Mass-radius constraints for compact stars and a critical endpoint

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We present two types of models for hybrid compact stars composed of a quark core and a hadronic mantle with an abrupt first order phase transition at the interface which are in accordance with the latest astrophysical measurements of two $2\,M_\odot$ pulsars. While the first is a schematic one, the second one is based on a QCD motivated nonlocal PNJL model with density-dependent vector coupling strength. Both models support the possibility of so called *twin* compact stars which have the same mass but different radius and internal structure at high mass ($\sim 2\,M_\odot$), provided they exhibit a large jump $\Delta\varepsilon$ in the energy density of the first order phase transition fulfilling $\Delta\varepsilon/\varepsilon_{\mathrm{crit}} > 0.6$. We conclude that the measurement of high-mass twin stars would support the existence of a first order phase transition in symmetric matter at zero temperature entailing the existence of a critical end point in the QCD phase diagram.

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1. Introduction

The quest for the location of the critical endpoint (CEP) in the QCD phase diagram (PD) in heavy-ion collisions (HIC) is ongoing and has not yet yielded definite results. Suspicions are raised that there might be none(!) since that transition which is seen to be crossover at vanishing chemical potential $\mu = 0$ on the temperature axis of the PD (lattice QCD results of the Wuppertal-Budapest collaboration [1] and the HotQCD Collaboration [2] may remain of this type also at $\mu \neq 0$. Such a behaviour has been obtained in local Polyakov-Nambu-Jona-Lasinio (PNJL) models with a vector meson coupling [3, 4]. In particular, when this coupling is fixed so that the slope of the pseudocritical temperature obtained in lattice QCD simulations [5] at small μ is reproduced on the meanfield level of description [6]. In nonlocal generalizations of the PNJL model the situation is not so clear; it depends on the details of the mdel setup [7, 8]. However, as understanding confinement within these models is not possible or questionable and also the backreaction of hadronic fluctuations on the phase structure is not accomplished, no firm satements about the location and even the existence of the CEP in the PD can be made at present.

This situation is rather unsatisfactory on the background of large scale experimental programmes to experimentally identify signatures of the CEP and identify its location which are operating at RHIC Brookhaven (STAR BES programme) and CERN-SPS (NA61) and are in preparation (NICA at JINR Dubna and CBM at FAIR Darmstadt).

Therefore, at this workshop an appeal was issued to the community of compact star astrophysicists to come up with suggestions how to prove the existence of a first order QCD phase transition in compact star interiors. Would it be possible to gain evidence for a strong first order deconfinement transition in compact stars this would prove the existence of at least one CEP in the QCD phase diagram since in the high temperature region explored by lattice QCD simulations [5] the transition is crossover.

One confirmative answer to the quest for a first-order phase transition in compact stars will be discussed here. If nature is so kind let cold, high-density compact star matter undergo a strong first-order phase transition which fulfills certain conditions to be detailed here, we show that the resulting mass-radius diagram for compact stars will provide besides an almost vertical hadronic star branch also a disconnected, hybrid star branch. If the onset of quark deconfinement, marking the end of this hadronic branch will occur at the presently known upper limit of compact star masses, around 2 $M_\odot$, then the "third family" of compact stars, the quark core hybrid stars, will occur also in this mass range provided the quark matter EoS is stiff enough to carry the hadronic mantle.

What would it require to prove that this situation occurs in nature? As we know already two of these high-mass compact stars: PSR J1614-2230 with 1.97 ± 0.04 $M_\odot$ [9] and PSR J0348+0432 with 2.01 ± 0.04 $M_\odot$ [10], the measurement of their radii to an accuracy of about 5% (i.e., $\delta R \sim$ 600m) bears the potential proof. If their radii would differ significantly, e.g., by 2 km, whie their masses are about the same, they would present an example of the high-mass twin stars, the existence of which would prove the presence of a strong first order phase transition in old compact stars and thus the existence of a CEP in the QCD phase diagram.

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2. Massive hybrid stars & twins

In the following we want to demonstrate on the examples of two classes of hybrid EoS models under which conditions the interesting phenomenon of twins at high compact star masses of $\sim 2\,M_\odot$ may be obtained. The first one is a phenomenological ansatz Zdunik, Haensel [11], Alford, Han and Prakash [12] which we call the ZHAHP scheme. The second one is based on a QCD motivated, microscopic EoS obtained within a nonlocal Polyakov-NJL model, see [13] and references therein.

