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Gauge theories with fermions in two-index representations

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After some introductory comments on the peculiar features of slowly running theories, we describe our results for the nonperturbative beta function and the mass anomalous dimension in two theories with fermions in a two-index representation. These are the SU(3) theory with two Dirac fermions in the adjoint representation, and the SU(4) theory with six Dirac fermions in the antisymmetric two-index representation.

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1. Introduction

Asymptotically free gauge theories can differ from QCD in several ways: the number of colors N_c, the number of flavors N_f, and the fermions' representation. These theories are interesting for purely theoretical reasons, and also as templates for physics beyond the Standard Model. There is a growing body of numerical work devoted to them (for a recent review, see Ref. [1]).

As the number of flavors is increased, typically the two-loop coefficient of the beta function

$$\beta(g^2) = -b_1 \frac{g^4}{16\pi^2} - b_2 \frac{g^6}{(16\pi^2)^2} + \cdots, \tag{1.1}$$

will flip sign before the one-loop coefficient. The range of N_f-values where $b_1 > 0 > b_2$ defines perturbatively the conformal window, where the running coupling is driven to an infrared-attractive fixed point (IRFP). Unlike QCD, in that case no physical scale is generated dynamically, and the long-distance behavior of all correlation functions is predicted to follow a power law.

The existence of an IRFP requires nonperturbative confirmation. Particularly interesting are borderline theories in which N_f is close to the critical value where the perturbative conformal window is entered.

We have carried out a long-term program of studying gauge theories with varying number of colors, and with fermions in various two-index representations. We concentrate on two observables. The first is the nonperturbative beta function, which we define and measure through the Schrödinger functional (SF) scheme. The second is the mass anomalous dimension γ_m, which we define as usual from the scaling behavior of $\overline{\psi}\psi$. Thanks to chiral symmetry (of the massless continuum theory), we may in fact extract γ_m from the scaling of the isospin-triplet pseudoscalar density, which in turn is much better behaved on the lattice, and which we have measured on the same ensembles used to determine the running coupling.

Previously, we studied gauge theories with fermions in the symmetric two-index representation [2, 3, 4, 5]. Here we will report on two more theories with fermions in a two-index representation [6]. These are the SU(3) theory with $N_f = 2$ Dirac fermions in the adjoint representation, and the SU(4) theory with $N_f = 6$ Dirac fermions in the antisymmetric representation, which is a sextet. Choosing $N_f = 6$ places that theory near the bottom of the perturbative conformal window.

2. Slow running

The SF setup was originally developed aiming for a precise numerical determination of the evolution of the QCD coupling. Using the SF setup in a different gauge theory is straightforward. But our analysis tools must be adapted to a new situation where the coupling hardly runs at all.

To appreciate this difference consider Fig. 1, where, as an example, we show the two-loop beta function for two different SU(2) theories, each containing two Dirac fermions in a given representation. In the left panel the fermions are in the adjoint representation. The (perturbative) fixed point is clearly visible. In the right panel, the downward pointing curve is the beta function of the theory with fermions in the fundamental representation. Much like QCD, this beta function is always negative, and grows in absolute value with increasing coupling.

The other curve in the right panel, which embraces the horizontal axis, is once again the beta function of the adjoint-fermions theory. The visual difference relative to the left panel comes from

