}\sum_s \psi_{\vec{p},s,\downarrow}(t_1)\,\bar{\psi}_{\vec{p},s,\downarrow}(t_2) \,. \tag{3.14b}$$

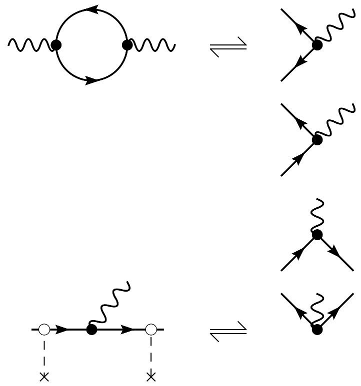

Figure 4: The photon self-energy, $\mathrm{i}\Pi^{<}_{\mathrm{T}}(\vec{k},t_1,t_2)$, is given by the one-loop approximation including the first-order QED processes. These processes are quark Bremsstrahlung (a), antiquark Bremsstrahlung (b), quark-antiquark pair annihilation into a single photon (c), and the spontaneous creation of a quark-antiquark pair together with a photon out of the vacuum (d). These processes are possible because of the coupling of the quarks and antiquarks to the background field, $\phi(t)$. This coupling is depicted by the open circles as to distinguish it from the electromagnetic coupling depicted by the full circles.

These wavefunctions fulfill the equation of motion

$$\left[\mathrm{i}\gamma^0\partial_t+\gamma^i p_i-m(t)\right]\psi_{\vec{p},s,\uparrow\downarrow}(t)\ , \tag{3.15}$$

with the initial conditions

$$\psi_{\vec{p},s,\uparrow}(t)\to\psi^c_{\vec{p},s,\uparrow}(t)\quad\text{for}\quad t\le t_0^{'}\ , \tag{3.16a}$$

$$\psi_{\vec{p},s,\downarrow}(t)\to\psi^c_{\vec{p},s,\downarrow}(t)\quad\text{for}\quad t\le t_0^{'}\ . \tag{3.16b}$$

Here $\psi^c_{\vec{p},s,\uparrow\downarrow}(t)$ denote the positive- and negative-energy spinors with respect to the initial, constituent mass, m_c. Upon insertion of (3.8) and relations (3.14) into (3.13), we can finally rewrite the photon number-density as

$$2\omega_{\vec{k}}\frac{\mathrm{d}^6n^{\varepsilon}_{\gamma}(t)}{\mathrm{d}^3x\mathrm{d}^3k}=e^2\sum_{r,s,\lambda}\frac{\mathrm{d}^3p}{(2\pi)^3}\left|\int_{-\infty}^{t}\underline{\mathrm{d}u}\ \varepsilon^{\mu}(\vec{k},\lambda)\cdot\bar{\psi}_{\vec{p},r,\uparrow}(u)\gamma_{\mu}\psi_{\vec{p}+\vec{k},s,\downarrow}(u)\mathrm{e}^{\mathrm{i}\omega_{\vec{k}}u}\right|^2\ . \tag{3.17}$$

This absolute-square representation ensures that (3.7) cannot acquire unphysical negative values. The physical photon number-density is extracted from (3.7) by taking the successive limits $t\to\infty$ and $\varepsilon\to 0$ after the time integrations have been performed, i.e.,

$$2\omega_{\vec{k}}\frac{\mathrm{d}^6n_{\gamma}}{\mathrm{d}^3x\mathrm{d}^3k}=\lim_{\varepsilon\to 0}\frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\underline{\mathrm{d}t_1}\int_{-\infty}^{\infty}\underline{\mathrm{d}t_2}\,\mathrm{i}\Pi^{<}_{\mathrm{T}}(\vec{k},t_1,t_2)\mathrm{e}^{\mathrm{i}\omega_k(t_1-t_2)}\ . \tag{3.18}$$

$\mathrm{i}\Pi^{<}_{\mathrm{T}}(\vec{k},t_1,t_2)$ reduces to the vacuum polarization if both time arguments are taken from the domain where the quark/antiquark mass is still at its initial constituent value, m_c,

$$\begin{aligned}\mathrm{i}\Pi^{<}_{\mathrm{T},0}(\vec{k},t_1,t_2)\ &=\ \mathrm{i}\Pi^{<}_{\mathrm{T},0}(\vec{k},t_1-t_2)\\ &=\ 2e^2\int\frac{\mathrm{d}^3p}{(2\pi)^3}\left\{1+\frac{px(px+k)+m_c^2}{E^c_{\vec{p}}E^c_{\vec{p}+\vec{k}}}\right\}\mathrm{e}^{\mathrm{i}\left(E^c_{\vec{p}+\vec{k}}+E^c_{\vec{p}}\right)(t_1-t_2)}\ .\end{aligned} \tag{3.19}$$