2.1 The ZHAHP scheme

A first order phase transition in neutron star matter can take place just as in symmetric matter where it is searched for in heavy ion collisions. Adopting the setting of the ZHAHP scheme, we construct hybrid stars with a hybrid EoS composed of a given hadronic EoS, here DD2 [14], and a quark matter EoS parametrized by its squared speed of sound $c_{\rm QM}^2$ which pretty well describes [11] results of a color superconducting NJL model [15]

$$P(\varepsilon) = P_{\rm DD2}(\varepsilon)\Theta(\varepsilon_{\rm crit} - \varepsilon) + c_{\rm QM}^2\,\varepsilon\,\Theta(\varepsilon - \varepsilon_{\rm crit} - \Delta\varepsilon)\,. \qquad (2.1)$$

The critical energy density $\varepsilon_{\rm crit}$ and the discontinuity $\Delta\varepsilon$ complete this three-parameter EoS model which is capable of describing compact star sequences with a third family of stars in the mass-radius diagram. For early works on the disconnected, third branch of stable compact stars and the related mass-twin phenomenon, see Refs. [16, 17, 18, 19]. Searching for sequences with twins obeying the $2\,M_\odot$ mass constraint [9, 10] we obtain a quasi-horizontal hybrid star branch disconnected by an unstable branch from the almost vertical hadron star branch, as a consequence of a strong phase transition.

Fig. 1 (left) shows the corresponding EoS (2.1) for the parameters: $c_{\rm QM}^2 = 0.94$, $\Delta\varepsilon = 0.67\,\varepsilon_{\rm crit}$ and $\varepsilon_{\rm crit} = 485$ MeV/fm^3. The latter corresponds to $P(\varepsilon_{\rm crit}) = 100$ MeV/fm^3 and a baryon density at the quark matter onset of $n_{\rm crit} = 2.9\,n_0$ with $n_0 = 0.16$ fm^{-3}. Fig. 1 (right) shows the mass-radius relation for this hybrid EoS.

The ZHAHP scheme allows also to study the dependence of the $M-R$ relation on the strength of the transition, i.e. on the parameter $\Delta\varepsilon$ which can be also set to zero, thus mimicking the situation of a crossover transition. When hadronic and high-density phases behave rather similar, this case is also called "masquerade" [20]. In Fig. 1, we show the EoS (left) and the $M-R$ relation (right) in this situation for the two cases: a transition at a low density $n_{\rm crit} = 1.8\,n_0$ corresponding to $\varepsilon_{\rm crit} = 290$ MeV, $c_s = 0.55$; and at a high density $n_{\rm crit} = 2.9\,n_0$ corresponding to $\varepsilon_{\rm crit} = 487$ MeV, $c_s = 0.76$.

2.2 Nonlocal PNJL model

In this microscopic scheme for obtaining high-mass twins we use the standard two-phase construction of the deconfinement phase transition, adopting separate EoS models for the $\beta-$ equilibrated $T = 0$ compact star matter: $P_H(\mu)$ for the hadronic phase and $P_Q(\mu;\eta_v)$ for the quark phase. For the former we employ standard nuclear matter EoS like APR [21] and DD2 [14] (see also [22, 23]) which we modify at high densities by adopting an excluded volume with a nonlinear dependence on the chemical potential [24]

