

Limitation on the luminosity of e^+e^- storage rings due to beamstrahlung

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Particle loss due to the emission of single energetic beamstrahlung photons in beam collisions is shown to impose a fundamental limit on storage-ring luminosities at energies greater than $2E_0 \sim 140$ GeV for head-on collisions and $2E_0 \sim 40$ GeV for crab-waist collisions. Above these threshold energies, the suppression factor due to beamstrahlung scales as $1/E_0^{4/3}$, and for a fixed power of synchrotron radiation, the luminosity $\mathcal{L} \propto R/E_0^{13/3}$, where R is the collider radius. For $2E_0 \gtrsim 150$ GeV, both collision schemes have similar luminosity limits. The luminosities attainable at storage-ring and linear-collider (LC) $2E_0 \sim 240$ GeV Higgs factories are comparable; at higher energies, LCs are preferable. This conference paper is based on my recent PRL publication [1] supplemented with additional comments on linac-ring e^+e^- colliders and ring e^+e^- colliders with charge compensation (four-beam collisions).

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1. Introduction

The ATLAS and CMS experiments at the LHC recently discovered [2, 3] the long-sought Higgs boson with the mass $M \approx 126 \text{ GeV}/c^2$. The precision study of the Higgs boson's properties would require the construction of an energy- and luminosity-frontier e^+e^- collider [4].

With the sole exception of the $2E_0 = 91 \text{ GeV}$ SLAC Linear Collider (SLC), all e^+e^- colliders ever built have been based on storage rings. The $2E_0 = 209 \text{ GeV}$ LEP collider at CERN is generally considered to have been the last energy-frontier e^+e^- storage-ring collider (SRC). Indeed, synchrotron-radiation energy losses, which are proportional to E_0^4/R , make the construction and operation of higher-energy SRCs excessively expensive. Energy-frontier linear e^+e^- colliders (LC), which have been in development for over 40 years, are free from this limitation and allow multi-TeV energies to be reached. Two LC projects are in advanced stages of development: the $2E_0 = 500 \text{ GeV}$ ILC [5] and the $2E_0 = 500\text{--}3000 \text{ GeV}$ CLIC [6].

Nevertheless, several proposals [7, 8, 9, 10, 11] for a $2E_0 = 240 \text{ GeV}$ SRC for the study of the Higgs boson in $e^+e^- \rightarrow HZ$ have recently been put forward [12]. Lower cost and reliance on firmly established technologies are cited as these projects' advantages over an LC. Moreover, it has been proposed [8] that a $2E_0 = 240 \text{ GeV}$ SRC can provide superior luminosity, and that the "crab waist" collision scheme [15, 16] can be adapted to energy-frontier SRCs, allowing them to exceed the ILC and CLIC luminosities even at $2E_0 = 400\text{--}500 \text{ GeV}$. Parameters of SRCs proposed in Refs. [7, 8] are summarized in Table 1.

In this conference paper, which is based on [1], we examine the role of *beamstrahlung*, i.e., synchrotron radiation in the electromagnetic field of the opposing beam, in high-energy e^+e^- SRCs. First discussed in [13], beamstrahlung has been well-studied only in the LC case [14]. As we shall see, at energy-frontier e^+e^- SRCs beamstrahlung determines the beam lifetime through the emission of single photons in the tail of the beamstrahlung spectra, thus severely limiting the luminosity. Unlike the LC case, beamstrahlung has little effect on the SRC energy spread.

At LCs, flat beams are employed to suppress beamstrahlung. Each colliding particle radiates $n_\gamma = 1\text{--}2$ beamstrahlung photons with the total energy averaging 3–5% of the beam energy. The long tails of the beamstrahlung energy-loss spectrum are not a problem for LCs because beams are used only once.

In contrast, at SRCs the particles that lose a certain fraction of their energy in a beam collision leave the beam and strike the vacuum chamber's walls; this fraction η is typically around 0.01 (0.012 at LEP) and is known as the ring's energy acceptance. Beamstrahlung was negligible at all previously built SRCs because of their relatively large beam sizes. Its importance considerably increases with energy. Table 1 lists the beamstrahlung characteristics of the newly proposed SRCs assuming a 1% energy acceptance: the critical photon energy for the maximum beam field $E_{c,\text{max}}$, the average number of beamstrahlung photons per electron per beam crossing n_γ , and the beamstrahlung-driven beam lifetime. Please note that once beamstrahlung is taken into account, the beam lifetimes drop to unacceptably low values, from a fraction of a second to as low as a few revolution periods.