From the mass-shift effects, $\mathrm{i}\Pi_{\mathrm{T}}^{<}(\vec{k},t_1,t_2)$ will acquire an additional non-stationary contribution, i.e.,

$$\mathrm{i}\Pi_{\mathrm{T}}^{<}(\vec{k},t_1,t_2) = \mathrm{i}\Pi_{\mathrm{T},0}^{<}(\vec{k},t_1-t_2) + \mathrm{i}\Delta\Pi_{\mathrm{T}}^{<}(\vec{k},t_1,t_2) \ , \tag{3.20}$$

depending on both time arguments explicitly. When performing the successive limits $t \to \infty$ and $\varepsilon \to 0$ the contribution from the vacuum polarization (3.19) is eliminated. This can be seen by inserting (3.19) into (3.18), which leads to

$$\begin{aligned} \omega_{\vec{k}} \frac{\mathrm{d}^6 n_\gamma}{\mathrm{d}^3x\mathrm{d}^3k}\bigg|_{\mathrm{VAC}} &= \lim_{\varepsilon\to 0} \frac{e^2}{(2\pi)^3} \int \frac{\mathrm{d}^3p}{(2\pi)^3} \left\{1 + \frac{px(px+\omega_{\vec{k}}) + m_c^2}{E^c_{\vec{p}} E^c_{\vec{p}+\vec{k}}}\right\} \left[\frac{2\varepsilon}{\varepsilon^2 + \left(E^c_{\vec{p}+\vec{k}} + E^c_{\vec{p}} + \omega_{\vec{k}}\right)^2}\right]^2 \\ &< \lim_{\varepsilon\to 0} \frac{4e^2}{(2\pi)^3} \int \frac{\mathrm{d}^3p}{(2\pi)^3} \left\{1 + \frac{px(px+\omega_{\vec{k}}) + m_c^2}{E^c_{\vec{p}} E^c_{\vec{p}+\vec{k}}}\right\} \frac{\varepsilon^2}{\left(E^c_{\vec{p}+\vec{k}} + E^c_{\vec{p}} + \omega_{\vec{k}}\right)^4} \\ &= 0 \ , \end{aligned} \tag{3.21}$$

where we have taken into account that $E^c_{\vec{p}+\vec{k}} + E^c_{\vec{p}} + \omega_{\vec{k}}$ is positive definite in the second step. Therefore, only mass-shift contributions to (3.18) characterized by $\mathrm{i}\Delta\Pi_{\mathrm{T}}^{<}(\vec{k},t_1,t_2)$ remain. In this context, we would like to point out that adhering to the exact sequence of limits, i.e., taking first $t \to \infty$ and then $\varepsilon \to 0$, is indeed crucial to eliminate the contribution from the vacuum polarization. When first taking $\varepsilon \to 0$ at some finite time and $t \to \infty$ afterwards, the contribution from (3.18) instead turns into

$$\omega_{\vec{k}} \frac{\mathrm{d}^6 n_\gamma(t)}{\mathrm{d}^3x\mathrm{d}^3k}\bigg|_{\mathrm{VAC}} = \frac{e^2}{(2\pi)^3} \int \frac{\mathrm{d}^3p}{(2\pi)^3} \left\{1 + \frac{px(px+\omega_{\vec{k}}) + m_c^2}{E^c_{\vec{p}} E^c_{\vec{p}+\vec{k}}}\right\} \frac{1}{\left(E^c_{\vec{p}+\vec{k}} + E^c_{\vec{p}} + \omega_{\vec{k}}\right)^2} \ , \tag{3.22}$$

with the integral over d^3p being linearly divergent for given photon energy, $\omega_{\vec{k}}$. In the course of our numerical investigations, we furthermore demonstrate that the correct sequence of limits is also essential to obtain physically reasonable results from the mass-shift effects. Together with (3.10) and (3.11), expression (3.18) describes photon emission arising from the chiral mass shift at first order in α_e but to all orders in g.

