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Walking signals in Nf=8 QCD on the lattice

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We investigate walking signals of $N_f = 8$ QCD through meson spectrum using the HISQ (highly improved staggered quark) action. Our data (the pion decay constant, the π and ρ meson masses and the chiral condensate) for the $N_f = 8$ QCD are consistent with the spontaneously broken chiral symmetry in the chiral limit extrapolation of the chiral perturbation theory (ChPT). Remarkably enough, while the $N_f = 8$ data near the chiral limit are well described by the ChPT, those for the relatively large fermion bare mass m_f away from the chiral limit actually exhibit a finite-size hyperscaling relation, suggesting a large anomalous dimension $\gamma_m \sim 1$. This implies that there exists a remnant of the infrared conformality, and suggests that a typical technicolor, "one-family model", as modeled by the $N_f = 8$ QCD can be a walking technicolor theory.

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1. Introduction

The origin of mass is the most urgent issue of the particle physics today. Although the LHC has discovered a 125 GeV boson, there still remain many unsolved problems with the Standard Model (SM), which would require physics beyond the SM. One of the candidates for the theory beyond the SM towards that problem is the Walking Technicolor (WTC) [1] having an approximate scale invariance with a large anomalous dimension $\gamma_m \simeq 1$ due to the "walking" coupling (Fig. 1), which is based on the scale-invariant gauge dynamics [2]. The walking behavior can in fact be realized in the "large N_f QCD". Actually, the $N_f = 8$ is particularly interesting from the model-building point of view: so-called "one-family model" (Farhi-Susskind model [3]). Thus if the $N_f = 8$ turns out to be a walking theory, it would be a great message for the phenomenology, which is to be tested by the on-going LHC. (See Ref. [4] and references therein for the review of the recent lattice studies.)

As in Fig. 1 , the fermion bare mass m_f obviously distorts the ideal behavior of the breaking of the scale symmetry. Then, disregarding the effects of the lattice parameters L and a for the moment[1], we may imagine possible effects of m_f on the walking coupling of our target of study. There is a caveat about the approximate hyperscaling (HS) relation to be expected in Case 2 in Fig. 1. There are two infrared (IR) mass parameters m_D and m_f which violate the IR conformality and hence the possible HS relations for the physical mass quantities should not be universal but do depend on both of them in non-universal ways, in sharp contrast to the HS relation in the conformal window [6].

Remarkably enough in our exploration by the numerical simulation [7], while the $N_f = 8$ data near the chiral limit are well described by the ChPT, those for the relatively large fermion bare mass m_f away from the chiral limit actually exhibit a finite-size hyperscaling (FSHS) relation, suggesting a large anomalous dimension $\gamma_m \sim 1$. It is the first time that the HS relation is observed in a theory with the spontaneous chiral symmetry breaking (SχSB).

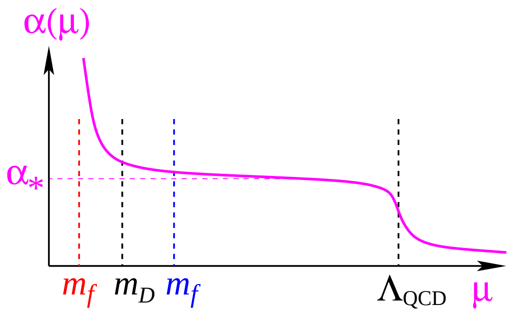

Figure 1: Schematic two-loop/ladder picture of the gauge coupling of the massless large N_f QCD as a walking gauge theory in the SχSB phase near the conformal window. m_D is the dynamical mass of the fermion generated by the SχSB. The effects of m_f would be qualitatively different depending on the cases: Case 1: $m_f \ll m_D$ (red dotted line) well described by ChPT, and Case 2: $m_f \gg m_D$ (blue dotted line) well described by the hyperscaling. The SχSB order parameter on the lattice is not m_D but would be F_π extrapolated to the chiral limit: $F = F_\pi(m_f = 0)$ which would be expected roughly as $m_D = \mathcal{O}(F)$.

[1] In our simulation we use the parameter region where the effect of the system size is subdominant compared to the mass effect. This strategy is different from the one which is advocated by the authors of Refs. [5].

2. Lattice simulation and the results

In our simulation, we use the tree-level Symanzik/HISQ action without the tadpole improvement and the mass correction in the Naik term. It is expected that the flavor symmetry in $N_f = 8$ case [7] and the behavior towards the continuum limit are improved. We carry out the simulation by using the standard Hybrid Monte-Carlo (HMC) algorithm. We measure the mass of the pion M_π, ρ-meson M_ρ, the decay constant of the pion F_π and the chiral condensate $\langle\bar{\psi}\psi\rangle$ as the basic observable to explore the large-N_f QCD.

The pilot study reported in Refs. [8, 9] for $N_f = 8$ is carried out at $\beta(= 6/g^2)$=3.6, 3.7, 3.8, 3.9, 4.0 for various quark masses and on various lattices, $L^3 \times T$ with the fixed aspect ratio as $T/L = 4/3$ for $L =$ 12, 18, 24, 30 and 36. We need to choose as small value of β as possible to obtain a large enough physical volume to minimize the finite-volume effect. From the pilot study mentioned above, we found that $\beta < 3.8$ is too strong to carry out the HMC simulation with HISQ. Therefore, we choose $\beta = 3.8$ in this article. The boundary condition in the temporal direction is the anti-periodic for fermions in HMC. The well-known technique of combining the valence quark propagators solved with periodic and anti-periodic boundary conditions in the temporal direction enables us to have sufficient range for the fitting due to the effectively doubled temporal size.