At the SRCs considered in Table 1, the beam lifetime due to the unavoidable radiative Bhabha scattering is 10 minutes or longer. One would therefore want the beam lifetime due to beamstrahlung to be at least 30 minutes. The simplest (but not optimum) way to suppress beamstrahlung

	LEP	LEP3	DLEP	STR1	STR2	STR3 cr-w	STR4 cr-w	STR5 cr-w	STR6 cr-w
$2E_0$, GeV	209	240	240	240	240	240	400	400	500
Circumference, km	27	27	53	40	60	40	40	60	80
Beam current, mA	4	7.2	14.4	14.5	23	14.7	1.5	2.7	1.55
Bunches/beam	4	3	60	20	49	15	1	1.4	2.2
N , 10^{11}	5.8	13.5	2.6	6	6	8.3	12.5	25.	11.7
σ_z , mm	16	3	1.5	3	3	1.9	1.3	1.4	1.9
ϵ_x , nm	48	20	5	23.3	24.6	3	2	3.2	3.4
ϵ_y , nm	0.25	0.15	0.05	0.09	0.09	0.011	0.011	0.017	0.013
β_x , mm	1500	150	200	80	80	26	20	30	34
β_y , mm	50	1.2	2	2.5	2.5	0.25	0.2	0.32	0.26
σ_x , μm	270	54	32	43	44	8.8	6.3	9.8	10.7
σ_y , μm	3.5	0.42	0.32	0.47	0.47	0.05	0.047	0.074	0.06
SR power, MW	22	100	100	100	100	100	100	100	100
Energy loss per turn, GeV	3.4	7	3.47	3.42	2.15	3.42	33.9	18.5	32.45
\mathcal{L} , $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$	0.013	1.3	1.6	1.7	2.7	17.6	4	7	2.2
$E_{c,\text{max}}/E_0$, 10^{-3}	0.09	6.3	4.2	3.5	3.4	38	194	232	91
n_γ per electron	0.09	1.1	0.37	0.61	0.6	4.2	8.7	11.3	4.8
lifetime(SR@IP), s	$\sim \infty$	0.02	0.3	0.2	0.4	0.005	0.001	0.0005	0.005
$\mathcal{L}_{\text{corr}}$, $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$	0.013	0.2	0.4	0.5	0.8	0.46	0.02	0.03	0.024

Table 1: Parameters of LEP and several recently proposed storage-ring colliders [7, 8]. ‘‘STR’’ refers to ‘‘SuperTRISTAN’’ [8]. Use of the crab-waist collision scheme [15, 16] is denoted by ‘‘cr-w’’. The luminosities and the numbers of bunches for all projects are normalized to the total synchrotron-radiation power of 100 MW. Beamstrahlung-related quantities derived in this paper are listed below the double horizontal line.

is to decrease the number of particles per bunch with a simultaneous increase in the number of colliding bunches. As explained below, $E_{c,\text{max}}$ should be reduced to $\approx 0.001E_0$. Thus, beamstrahlung causes a great drop in luminosity, especially at crab-waist SRCs: compare the proposed \mathcal{L} and corrected (as suggested above) $\mathcal{L}_{\text{corr}}$ rows in Table 1.

To achieve a reasonable beam lifetime, one must make small the number of beamstrahlung photons with energies greater than the threshold energy $E_{\text{th}} = \eta E_0$ that causes the electron to leave the beam. These photons belong to the high-energy tail of the beamstrahlung spectrum and have energies much greater than the critical energy. It will be clearly shown below that the beam lifetime is determined by such single high-energy beamstrahlung photons, not by the energy spread due to the emission of multiple low-energy photons.