$$v_{\rm ex}(\mu) = (4\pi/3)r^3(\mu)\,,\quad r^3(\mu) = r_0 + r_1\mu + r_2\mu^2\,. \qquad (2.2)$$
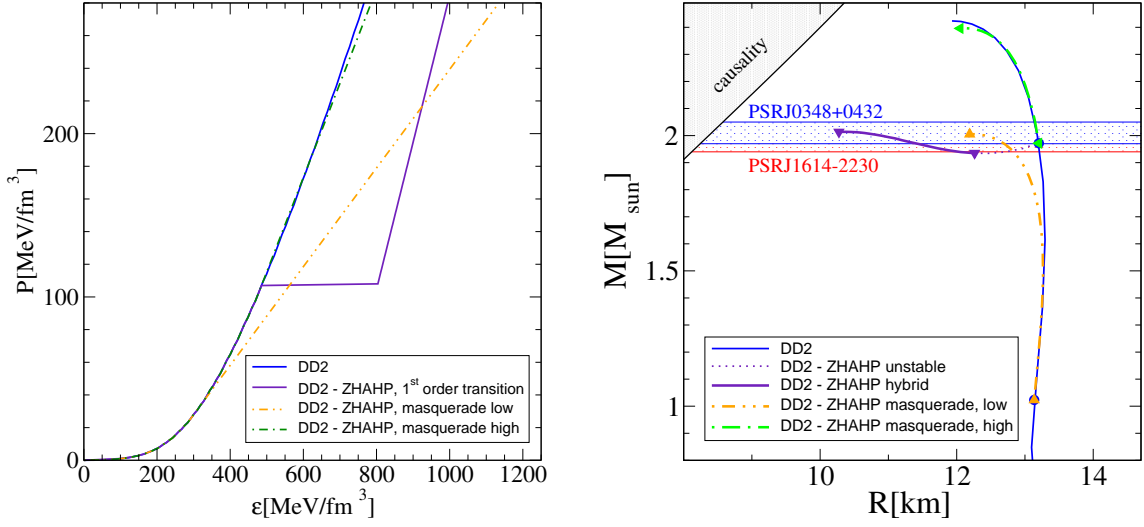
Figure 1: Left panel: Hadronic EoS (DD2, blue solid line) vs. hybrid one in the ZHAHP scheme of Eq. (2.1) with a strong first order transition (indigo solid line) and with a crossover behaviour ($\Delta\varepsilon = 0$) at a low critical density ($\varepsilon_{\text{crit}} = 487$ MeV, orange dash-double dotted line) and at a high one ($\varepsilon_{\text{crit}} = 290$ MeV, green dash-dotted line). Right panel: Mass-radius diagram for hybrid star EoS in the ZHAHP scheme (2.1). The almost vertical branch of pure hadronic stars (blue solid line) is separated from the almost horizontal hybrid star branch (indigo solid line) by an unstable branch (indigo dashed line) for a strong first order transition to stiff high-density matter. For comparison, we show two crossover situations, at low density (orange dash-double dotted line) and at high density (green dash-dotted line). The triangles mark the limits of the stable hybrid star branches. For parameters, see text. Recently measured masses for two 2 $M_\odot$ pulsars shown by red and blue hatched regions.

For the quark phase, we use the nonlocal PNJL model EoS of Refs. [7, 25] where the 4-momentum dependence of the formfactors is adjusted to describe the dynamical mass function and wave function renormalization of the $T = 0$ quark propagator from lattice QCD simulations [26, 27] and the vector meson coupling η_v is adjusted to obtain the slope of the chemical potential dependence of the pseudocritical temperature $T_c(\mu)$ in accordance with lattice QCD [5]. In addition, we follow the procedure suggested in [13] to implement a μ dependence of η_v by interpolating between zero temperature pressures $P_< = P_Q(\mu; \eta_<)$ and $P_> = P_Q(\mu; \eta_>)$ in $\beta-$ equilibrium which are calculated at different, but fixed values $\eta_< = \eta_v(\mu \le \mu_c)$ and $\eta_> = \eta_v(\mu > \mu_c)$. The critical chemical potential μ_c is found from the Gibbs condition for the phase equilibrium $P_H(\mu_c) = P_<(\mu_c)$, where $P_H(\mu)$ stands for the pressure of one of the hadronic EoS models in $\beta-$equilibrium. Different from [13] we use here a Gaussian function for this interpolation and obtain for the resulting hybrid star matter EoS

$$P(\mu) = P_H(\mu)\Theta(\mu_c - \mu) + \left[(P_< - P_>)\exp\left[-(\mu - \mu_c)^2/\Gamma^2\right] + P_>\right]\Theta(\mu - \mu_c) . \quad (2.3)$$

The density jump at the first order phase transition is given by the change in the slope of the pressure at the critical chemical potential $\Delta n = \partial P_</\partial\mu|_{\mu=\mu_c} - \partial P_H/\partial\mu|_{\mu=\mu_c}$. Note that with the choice $\eta_< < \eta_>$ one achieves a strong first order phase transition with large Δn on the one hand and stable hybrid star configurations even at high central energy densities on the other. In particular, one

model	r_0 [fm]	r_1 [fm/GeV]	r_2 [fm/GeV^2]	$\varepsilon_{\rm crit}$ [MeV/fm^3]	μ_c [MeV]	Γ [MeV]	$\eta_<$	$\eta_>$
DD2v$_{\rm ex}$ - nlPNJL	0.3	0.35	0.05	225.3	1420	360	0.10	0.20
APRv$_{\rm ex}$ - nlPNJL	0.65	0.01	0.0	373.0	1400	400	0.05	0.25

Table 1: Parameters of the hybrid EoS model. For details, see text.