4. Quark-antiquark pair production

Before we turn to our numerical investigations on chiral photon production, we first provide an insertion on quark-antiquark pair production arising from the chiral mass shift. Thereby, we consider the pair production arising due to the chiral mass shift only. The starting point of our investigations hence is the fermionic part of the Hamiltonian

$$\hat{H}(t) = \int \mathrm{d}^3x \ \hat{\bar{\psi}}(x) \left[-\mathrm{i}\vec{\gamma}\cdot\vec{\nabla} + m(t)\right] \hat{\psi}(x) \ , \tag{4.1}$$

with the mass function, $m(t)$, given by (3.2). As the next step, we expand the fermion-field operator, $\hat{\psi}(x)$, in terms of the positive- and negative-energy wavefunctions, $\psi_{\vec{p},s\uparrow\downarrow}(t)$, i.e.,

$$\hat{\psi}(x) = \sum_s \int \frac{\mathrm{d}^3p}{(2\pi)^3} \left[\psi_{\vec{p},s,\uparrow}(t)\hat{b}_{\vec{p},s} + \psi_{\vec{p},s,\downarrow}(t)\hat{d}^\dagger_{-\vec{p},s}\right] \mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}} \ , \tag{4.2}$$

with $\hat{b}_{\vec{p},s}$ and $\hat{d}_{-\vec{p},s}$ both annihilating the initial fermionic vacuum state, $|0_{q\bar{q}}\rangle$. Upon insertion of (4.2) into (4.1) we obtain

$$\hat{H}(t) = \sum_s \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \left\{ \Omega(t) \left[\hat{b}^\dagger_{\vec{p},s}\hat{b}_{\vec{p},s} - \hat{d}_{-\vec{p},s}\hat{d}^\dagger_{-\vec{p},s} \right] + \Lambda(t)\hat{b}^\dagger_{\vec{p},s}\hat{d}^\dagger_{-\vec{p},s} + \Lambda^*(t)\hat{d}_{-\vec{p},s}\hat{b}_{\vec{p},s} \right\} . \tag{4.3}$$

In order to keep the notation short, we have introduced

$$\Omega(t) = \bar{\psi}_{\vec{p},s,\uparrow}(t)\left[\gamma^i p_i + m(t)\right]\psi_{\vec{p},s,\uparrow}(t) = -\bar{\psi}_{\vec{p},s,\downarrow}(t)\left[\gamma^i p_i + m(t)\right]\psi_{\vec{p},s,\downarrow}(t) , \tag{4.4a}$$

$$\Lambda(t) = \bar{\psi}_{\vec{p},s,\uparrow}(t)\left[\gamma^i p_i + m(t)\right]\psi_{\vec{p},s,\downarrow}(t) . \tag{4.4b}$$

The particle number-density of quarks and antiquarks, which corresponds to the number of produced quark-antiquark pairs per unit volume, is extracted from (4.3) by diagonalizing this expression via a Bogolyubov transformation

$$\hat{\tilde{b}}_{\vec{p},s}(t) = \xi_{\vec{p},s}(t)\hat{b}_{\vec{p},s} + \eta_{\vec{p},s}(t)\hat{d}^\dagger_{-\vec{p},s} , \tag{4.5a}$$

$$\hat{\tilde{d}}^\dagger_{-\vec{p},s}(t) = \xi^*_{\vec{p},s}(t)\hat{d}^\dagger_{-\vec{p},s} - \eta^*_{\vec{p},s}(t)\hat{b}_{\vec{p},s} . \tag{4.5b}$$

As discussed in greater detail in [38], this procedure corresponds to a re-expansion of the fermion-field operator (4.2) in terms of the instantaneous eigenstates of the Hamilton density operator

$$\hat{h}_{\mathrm{D}}(t) = -\mathrm{i}\vec{\gamma}\cdot\vec{\nabla} + m(t) .$$

As a consequence, the Hamiltonian (4.1) reads in terms of the transformed creation and annihilation operators (4.5):

$$\begin{aligned}\hat{H}(t) =& \sum_s \int \frac{\mathrm{d}^3 p}{(2\pi)^3} E_{\vec{p}}(t) \left[\hat{\tilde{b}}^\dagger_{\vec{p},s}(t)\hat{\tilde{b}}_{\vec{p},s}(t) - \hat{\tilde{d}}_{-\vec{p},s}(t)\hat{\tilde{d}}^\dagger_{-\vec{p},s}(t) \right] \\ \to& \sum_s \int \frac{\mathrm{d}^3 p}{(2\pi)^3} E_{\vec{p}}(t) \left[\hat{\tilde{b}}^\dagger_{\vec{p},s}(t)\hat{\tilde{b}}_{\vec{p},s}(t) + \hat{\tilde{d}}^\dagger_{-\vec{p},s}(t)\hat{\tilde{d}}_{-\vec{p},s}(t) \right] .\end{aligned} \tag{4.6}$$