2. Beam lifetime due to beamstrahlung, restriction on beam parameters [1]

The critical energy for synchrotron radiation

$$E_c = \hbar\omega_c = \hbar \frac{3\gamma^3 c}{2\rho}, \quad (2.1)$$

where ρ is the bending radius and $\gamma = E_0/mc^2$. The spectrum of photons per unit length with energy well above the critical energy

$$\frac{dn}{dx} = \sqrt{\frac{3\pi}{2}} \frac{\alpha\gamma}{2\pi\rho} \frac{e^{-u}}{\sqrt{u}} du, \quad (2.2)$$

where $u = E_\gamma/E_c$, $\alpha = e^2/\hbar c$.

Using these simple formulas, one can find the critical energy of beamstrahlung photons (for the maximum beam field) corresponding to a beam lifetime of 30 minutes:

$$u = \eta E_0/E_c \approx 8.5; \quad E_c \approx 0.12\eta E_0 \sim 0.1\eta E_0. \quad (2.3)$$

The accuracy of this expression is quite good for any SRC because it depends on collider parameters (R , E_0 , σ_z) as well as on the assumed lifetime only logarithmically.

The critical energy is related to the beam parameters as follows:

$$\frac{E_c}{E_0} = \frac{3\gamma r_e^2 N}{\alpha \sigma_x \sigma_z}, \quad (2.4)$$

where $r_e = e^2/mc^2$ is the classical radius of the electron. Combined with Eq. 2.3, this imposes a restriction on the beam parameters,

$$\frac{N}{\sigma_x \sigma_z} < 0.1\eta \frac{\alpha}{3\gamma r_e^2}, \quad (2.5)$$

where N is the number of particles in the beam, $\alpha = e^2/\hbar c \approx 1/137$, and σ_x and σ_z are the rms horizontal and longitudinal beam sizes, respectively. *This formula is the basis for the discussion that follows.* This constraint on beam parameters should be taken into account in luminosity optimization.

It can be shown that the beam lifetime given the above conditions is determined by the emission of beamstrahlung photons with energies a factor of ~ 65 greater than $\langle E_\gamma \rangle$.

3. The beam energy spread

In Ref. [1], the rms beam energy spread due to beamstrahlung was compared to that due to synchrotron radiation in bending magnets. It was shown that in rings with large energy acceptance the energy spread due to beamstrahlung could be larger than due to SR; however, the lifetime is always determined by the emission of energetic single photons.

4. Head-on and crab-waist collisions [1]

In the ‘‘crab waist’’ collision scheme [15, 16], the beams collide at an angle $\theta \gg \sigma_x/\sigma_z$, in contrast with the usual head-on collisions, where $\theta \ll \sigma_x/\sigma_z$. The crab-waist scheme allows for higher luminosity when it is restricted only by the tune shift, characterized by the beam-beam strength parameter. One should work at a beam-beam strength parameter smaller than a certain threshold value, ≈ 0.15 for high-energy SRCs [7].

In head-on collisions, the vertical beam-beam strength parameter (hereinafter, the ‘‘beam-beam parameter’’)

$$\xi_y = \frac{Nr_e\beta_y}{2\pi\gamma\sigma_x\sigma_y} \approx \frac{Nr_e\sigma_z}{2\pi\gamma\sigma_x\sigma_y} \text{ for } \beta_y \approx \sigma_z. \quad (4.1)$$

In the crab-waist scheme [15],

$$\xi_y = \frac{Nr_e\beta_y^2}{\pi\gamma\sigma_x\sigma_y\sigma_z} \text{ for } \beta_y \approx \sigma_x/\theta. \quad (4.2)$$

The luminosity in head-on collisions

$$\mathcal{L} \approx \frac{N^2 f}{4\pi\sigma_x\sigma_y} \approx \frac{Nf\gamma\xi_y}{2r_e\sigma_z}; \quad (4.3)$$

in crab-waist collisions,

$$\mathcal{L} \approx \frac{N^2 f}{2\pi\sigma_y\sigma_z\theta} \approx \frac{N^2\beta_y f}{2\pi\sigma_x\sigma_y\sigma_z} \approx \frac{Nf\gamma\xi_y}{2r_e\beta_y}. \quad (4.4)$$

In the crab-waist scheme, one can make $\beta_y \ll \sigma_z$, which enhances the luminosity by a factor of σ_z/β_y compared to head-on collisions. For example, at the proposed Italian SuperB factory [16] this enhancement could be a factor of 20–30.