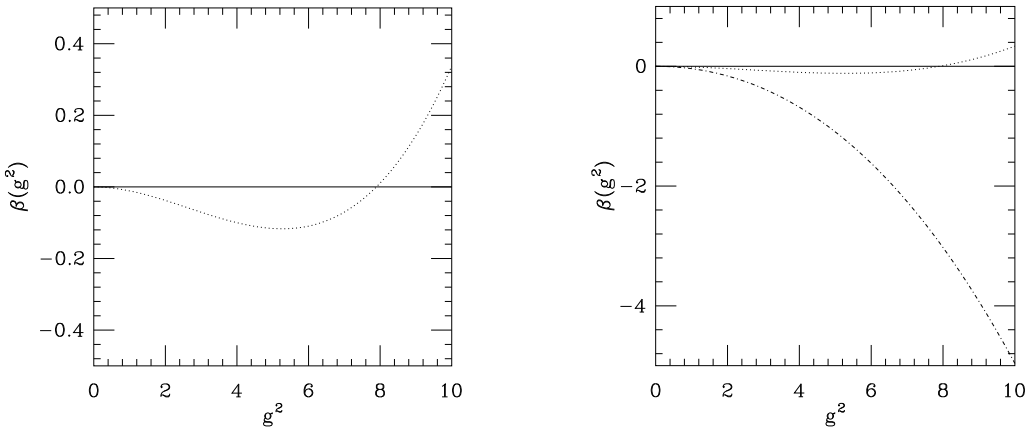

Figure 1: Two-loop beta function in SU(2) gauge theories. See text for explanation.

the different vertical scales. The lesson from this comparison is that measuring a beta function so much smaller than that of a QCD-like theory is bound to be substantially more difficult.

The prime dynamical question about any massless asymptotically free theory is whether its infrared physics is conformal, or, alternatively, confining and with broken chiral symmetry. When the coupling runs slowly, in order to probe interesting values of the renormalized coupling already the bare coupling must be quite strong. As a result, unlike in QCD simulations, lattice perturbation theory is not applicable at the lattice scale in our simulations, and cannot provide us with any guidance. At the same time, as we will see, new analysis methods can be developed that are especially tailored to slow running.

3. Nonperturbative beta function

We use Wilson-clover fermions. The links in the Dirac operator are nHYP smeared links that are subsequently promoted to the fermions' representation. The geometry of our SF lattices is hypercubical with equal size $L = Na$ in all four directions. For most values of the bare parameters studied, we performed simulations for $N = 6, 8, 10, 12, 16$. Full details can be found in Ref. [6].

Instead of the usual beta function (1.1), it is convenient to introduce the beta function $\tilde{\beta}(u)$ for $u \equiv 1/g^2$, define as

$$\tilde{\beta}(u) \equiv \frac{d(1/g^2)}{d\log L} = 2\beta(g^2)/g^4 = 2u^2\beta(1/u). \tag{3.1}$$

Were the beta function $\tilde{\beta}(u)$ constant, the running coupling would take the form

$$u(L) = c_0 + c_1 \log\big(L/(8a)\big)\ , \tag{3.2}$$

where c_0 is $u(L = 8a)$, and c_1 is the constant value of $\tilde{\beta}(u)$. In Fig. 2 we show our results for the running coupling in the two theories. The straight lines are fits to Eq. (3.2) of the results from all volumes at each fixed set of bare parameters. It is evident from this figure that, over the range of volumes we studied, a constant beta function is a reasonable first approximation of the data. As we go upwards in the figure, both the bare and the renormalized couplings get smaller. For reference,

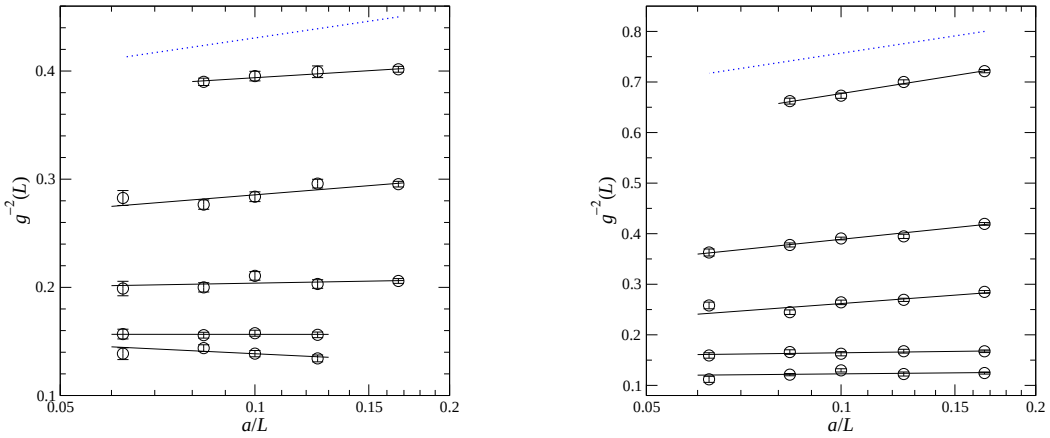

Figure 2: A semilog plot of the running coupling $1/g^2$ vs. a/L in the SU(3)/adjoint theory (left) and the SU(4)/sextet theory (right).