In the second step, $\hat{H}(t)$ has been normal-ordered with respect to (4.5) as to avoid an infinite negative vacuum energy. Moreover, we have introduced the dispersion relation

$$E_{\vec{p}}(t) = \sqrt{\vec{p}^2 + m^2(t)} . \tag{4.7}$$

The Bogolyubov particle number-density and thus the number of produced quark/antiquark pairs is then defined as [41]:

$$\begin{aligned}\frac{\mathrm{d}^6 n_{q\bar{q}}(t)}{\mathrm{d}^3 x \mathrm{d}^3 p} &= \frac{1}{(2\pi)^3 V} \sum_s \langle 0_{q\bar{q}} | \hat{\tilde{b}}^\dagger_{\vec{p},s}(t)\hat{\tilde{b}}_{\vec{p},s}(t) | 0_{q\bar{q}} \rangle \\ &= \frac{1}{(2\pi)^3 V} \sum_s \langle 0_{q\bar{q}} | \hat{\tilde{d}}^\dagger_{-\vec{p},s}(t)\hat{\tilde{d}}_{-\vec{p},s}(t) | 0_{q\bar{q}} \rangle \\ &= \frac{1}{(2\pi)^3} \sum_s \left| \eta_{\vec{p},s}(t) \right|^2 ,\end{aligned} \tag{4.8}$$

For our numerical investigations on the number of produced quark-antiquark pairs, we consider different mass parameterizations, which are depicted in Figure 5.

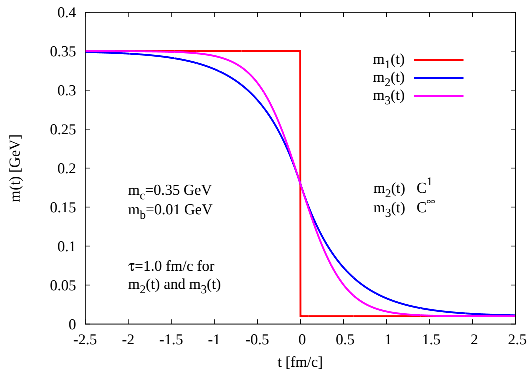

Figure 5: The change of the quark/antiquark mass from its constituent value, m_c, to its bare value, m_b, during the chiral phase transition is modeled by different mass parameterizations, $m_i(t)$.

Thereby, $m_1(t)$ describes an instantaneous change from the constituent mass, m_c, to the bare mass, m_b, at $t = 0$, whereas $m_2(t)$ and $m_3(t)$ each describe a change over a finite interval of time. The latter two mass parameterizations differ with respect to their order of differentiability. $m_2(t)$ is continuously differentiable once, whereas $m_3(t)$ is continuously differentiable infinitely many times. For both parameterizations, we assume a transition time of $\tau = 1$ fm$/c$. Figure 6 compares the asymptotic particle spectra for the different mass parameterizations, $m_i(t)$.

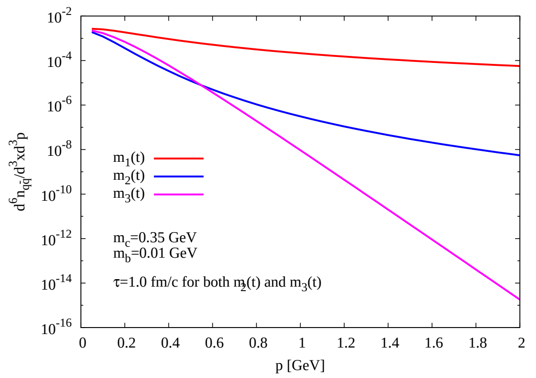

Figure 6: The decay behavior of the asymptotic particle spectra shows a strong sensitivity on the 'smoothness' of considered mass parametrization, $m_i(t)$.

For $m_1(t)$, the quark/antiquark occupation numbers scale as $1/p^2$ for $p \gg m_c, m_b$, which implies that the total number density and the total energy density of the produced quarks and antiquarks are linearly and quadratically divergent, respectively. However, this artifact is cured when turning from instantaneous mass change to a mass change over a finite interval of time. In particular, the quark/antiquark occupation numbers are suppressed to $1/p^6$ for $m_2(t)$ such that both the total number density and the total energy density are rendered finite. Moreover, if one turns from $m_2(t)$ to $m_3(t)$, which is continuously differentiable infinitely many times and hence describes the most physical scenario, the quark/antiquark occupation numbers are suppressed even further to an exponential decay. One encounters the same sensitivity on the 'smoothness' of the mass parameterization if the mass is not only changed from its constituent value, m_c, to its bare value, m_b, but also back to its constituent value after a certain interval of time as to take into account the finite lifetime of the chirally restored phase [38].

In addition to the asymptotic particle spectra, we also consider the time evolution of the quark/antiquark occupation numbers, which is shown in Figure 7.