5. The beam energies where beamstrahlung is important [1]

Using Eqs. 4.1 and 4.2 and the restriction in Eq. 2.5, we find the minimum beam energy at which beamstrahlung becomes important. For head-on collisions,

$$\gamma_{\min} = \left(\frac{0.1\eta\alpha\sigma_z^2}{6\pi r_e \xi_y \sigma_y} \right)^{1/2} \propto \frac{\sigma_z^{3/4}}{\xi_y^{1/2} \varepsilon_y^{1/4}}; \quad (5.1)$$

for crab-waist collisions,

$$\gamma_{\min} = \left(\frac{0.1\eta\alpha\beta_y^2}{3\pi r_e \xi_y \sigma_y} \right)^{1/2} \propto \frac{\beta_y^{3/4}}{\xi_y^{1/2} \varepsilon_y^{1/4}}. \quad (5.2)$$

In the crab-waist scheme, beamstrahlung becomes important at much lower energies because $\beta_y \ll \sigma_z$. This can be understood from Eq. 4.2: smaller β_y corresponds to denser beams, leading to a higher beamstrahlung rate.

For typical beam parameters presented in Table 1, beamstrahlung becomes important at energies $E_0 \gtrsim 70\text{ GeV}$ for e^+e^- storage rings with head-on collisions; when the crab-waist scheme is employed, this changes to the more strict $E_0 \gtrsim 20\text{ GeV}$. All newly proposed projects listed in Table 1 are affected as they are designed for $E_0 \geq 120\text{ GeV}$.

6. Luminosities for the head-on and crab-waist schemes in a beamstrahlung-dominated regime [1]

Now, let us find the luminosity \mathcal{L} when it is restricted both by the tune shift (beam-beam strength parameter) and beamstrahlung. For head-on collisions,

$$\mathcal{L} \approx \frac{(Nf)N}{4\pi\sigma_x\sigma_y}, \quad \xi_y \approx \frac{Nr_e\sigma_z}{2\pi\gamma\sigma_x\sigma_y}, \quad \frac{N}{\sigma_x\sigma_z} \equiv k \approx 0.1\eta \frac{\alpha}{3\gamma r_e^2} \quad (6.1)$$

and $\sigma_y \approx \sqrt{\varepsilon_y\sigma_z}$. This can be rewritten as

$$\mathcal{L} \approx \frac{(Nf)k\sigma_z}{4\pi\sigma_y}, \quad \xi_y \approx \frac{kr_e\sigma_z^2}{2\pi\gamma\sigma_y}, \quad \sigma_y \approx \sqrt{\varepsilon_y\sigma_z}. \quad (6.2)$$

Thus, in the beamstrahlung-dominated regime the luminosity is proportional to the bunch length, and its maximum value is determined by the beam-beam strength parameter. Together, these equations give

$$\mathcal{L} \approx \frac{Nf}{4\pi} \left(\frac{0.1\eta\alpha}{3} \right)^{2/3} \left(\frac{2\pi\xi_y}{\gamma r_e^5 \varepsilon_y} \right)^{1/3}, \quad (6.3)$$

$$\sigma_{z,\text{opt}} = \varepsilon_y^{1/3} \left(\frac{6\pi\gamma^2 r_e \xi_y}{0.1\eta\alpha} \right)^{2/3}. \quad (6.4)$$

Similarly, for the crab-waist collision scheme,

$$\mathcal{L} \approx \frac{(Nf)N\beta_y}{2\pi\sigma_x\sigma_y\sigma_z}, \quad \xi_y \approx \frac{Nr_e\beta_y^2}{\pi\gamma\sigma_x\sigma_y\sigma_z}, \quad \frac{N}{\sigma_x\sigma_z} \equiv k \approx 0.1\eta \frac{\alpha}{3\gamma r_e^2} \quad (6.5)$$

and $\sigma_y \approx \sqrt{\varepsilon_y\beta_y}$. Substituting, we obtain

$$\mathcal{L} \approx \frac{(Nf)k\beta_y}{2\pi\sigma_y}, \quad \frac{kr_e\beta_y^2}{\pi\gamma\sigma_y} \approx \xi_y, \quad \sigma_y \approx \sqrt{\varepsilon_y\beta_y}. \quad (6.6)$$