the dotted blue lines show the slopes of one-loop running (note that $\tilde{\beta}(u)$ is constant in the one-loop approximation).

In order to extrapolate our results to the continuum limit we make use of the following observation. If the coupling did not run at all, the only obstruction to Eq. (3.2) would be discretization errors. Much like in a free theory, in the absence of any dynamical scale the discretization errors would necessarily depend on a/L only. Indeed we may then identify the lattice spacing a with $1/L_{min}$, where L_{min} is the smallest lattice size included in the fit (3.2). By repeatedly dropping the smallest lattice, we should get better and better estimates of the continuum-limit value. Ordering all lattice sizes as $L_1 < L_2 < \ldots < L_n$, we denote by $c_0^{(k)}, c_1^{(k)}$ the parameters obtained from a fit in which the smallest size kept was L_k. We can then extrapolate to $a/L = 0$ either linearly,

$$c_1^{(k)} = \tilde{\beta}(u) + C(a/L_k) \tag{3.3}$$

or quadratically,

$$c_1^{(k)} = \tilde{\beta}(u) + C(a/L_k)^2. \tag{3.4}$$

The results of both types of extrapolation, along with the results of the simple fit (3.2), are shown in Fig. 3. As can be seen, substantially bigger computation resources and/or better observables would be required to establish the presence or absence of an IRFP in these theories.

For a very small lattice spacing a, or, equivalently, for very large L, ultimately the linear discretization error must dominate. As it turns out, even in the one-loop approximation linear and quadratic discretization errors remain comparable in size over the entire range of volumes we have. (We discussed this in some detail regarding a different slowly-running theory in Ref. [4].) Therefore there is no good reason to prefer one type of error over the other, and, in principle, we must allow for linear and quadratic discretization errors simultaneously. Since our data are not precise enough to allow for such a combined extrapolation, the results of both types of extrapolation must be considered as models.

Equation (3.2) is exact only in the limit of a constant beta function $\tilde{\beta}(u)$. In reality, as we have discussed in Ref. [2], the slow change in $\tilde{\beta}(u)$ will give rise to higher powers of $\log L$. As a better

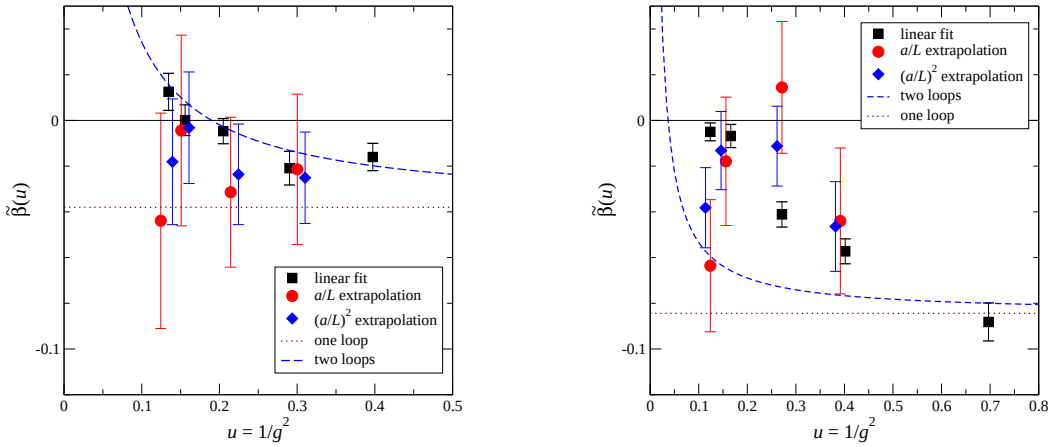

Figure 3: Beta function $\tilde{\beta}(u)$ of the SU(3)/adjoint theory (left) and the SU(4)/sextet theory (right), extrapolated to the continuum limit. The points for the extrapolations have been displaced slightly for clarity.