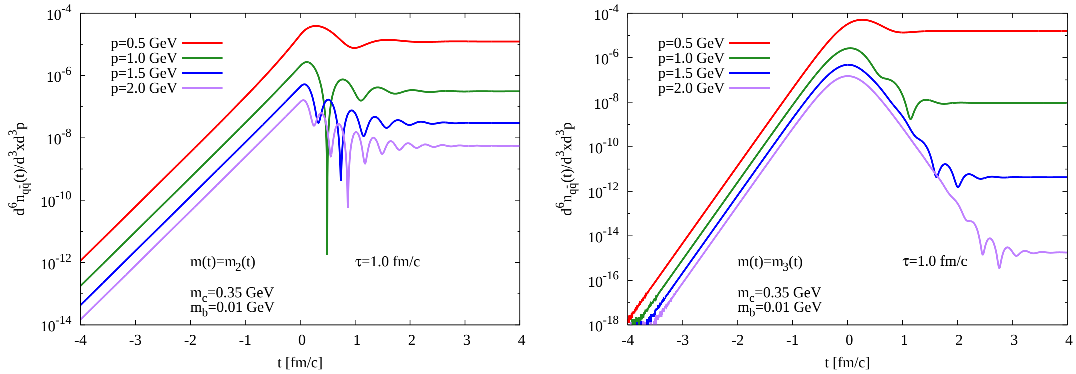

Figure 7: Time dependence of the quark/antiquark occupation numbers for $m_2(t)$ (left panel) and $m_3(t)$ (right panel) for $\tau = 1.0$ fm/c. For large values of p, they exhibit an 'overshoot' over their asymptotic value around $t = 0$, which is particularly distinctive for $m_3(t)$.

As one can see, the quark/antiquark occupation numbers exhibit a strong 'overshoot' over their asymptotic value in the region of strong mass gradients, which is especially distinctive for $m_3(t)$. In particular, the quark/antiquark occupation numbers scale as $1/p^4$ for large p when the mass is being changed from m_c to m_b. Such a scaling behavior means that the total number density is finite, whereas the total energy density is divergent. This divergence, however, only shows up in the region of strong mass gradients and disappears again as soon as the quark/antiquark mass has reached its final bare value, m_b.

At first sight, the temporary logarithmic divergence might be disturbing. In this context, it is, however, important to point out that only the asymptotic energy density, i.e., for $t \to \infty$ constitutes a well-defined physical quantity since the interpretation of (4.8) as a particle number-density is only justified for asymptotic times where $\dot{m}(t) = 0$. The reason is that the dispersion relation (4.7) then actually characterizes free and thus detectable particles, wheres it describes quasiparticles for $\dot{m}(t) \neq 0$. In particular, the asymptotic energy density is finite as long as the mass parameterization, $m(t)$, is chosen smooth enough, which represents a physically reasonable condition. Moreover, the temporary logarithmic divergence in the energy density does not manifest itself in form of a similar pathology neither in the total number density nor in the total energy density of the emitted photons.

5. Photon production

Our investigations on pair production have shown that the asymptotic quark/antiquark occupation numbers exhibit a very strong sensitivity on the 'smoothness' of the mass parameterization, $m(t)$. In particular, the asymptotic particle spectra are rendered UV integrable if the mass change is assumed to take place over a finite interval of time, τ. We now determine whether the asymptotic photon number-density exhibits a similar sensitivity. In this context, we recall that only asymptotic quantities constitute observables whereas the corresponding expressions at finite times do, in general, not allow for a similar interpretation.

For our investigations on chiral photon production, we again consider the different mass parameterizations shown in Figure 5. For the special case of an instantaneous mass shift, the loop integral entering (3.18) features a linear divergence. This divergence arises from the Bogolyubov particle numbers $\propto 1/p^2$ for large p [38]. This behavior is an artifact from the instantaneous mass shift and is removed if the mass shift is assumed to take place over a finite interval of time. Hence, from the conceptional point of view this divergence does not require a specific kind of renormalization and is regulated by a cutoff at $p = \Lambda_C$. Figure 7 shows the resulting photon spectra for the different mass parameterizations. Thereby, we have chosen $\Lambda_C = 20$ GeV for $m(t) = m_1(t)$.