The corresponding solutions are

$$\mathcal{L} \approx \frac{Nf}{4\pi} \left(\frac{0.2\eta\alpha}{3} \right)^{2/3} \left(\frac{2\pi\xi_y}{\gamma r_e^5 \varepsilon_y} \right)^{1/3}, \quad (6.7)$$

$$\beta_{y,\text{opt}} = \varepsilon_y^{1/3} \left(\frac{3\pi\gamma^2 r_e \xi_y}{0.1\eta\alpha} \right)^{2/3}. \quad (6.8)$$

We have obtained a very important result: *in the beamstrahlung-dominated regime, the luminosities attainable in crab-waist and head-on collisions are practically the same.* In fact, the gain from using the crab-waist scheme is only a factor of $2^{2/3} \sim 1$, contrary to the low-energy case, where the gain may be greater than one order of magnitude. For this reason, from this point on we will consider only the case of head-on ($\theta \ll \sigma_x/\sigma_z$) collisions.

From the above considerations, one can find the ratio of the luminosities with and without taking beamstrahlung into account: it is equal to $\sigma_z/\sigma_{z,\text{opt}}$ for head-on collisions and $\beta_y/\beta_{y,\text{opt}}$ for crab-waist collisions and scales as $1/E_0^{4/3}$ for $\gamma > \gamma_{\text{min}}$.

In practical units,

$$\frac{\sigma_{z,\text{opt}}}{\text{mm}} \approx \frac{2\xi_y^{2/3}}{\eta^{2/3}} \left(\frac{\varepsilon_y}{\text{nm}} \right)^{1/3} \left(\frac{E_0}{100\text{GeV}} \right)^{4/3}. \quad (6.9)$$

For example, for $\xi_y = 0.15$, $\eta = 0.01$, $E_0 = 100\text{GeV}$ and the vertical emittances from Table 1 ($\varepsilon_y = 0.01$ to 0.15nm), we get $\sigma_{z,\text{opt}} = 2.5$ to 6.4mm .

According to Eq. 6.3, the maximum luminosity at high-energy SRCs with beamstrahlung taken into account

$$\mathcal{L} \approx h \frac{N^2 f}{4\pi\sigma_x\sigma_y} = h \frac{Nf}{4\pi} \left(\frac{0.1\eta\alpha}{3} \right)^{2/3} \left(\frac{2\pi\xi_y}{\gamma r_e^5 \varepsilon_y} \right)^{1/3}, \quad (6.10)$$

where h is the hourglass loss factor, $f = n_b c / 2\pi R$ is the collision rate, R the average ring radius, and n_b the number of bunches in the beam.

The energy loss by one electron in a circular orbit $\delta E = 4\pi e^2 \gamma^4 / 3R_b$. Then, the power radiated by the two beams in the ring

$$P = 2\delta E \frac{cNn_b}{2\pi R} = \frac{4e^2 \gamma^4 cNn_b}{3RR_b}. \quad (6.11)$$

Substituting Nn_b from Eq. 6.11 to Eq. 6.10, we obtain

$$\mathcal{L} \approx h \frac{(0.1\eta\alpha)^{2/3} PR}{32\pi^2 \gamma^{13/3} r_e^3} \left(\frac{R_b}{R}\right) \left(\frac{6\pi\xi_y r_e}{\varepsilon_y}\right)^{1/3}, \quad (6.12)$$

or, in practical units,

$$\frac{\mathcal{L}}{10^{34} \text{ cm}^{-2} \text{ s}^{-1}} \approx \frac{100h\eta^{2/3} \xi_y^{1/3}}{(E_0/100 \text{ GeV})^{13/3} (\varepsilon_y/\text{nm})^{1/3}} \left(\frac{P}{100 \text{ MW}}\right) \left(\frac{2\pi R}{100 \text{ km}}\right) \frac{R_b}{R}. \quad (6.13)$$

Once the vertical emittance is given as an input parameter, we find the luminosity and the optimum bunch length by applying Eq. 6.9. Beamstrahlung and the beam-beam strength parameter determine only the combination N/σ_x ; additional technical arguments are needed to find N and σ_x separately. When they are fixed, the optimal number of bunches n_b is found from the total SR power, Eq. 6.11.