approximation of the continuum evolution we may therefore take

$$u(L) = c_0 + c_1 \log\big(L/(8a)\big) + c_2 \log^2\big(L/(8a)\big) \ . \tag{3.5}$$

In principle we could then perform a similar continuum-extrapolation procedure by repeatedly dropping the smallest volumes, now using Eq. (3.5) as our basic fit. Once again, our data are not precise enough to obtain meaningful results this way. We stress, however, that the extrapolations (3.3) and (3.4) based on Eq. (3.2) both have good quality, showing that a term like $\log^2$ is unnecessary given our statistical error.

4. Mass anomalous dimension

In the approximation that the coupling does not run at all, the pseudoscalar renormalization constant Z_P follows a power law. Accordingly, for each set of bare parameters, we fit

$$\log Z_P(L) = c_0 + c_1 \log\big((8a)/L\big) \ , \tag{4.1}$$

where now c_1 gives an estimate for the mass anomalous dimension γ_m. We plot the results of these fits in Fig. 4, together with the results of linear and quadratic continuum extrapolations following the same procedure as before. Unlike the beta function, here the error bars remain quite small even after the continuum extrapolation.

Focusing first on the SU4/sextet theory, we see that at weak coupling our results agree with one-loop perturbation theory. But for $g^2 \gtrsim 3$, γ_m levels off, becoming practically independent of g^2. A similar behavior, although a bit noisier, is seen in the SU(3)/adjoint theory. [In the case of the rightmost (strongest coupling) point, we could not overcome the long autocorrelations of the observable. The results marked by the orange brackets come from 3 streams that agreed with each other, after discarding an outlier stream [6].]

The leveling off of γ_m is a remarkable feature, common to all of the theories with fermions in two-index representation we have studied [2, 3, 4, 6]. This is a surprising result, that, to our knowledge, was not predicted by any perturbative calculation.

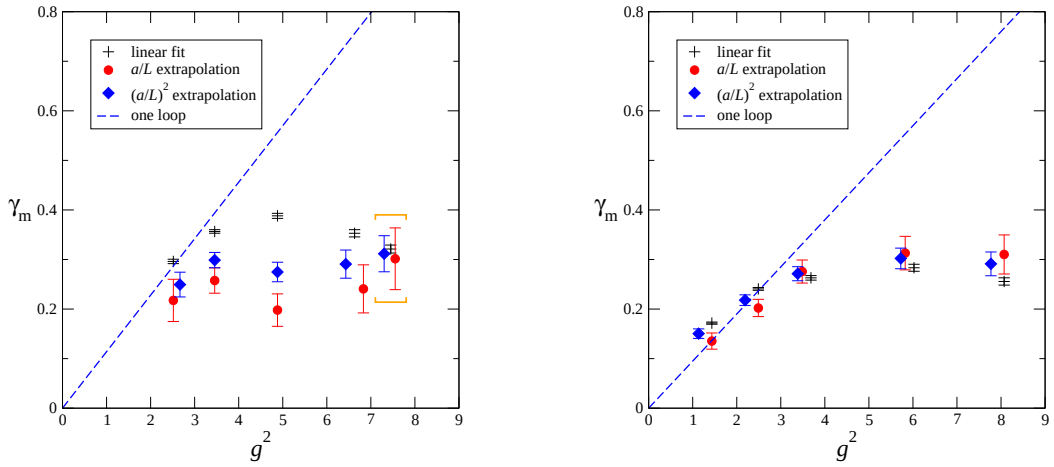

Figure 4: Mass anomalous dimension $\gamma_m(g^2)$ of the SU(3)/adjoint theory (left) and the SU(4)/sextet theory (right), plotted as a function of $g^2(L = 8a)$.