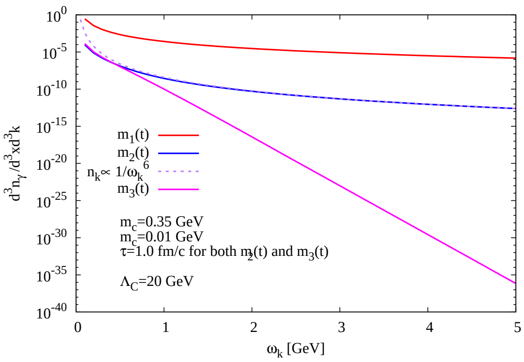

Figure 8: The decay behavior of the asymptotic photon spectra in the UV domain is highly sensitive to the 'smoothness' of the mass parameterization. In particular, they are rendered integrable in this domain if the change of the quark/antiquark mass is assumed to take place over a finite interval of time, τ.

We see that for an instantaneous mass change, the photon number-density scales as $1/\omega_{\vec{k}}^3$ for large $\omega_{\vec{k}}$, which means that for given given Λ_C, the total number density and the total energy density of the emitted photons are logarithmically and linearly divergent, respectively. In contrast to our earlier model approach [37], this artifact is now fully removed if we turn from an instantaneous mass shift to a mass shift over a finite interval of time. For $m_2(t)$, which is continuously differentiable once, the photon number-density is suppressed from $1/\omega_{\vec{k}}^3$ to $1/\omega_{\vec{k}}^6$ in the UV domain such that both the total number density and the total energy density of the emitted photons are UV finite. Moreover, if one turns from $m_2(t)$ to $m_3(t)$, which continuously differentiable infinitely many times, the photon number-density is suppressed even further to an exponential decay. The sensitivity of this quantity to the 'smoothness' of the mass parameterization is the same if the quark/antiquark mass is changed back to its constituent value, m_c, after a certain period of time as to take into account the finite lifetime of the chirally restored phase [38].

As the artifact with the UV behavior encountered in [30, 31, 32] and still partly in [37] is now resolved, it is convenient to compare our results to leading-order thermal contributions. This is done in Figure 9. Thereby, the thermal contributions have been obtained by integrating the leading-order thermal rate taken from [42] over the expected lifetime of the chirally restored phase of $\tau_L = 4.0$ fm/c at a temperature of $T = 0.2$ GeV. In this context, we note again that first-order photon production does not occur in thermal equilibrium. There the leading-order contributions are of linear order both in the electromagnetic coupling constant, α_e, and the the strong coupling constant α_s, and hence of overall second order.

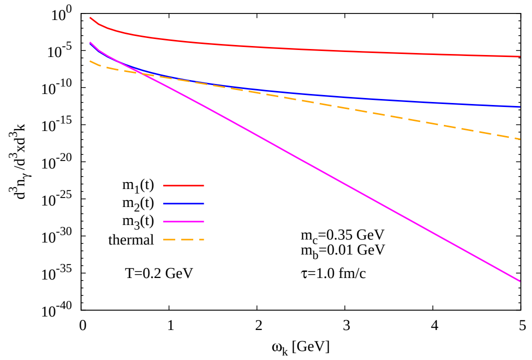

Figure 9: Comparison of first-order mass-shift contributions to leading-order thermal contributions. For $i = 3$ most likely characterizing a physical scenario, the contributions arising from the chiral mass shift are clearly subdominant for $\omega_{\vec{k}} \gtrsim 1$ GeV.

Nevertheless, we see that chiral photon production does not generally dominate over leading-order thermal photon production. For $m(t) = m_3(t)$, which describes the most physical scenario as it is continuously differentiable infinitely many times, we see that chiral photon production is even subdominant by several orders of magnitude in the UV domain.

For completeness, we would still like to demonstrate that adhering to the correct sequence of limits leading to (3.18), i.e., taking *first* $t \to \infty$ and *then* $\varepsilon \to 0$ is crucial not only to eliminate a possible unphysical contribution from the vacuum polarization, but also to obtain physically reasonable results from mass-shift effects. For this purpose, we consider the time evolution of the pure mass-shift contribution to (3.18) for different values of ε which is shown in Figure 10.

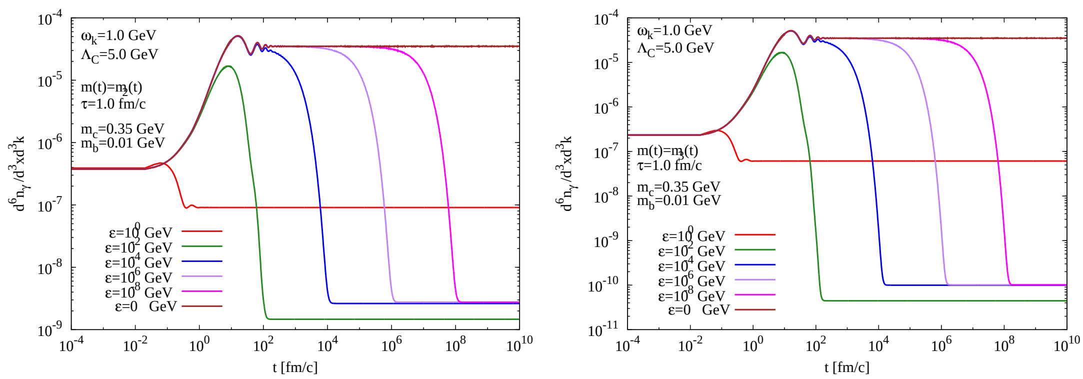

Figure 10: Time evolution of the mass-shift contribution for different values of ε for $m_2(t)$ (left panel) and $m_3(t)$ (right panel).