In Table 2, we present the realistic luminosities and beam parameters for the rings listed in Table 1 after both beamstrahlung and the beam-beam parameter are taken into account. The following assumptions are made: SR power $P = 100$ MW, $R_b/R = 0.7$, the hourglass factor $h = 0.8$, the beam-beam parameter $\xi_y = 0.15$, the energy acceptance of rings $\eta = 0.01$; the values of all other parameters (ε_y , ε_x and β_x) are taken from Table 1.

	LEP	LEP3	DLEP	STR1	STR2	STR3 cr-w	STR4 cr-w	STR5 cr-w	STR6 cr-w
$2E_0$, GeV	209	240	240	240	240	240	400	400	500
Circumference, km	27	27	53	40	60	40	40	60	80
Bunches/beam	~ 2	~ 7	70	24	53	240	36	45	31
N , 10^{11}	33	5.9	2.35	3.9	4.	0.4	0.34	0.6	0.65
σ_z , mm	8.1	8.1	5.7	6.9	6.9	3.4	6.7	7.8	9.6
σ_y , μm	1.4	1.1	0.53	0.78	0.78	0.19	0.27	0.36	0.35
\mathcal{L} , $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$	0.47	0.31	0.89	0.55	0.83	1.1	0.12	0.16	0.087

Table 2: Realistically achievable luminosities and other beam parameters for the projects listed in Table 1 at synchrotron-radiation power $P = 100$ MW. Only the parameters that differ from those in Table 1 are shown.

Comparing Tables 1 and 2, one can see that at $2E_0 = 240$ GeV, which is the preferred energy for the study of an $m = 126$ GeV Higgs boson, taking beamstrahlung into account lowers the luminosities achievable at storage-ring colliders with crab-waist collisions by a factor of 15. Nevertheless, these luminosities are comparable to those at the ILC, $\mathcal{L}_{\text{ILC}} \approx (0.55-0.7) \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ at $2E_0 = 240$ GeV [17]. However, at $2E_0 = 500$ GeV the ILC can achieve $\mathcal{L}_{\text{ILC}} \approx (1.5-2) \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$, which is a factor 15–25 greater than the luminosities achievable at storage rings.

7. Ways to increase luminosity of high-energy storage rings

7.1 Linear-ring collider with energy recuperation

At linear colliders, beams are used only once; one should accelerate new beams from zero to the maximum energy for each beam collisions; therefore, LCs are very energy-ineffective machines.

Storage rings are much better in this respect; however, at high beam energies they also need huge electrical power to compensate for SR energy losses. At first sight, these problems can be partially overcome in the collider scheme shown in Fig. 1. In this scheme, after the collision at the interaction point (IP), the electron (positron) bunch is decelerated in the second half of the linac down to the energy of about 15 GeV, makes one turn in the ring, then is accelerated again—and the process is repeated over and over again (for minutes) as in conventional storage rings but with much smaller SR losses (proportional to E^4/R per turn).

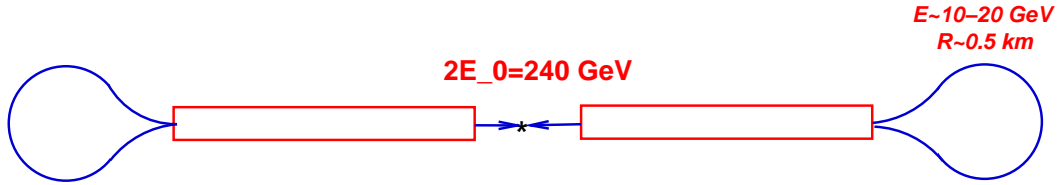


Figure 1: Linear-ring collider with energy recuperation

If η is the energy acceptance of the ring, the maximum energy of beamstrahlung photons should be ηE (not ηE_0). This reduces luminosity \mathcal{L} by a factor of $(E/E_0)^{2/3} \sim 0.25$. However, thanks to much lower SR losses, one can increase Nf by a very large factor, and thus increase the luminosity by 1-2 orders of magnitude.