5. N_c scaling

In the course of our work we have studied two gauge theories each containing two Dirac fermions in the adjoint representation: the SU(2) theory [2] and, here, the SU(3) theory. It is interesting to look for trends as N_c is changed [7].

Fig. 5 shows this comparison. Here we only compare the basic linear fits (3.2) for the beta function and (4.1) for the mass anomalous dimension. The results suggest that large-N_c scaling works quite well down to the smallest value $N_c = 2$ (including any discretization error that is present in the plots). We note, however, that unlike the SU(2) theory, where we established the existence of an IRFP, the SU(3) theory could be confining [8].

6. Conclusions

While somewhat disappointing, in view of the difficulties explained in Sec. 2 it is no surprise that the extrapolations of our data for the nonperturbative beta functions result in rather large errors.

Our results for the mass anomalous dimension are much nicer. They have fairly small errors even after the continuum extrapolation. The surprising leveling off at strong coupling leads to a scheme-independent universal bound $\gamma_m < 0.5$, a bound that applies to *all* the theories we have studied in the course of this research program.

A second look at the continuum extrapolations of the beta function of the SU(4)/sextet theory (Fig. 3, right panel) may reveal a hint of the behavior known as "walking," where the beta function first gets very close to zero, and then veers off. Accordingly, after many decades of almost no running, eventually the couplings grows strong enough to trigger chiral symmetry breaking and confinement. Walking theories can naturally accommodate a light composite scalar, which can arise as a pseudo Nambu–Goldstone boson of the spontaneously broken approximate dilatation symmetry. For a recent discussion of whether this scalar could be identified with the 125 GeV particle discovered at the LHC, see Ref. [1].

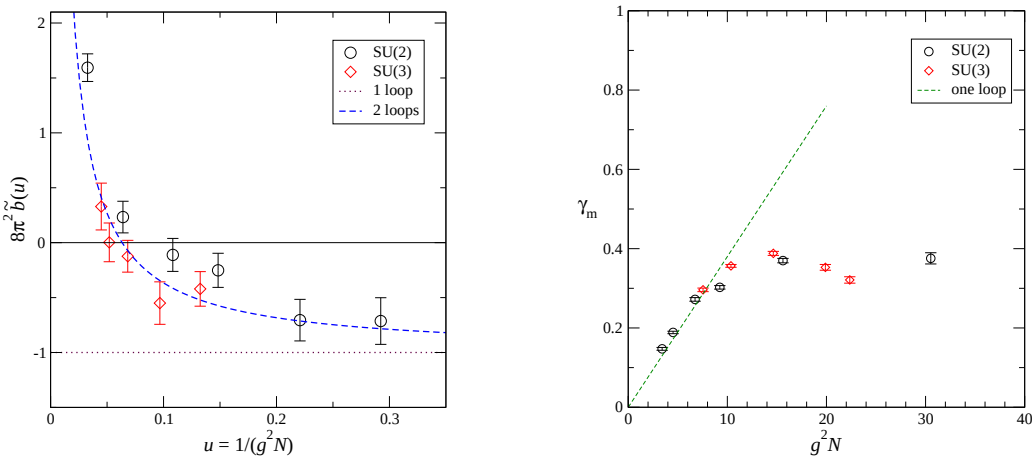

Figure 5: N_c scaling.

Even if walking technicolor can explain the existence of a light Higgs particle, to qualify as a successful theory of Electro-Weak symmetry breaking it has to provide a mechanism for the generation of lepton and quark masses as well. Traditionally, this was done by invoking an "extended" technicolor theory. For this mechanism to meet phenomenological constraints, typically a large mass anomalous dimension, $\gamma_m \approx 1$, was invoked. Our results for γ_m therefore cast doubt on the ability to use any of the theories we have studied as (extended) technicolor candidates.

Acknowledgments

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