Taking first $\varepsilon \to 0$ at some finite time corresponds to the curve labeled by $\varepsilon = 0$ in each case. If we then take the subsequent limit $t \to \infty$, we obtain an asymptotic value for the mass-shift contribution which is by several orders of magnitudes larger than the value against which the asymptotic values for finite ε converge in the limit $\varepsilon \to 0$ and which accordingly corresponds to (3.18). The reason why the asymptotic value obtained for an interchanged sequence of limits must be unphysical is that it coincides with the transient value of the mass-shift contribution for sufficiently small values of ε. On the other hand, expression (3.7) does not allow for an interpretation as a photon number-density at finite times since one either has no free asymptotic states or one has to introduce an artificial switching-off of the electromagnetic interaction at the point of time at which (3.7) is considered. By means of the latter procedure, a violation of the Ward-Takahashi identities furthermore comes into play again [38].

The same conceptual problem arises when only using an adiabatic switching-on of the electromagnetic interaction for $t \to -\infty$ but no switching-off for $t \to \infty$. Such a procedure has been introduced in [43] as to describe intial correlations at some $t = t_0$ evolving from an uncorrelated initial state at $t \to -\infty$. In fact, the excess of the mass-shift contribution over its asymptotic value at finite times arises from spurious transient contributions that again lead to an unphysical UV scaling behavior and must be eliminated by switching the electromagnetic interaction off again according to (3.6).

In addition to the vacuum contribution and the pure mass-shift contribution, the photon self-energy (3.10) also contains a contribution which describes the interference between the vacuum and the mass-shift effects. This contribution is also eliminated by the sequence of limits leading to (3.18). A more detailed consideration can be found in [38].

6. Summary, conclusions and outlook

In this work, we have investigated photon emission during the chiral phase transition in the early stage of a heavy-ion collision. During this phase transition, the quark mass is changed from its constituent value, m_c, to its bare value, m_b, which leads to the spontaneous and non-perturbative pair production of quarks and antiquarks [39, 40]. This effectively contributes to the creation of the QGP, and we have investigated the photon emission arising from this pair-creation process. In particular, our investigations are relevant in the context of the question how the finite lifetime of the QGP during a heavy-ion collision affects the photon emission from it.

Earlier investigations on this topic had been accompanied by two artifacts. In the first place, the photon number-density contained a divergent contribution from the vacuum polarization. Furthermore, the remaining contributions to this quantity were not integrable in the UV domain. In particular, the total number density and the total energy density of the emitted photons were logarithmically and linearly divergent, respectively. It has been our original motivation to resolve these conceptional problems in a satisfactory manner.

For this purpose, we have first pursued a model approach [37] where the finite lifetime of the QGP is simulated by time-dependent quark/antiquark occupation numbers in the photon self-energy. By means of this procedure, we have been able to renormalize the divergent contribution from the vacuum polarization in a consistent manner, but it has not been possible to solve the remaining problem with the UV behavior to full extent. We have assumed that this remaining shortcoming results from a violation of the Ward-Takahashi identities. This aspect has been our eventual motivation to consider photon production arising from the change of the quark/antiquark mass during the chiral phase transition since this scenario allows for a first-principle description by which the Ward-Takahashi identities are conserved.

In particular, the change of the quark/antiquark mass has been modeled by a Yukawa-like source term in the QED Lagrangian which couples the quarks and antiquarks to a scalar and time-dependent background field. As in [30, 31, 32, 37], we have restricted ourselves to first-order QED processes. These processes are kinematically possible since the background field acts as a source of additional energy. On the other hand, the coupling to the background field has been resummed to all orders as to properly take into account the non-perturbative nature of the pair-creation process.

In contrast to [30, 31, 32, 37], we have not considered the photon number-density at finite times but for free asymptotic states. Such states have been obtained by switching the electromagnetic interaction according to the Gell-Mann and Low theorem. The photon number-density has then been considered in the asymptotic time limit $t \to \infty$ and we have taken the switching parameter to zero at the very end of our calculation. By means of this procedure, the photon number-density does not contain any unphysical contributions from the vacuum polarization and is furthermore rendered UV integrable for physically reasonable mass parameterizations.