Unfortunately, there are many stoppers which make this scheme impractical:

- The required refrigeration power for the linac working in the continuous mode is about 150–200 MW (assuming an acceleration gradient of ~ 15 MeV/m, $Q = 2 \times 10^{10}$) [18].
- Parasitic collisions of beams inside the linac. One can separate the beams (the pretzel scheme), but beam attraction would lead to beam instabilities.
- The transverse wakefield problem for beams shifted from the axis.
- The energy difference between the head and tail becomes unacceptable after deceleration (beam loading helps during acceleration but is detrimental during deceleration).

Thus, this idea looks interesting but technically impossible. LC schemes with recuperation were considered in 1970s [19] and were also rejected then.

7.2 Charge-compensated (four-beam) collisions

The idea to collide four beams (e^+e^- with e^+e^-) is more than 40 years old. Beams are neutral at the IP, so there are no collision effects, which sounds nice. A four-beam e^+e^- collider, DCI, with the energy $2E \sim 2$ GeV, was build in 1970s in Orsay [20]. There were hopes the four-beam collision scheme would increase the luminosity by a factor of 100—but the result was confusing: the maximum luminosity was approximately the same as in two-beam collisions. This is due to the instability of neutral e^+e^- beams [21]: a small displacement of charges in one beam leads to charge separation in the opposing beam and subsequent development of beam instabilities as in two-beam collisions. The beam-beam parameter ξ_y attainable at DCI was approximately the same as without neutralization.

A similar four-beam approach was considered in 1980s for linear colliders in order to suppress beamstrahlung [22]. However, simulation and theory have shown that the kink instability that develops in collision limits the luminosity to a value that is lower than in two-beam collisions because the pinch effect is absent; beamstrahlung is suppressed only at rather low luminosities. For these reasons, the idea of charge neutralization at e^+e^- colliders was dismissed. Why are we discussing it again?

The above-mentioned beam neutralization exercises have shown that this method does not help to increase the beam-beam parameter ξ . However, it should work at values of ξ_y attainable in two-beam collisions. We have such a case at e^+e^- Higgs factories. Their luminosities are limited by beamstrahlung. Using charge compensation, one can suppress beamstrahlung and increase the luminosity while keeping the same ξ_y as for standard two-beam collisions.

What luminosity increase can be achieved this way? For head-on collisions, beam neutralization can increase the luminosity only by a factor of 2–3 at $2E_0 = 240$ GeV. However, in the crab-waist scheme (which, as demonstrated above, has no benefit in beamstrahlung-dominated ring colliders) the situation is much more attractive. Comparing the luminosities for crab-waist storage rings in Tables 1 and 2 (without and with beamstrahlung taken into effect), one can see the possible benefits from the suppression of beamstrahlung (in the inline table below, the subscript “nb” stands for “no beamstrahlung”, “b” stands for “with beamstrahlung”):

$\frac{2E_0, \text{ GeV}}{\mathcal{L}_{\text{nb}}/\mathcal{L}_{\text{b}}}$	240	400	400	500
	16	33	43	25

These numbers, corresponding to an ideal charge compensation, look quite attractive.

Let us find the charge compensation “quality” required to increase the luminosity by a factor of $\mathcal{L}_c/\mathcal{L}$, where the subscript “c” denotes “compensated”. From Eqs. 6.5 and 6.6, we have

$$\mathcal{L} \propto k^{2/3} \xi_y^{1/3}, \quad (7.1)$$

where $k \equiv N/(\sigma_x \sigma_z) \approx 0.1 \eta \alpha / (3 \gamma r_e^2)$. For the case of charge-compensated beams, the beamstrahlung condition should be rewritten as follows: $\Delta N_c / (\sigma_{x,c} \sigma_{z,c}) \equiv k$ or $N_c / (\sigma_{x,c} \sigma_{z,c}) \equiv k' = k(N_c / \Delta N_c)$. The expression for luminosity will be similar to Eq. 7.1, with k replaced by k' :

$$\mathcal{L}_c \propto \left(k \frac{N_c}{\Delta N_c} \right)^{2/3} \xi_{y,c}^{1/3}. \quad (7.2)$$

From these two equations, we get the required degree of charge compensation

$$\frac{\Delta N_c}{N_c} = \left(\frac{\mathcal{L}}{\mathcal{L}_c} \right)^{3/2} \left(\frac{\xi_c}{\xi} \right)^{1/2}. \quad (7.3)$$

For example, if $\xi_c = \xi$, then to increase the luminosity by a factor of $\mathcal{L}_c/\mathcal{L} = 10$ one needs $\Delta N_c/N_c = 0.03$. Gaining a factor of 30 requires $\Delta N/N = 0.006$. Thus, an order-of-magnitude luminosity increase seems realistic. Unfortunately, this method has many problems (and even stoppers):

1. Crab-waist collisions assume collisions at some horizontal angle, which requires two rings with electrostatic separators near the IP and inside the rings, see Fig. 2, plus one ring for injection; three rings in total.