can realize in this way microscopically motivated hybrid star EoS which also exhibit the feature of mass twin star configurations at high masses $M \sim 2M_\odot$. Two particular examples are shown in Fig. 2 compared to the ZHAHP scheme parametrization of Ref. [28], given also in Fig. 1. These examples show that it is possible to prove the presence of a strong first order phase transition in compact star matter provided nature would be so kind to allow the mass twin phenomenon to occur.
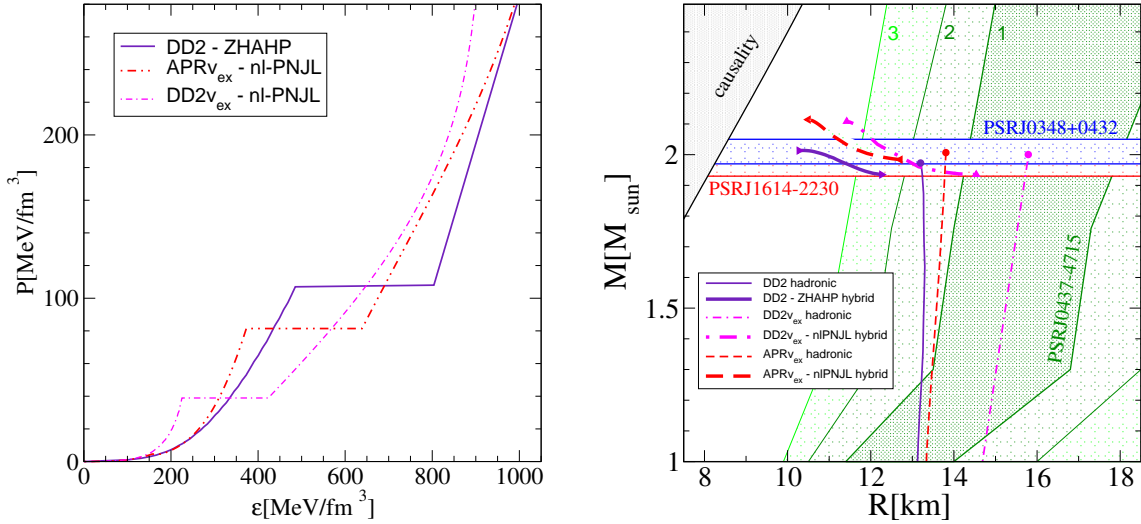
Figure 2: Left panel: Hadronic EoS (DD2, green dashed line) vs. hybrid one (Eq. 2.1, indigo solid line). Right panel: Mass-radius diagram for hybrid star EoS (2.1). The almost vertical branch of pure hadronic stars (green solid line) is separated from the almost horizontal hybrid star branch (indigo solid line) by an unstable branch (black dashed line). Recently measured masses for two $2\,M_\odot$ pulsars shown by red and blue hatched regions.

The two parameter sets are given in Table 1 One may ask for the motivation of the increasing stiffness of quark matter with chemical potential. This is a generally open question and should be answered within a fully nonperturbative QCD approach to dense matter in the vicinity of the deconfinement transition, which is not available yet. So we may speculate that at asymptotically high chemical potentials the ratio of vector to scalar coupling can be given by the value from Fierz transformation of one-gluon exchange, $\eta_v = 0.5$. Approaching the regio of the phase transition from above, the hadronic correlations might lead to a lowering of this value, eventually to as low values as we use here, $\eta_v(\mu_c) = 0.05...0.10$.

3. Conclusions

In this work we have demonstrated for a simple hybrid star EoS model in the ZHAHP scheme as well as for more elaborated, microscopically motivated models, that a strong first order phase transition in cold nuclear matter under neutron star constraints reveals itself by the mass twin phenomenon in the mass-radius diagram for compact star sequences. Particularly interesting for verification by observations is the case of high-mass twins at $M \sim 2M_\odot$. As our examples show, those twins have typically a difference in their radii of about 2 km. A further precondition for the existence of high-mass twins is a large radius for massive stars on the hadronic branch, as indicated in some recent analyses [29, 30]. Therefore, if radius measurements of compact stars in that mass range could be performed to an accuracy of less than 1 km, this would allow in principle to detect such high-mass compact star twins, if they exist. Their detection in turn would yield important impact to studies of the QCD phase diagram since the proof of a strong first order phase transition at $T = 0$ would imply the existence of a line of critical endpoints at finite temperatures. Then, there can be hope to find signatures of a strong first-order phase transition also in heavy-ion collision experiments with cool, strongly compressed baryon matter. A very concrete suggestion derived from this study would be an observational campaign to measure the radii of the known 2 $M_\odot$ pulsars, PSR J1614-2230 and PSR J0348+0432, for instance with the planned missions LOFT [31] and/or NICER.

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