In particular, we have shown that the consideration of this quantity for free asymptotic states is indeed crucial to obtain such physically reasonable results. The reason is that a consistent definition actually is only possible for free asymptotic states, whereas a similar interpretation of the respective formal expression is usually not justified at finite times, t, since one then does not have free asymptotic states. The same problem occurs if the electromagnetic interaction is only switched on adiabatically from $t \to -\infty$ but not switched off again for $t \to \infty$. This procedure has been suggested in [43] as to implement initial correlations at some $t = t_0$ evolving from an uncorrelated initial state at $t \to -\infty$.

Moreover, the consideration of the 'photon number-density' at finite times effectively comes along with a violation of the Ward-Takahashi identities. Our investigations hence support the respective concern raised in [33, 34] towards [30, 31, 32].

Consequently, our results indicate that the remaining artifact with the UV behavior within our model description [37] results from an inconsistent definition of the photon number-density at finite times. This in turn imposes the question whether this artifact is also removed if the photon number-density is instead considered for free asymptotic states, which are obtained by switching the electromagnetic interaction according to the Gell-Mann and Low theorem, i.e.,

$$\hat{H}_{\mathrm{EM}}(t) = e \int \mathrm{d}^3x \; \hat{\bar{\psi}}(x)\gamma_\mu \hat{\psi}(x)\hat{A}^\mu(x) \to \mathrm{e}^{-\varepsilon|t|}\hat{H}_{\mathrm{EM}}(t) \;, \quad \text{with} \quad \varepsilon > 0 \,. \tag{6.1}$$

An accordant revision of [37], for which the resulting photon spectra are depicted in Figure 11, shows that this is indeed the case.

As in [30, 31, 32], the photon number-density decays as $1/\omega_{\vec{k}}^3$ in the UV domain if the quark/antiquark occupation numbers are switched on instantaneously. If we turn from an instantaneous switching to a switching over a finite interval of time, however, the photon number-density is suppressed from $1/\omega_{\vec{k}}^3$ to $1/\omega_{\vec{k}}^7$ which means that the total number density and the total energy density of the radiated photons are both rendered UV finite. To the contrary, the UV scaling behavior is solely suppressed from $1/\omega_{\vec{k}}^3$ to $1/\omega_{\vec{k}}^{3.8}$ when turning from $f_1(t)$ to $f_2(t)$ if the 'photon number-density' is instead considered at finite times (see Figure 2). Our outlook to future investigations is hence as follows.

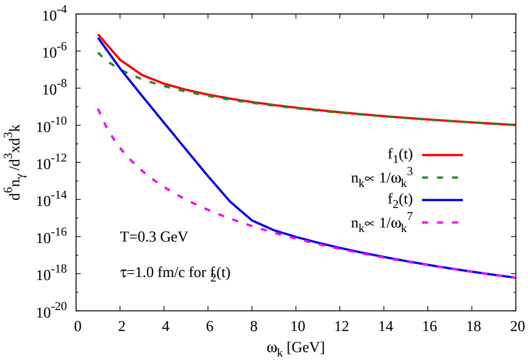

Figure 11: The photon spectra are also rendered UV integrable within our model description [37] if they are considered for free asymptotic states and if the time evolution of the QGP is modeled in a suitable manner, i.e., if its formation is assumed to take place over a finite interval of time.

In the first place, the actual role of the Ward-Takahashi identities, which are violated in [37] but conserved within our first-principle approach to chiral photon production requires deeper consideration. In particular, the fact that our model description [37] leads to UV integrable photon spectra despite the violation of these identities does not necessarily disprove our earlier conjecture concerning their role. The reason is that the Ward-Takahashi identities can be violated in two different ways:

- On the one hand, they can be violated directly by making ad-hoc assumptions on the two-time dependence of the photon self-energy, which has been the case in [37].
- On the other hand, they can also be violated indirectly by considering the 'photon number-density' at finite times even though they are formally fulfilled at first. This has been shown in [38].

Since our model description [37] leads to a UV integrable photon number-density if this quantity is considered for free asymptotic states, we hence have to determine if and, as the case may be, why only an indirect violation of the Ward-Takahashi identities, which occurs in addition to the direct violation when considering a transient 'photon number-density' within [37], leads to artificial results.

Moreover, it is of particular interest whether our previous asymptotic description allows for a consistent extension to finite times and/or which alternative quantities can be considered to describe the time evolution of the electromagnetic sector during a heavy-ion collision.

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