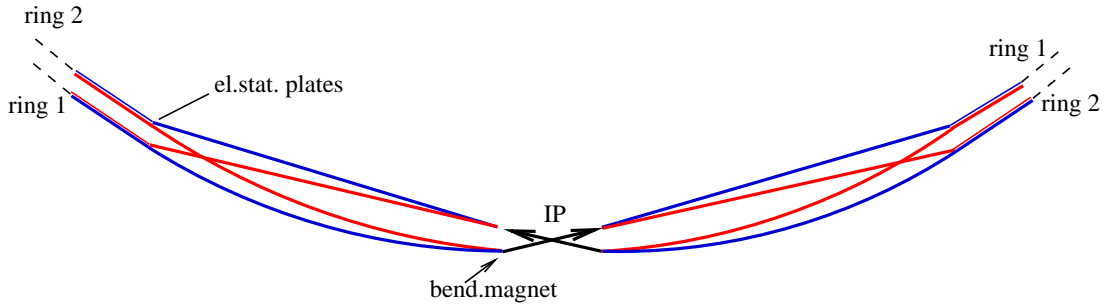


Figure 2: Ring collider with charge compensation. Blue: electron, red: positron beamlines.

2. The assembly of electron and positron bunches approaching the IP at a relative angle of about 10 mrad (determined by quads radius) into one neutral bunch using a bending magnet placed between the IP and the final-focus quads looks very problematic due to synchrotron radiation in the magnet. This leads to the increase of the vertical beam size at the IP (deflection in the solenoidal detector field due to the crossing angle). This problem is very serious and appears to be a stopper for high-energy rings.
3. Everything must be very close to ideal: even a small displacement of beams at the IP (e.g., $0.2\sigma_y$) would lead to a very short beam lifetime.
4. Immediately after refilling, bunches have slightly different sizes and some displacement. One should avoid collisions of such bunches at the IP until full damping has been achieved.
5. The additional cost.

8. Conclusion

We have shown that the beamstrahlung phenomenon must be properly taken into account in the design and optimization of high-energy storage rings colliders (SRCs). Beamstrahlung determines the beam lifetime through the emission of single photons in the tail of the beamstrahlung spectra, thus severely limiting the luminosity. We have demonstrated that beamstrahlung suppresses the luminosities of high-energy e^+e^- storage rings as $1/E_0^{4/3}$ at beam energies $E_0 \gtrsim 70\text{GeV}$ for head-on collisions and $E_0 \gtrsim 20\text{GeV}$ for crab-waist collisions. Very importantly, beamstrahlung makes the luminosities attainable in head-on and crab-waist collisions approximately equal above these threshold energies. At $2E_0 = 240\text{--}500\text{GeV}$, beamstrahlung lowers the luminosity of crab-waist rings by a factor of 15–40. Some increase in SRC luminosities can be achieved at rings with larger radius, larger energy acceptance, and smaller beam vertical emittance.

We also conclude that the luminosities attainable at e^+e^- storage rings (with one interaction point) and linear colliders are comparable at $2E_0 = 240\text{GeV}$. However, at $2E_0 = 500\text{GeV}$ storage-ring luminosities are by a factor of 15–25 smaller (this factor is smaller for rings with larger radius). Therefore, linear colliders remain the most promising instrument for studying the physics at energies $2E_0 \gtrsim 250\text{GeV}$.

We have also briefly considered two possible methods of increasing the luminosity of high-energy e^+e^- storage rings: the linac-ring storage ring scheme with the energy recuperation and the charge-compensated four-beam collider. The first scheme looks completely unrealistic; the second one is also very difficult to implement and suffers from a fundamental problem connected with radiation in beam-combining magnets.

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