We performed the analysis based on ChPT and the FSHS; If the simulation region of m_f is in the SχSB region, physical quantities in the spectroscopy, M_H for $H = \pi, \rho, \cdots$ and F_π, are described by the ChPT. The masses and F_π depend on m_f up to chiral log as $M_\pi^2 = C_1^\pi m_f + C_2^\pi m_f^2 + \cdots$, $\quad F_\pi = F + C_1^F m_f + C_2^F m_f^2 + \cdots$, where F is the value in the chiral limit. On the other hand, if the theory is in the conformal window, M_H and F_π on the finite volume are described by the FSHS [10, 11, 12] $\xi_H = \mathscr{F}_H(L m_f^{\frac{1}{1+\gamma_*}})$, where $\xi_p \equiv L M_p$ and $\xi_F \equiv L F_\pi$, for $H = \pi$, ρ or F. The function, $\mathscr{F}_H$, is a some function (unknown *a priori*) of the scaling variable $X = L m_f^{\frac{1}{1+\gamma_*}}$ in which γ_* denotes the mass anomalous dimension γ_m at the infrared fixed point and its value is universal for all channels.

3. Chiral perturbation Theory (ChPT) analysis

3.1 Quadratic fit of F_π and M_ρ

Based on the scenario explained in Fig. 1, we attempt the ChPT analysis in the small m_f. Let us analyze the behavior of F_π in detail, towards the chiral limit.

Table 1 is the result of the quadratic fit of F_π. (The fitted data are the result on the largest volume at each m_f. Namely the fit is done along the envelope towards the chiral limit.) Particularly for the small region, $0.015 \le m_f \le 0.04$, the polynomial fit gives the good χ^2/dof ($= 0.46$). When we include the data at $m_f = 0.05$, χ^2/dof jumps up and this jump might be caused by the instability due to small dof. This suggests that there is a bound, beyond which the ChPT does not describe the data well, and that bound is around $m_f \lesssim 0.05$. With this consideration and the good chiral behavior observed for other quantities (for instance M_ρ and $\langle\bar{\psi}\psi\rangle$) for $m_f = 0.015 - 0.04$, we chose $m_f = 0.015 - 0.04$ for the fitting range of all quantities (roughly corresponding to Case 1 in

Fig. 1). The above analysis[2] suggests that our result in $N_f = 8$ is consistent with SχSB phase with $F = 0.0310(13)$ up to chiral log. Figure 2 shows the ChPT fit[3] in the range $0.015 \le m_f \le 0.04$ and the HS fit in the range $0.05 \le m_f$. As shown in Fig. 2, the behavior in the small m_f is well-described by the ChPT. Therefore $N_f = 8$ QCD is consistent with that of SχSB from the analysis of F_π and M_ρ.

fit range (m_f)	F	$\mathscr{X}\,(m_f^{\rm min} = 0.015)$	$\mathscr{X}\,(m_f = m_{\rm max})$	χ^2/dof	dof
0.015–0.04	0.0310(13)	3.74	11.80	0.46	1
0.015–0.05	0.0278(8)	4.64	19.28	5.56	2
0.015–0.06	0.0284(6)	4.44	23.2	4.09	3
0.015–0.10	0.0311(3)	3.70	37.0	7.85	6
0.015–0.16	0.0349((2)	2.94	54.0	34.2	9

Table 1: Results of chiral fit of F_π with $F_\pi = F + C_1^F m_f + C_2^F m_f^2$ and the the expansion parameter in ChPT, $\mathscr{X} = N_f \left(\frac{M_\pi}{4\pi F/\sqrt{2}}\right)^2$, for various fit ranges.

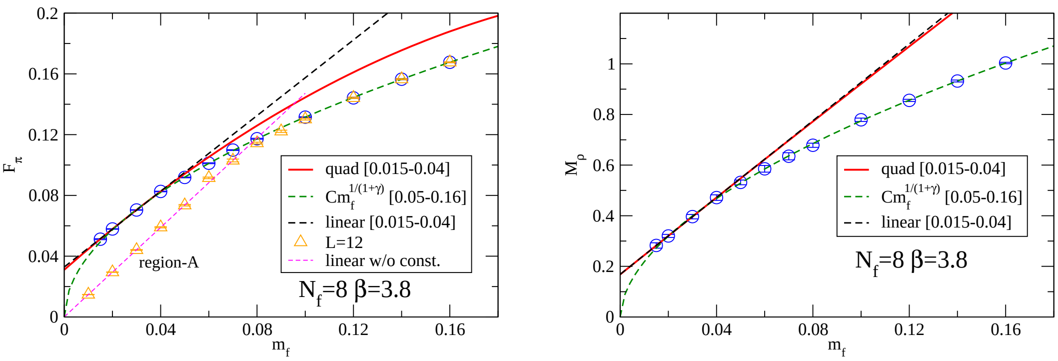

Figure 2: Fitting results of F_π (left) and M_ρ (right). Linear and quadratic fits in $0.015 \le m_f \le 0.04$, Power fit ($y = Cm_f^\alpha$) in $0.05 \le m_f \le 0.16$. The quadratic term does not affect the result in the fitted region of the ChPT. On the other hand, in $0.05 \le m_f \le 0.16$, $F_\pi = 0.431(2) \cdot m_f^{0.515(2)}$ ($\chi^2/\text{dof} = 0.66$) and $M_\rho = 2.75(5) \cdot m_f^{0.550(9)}$ ($\chi^2/\text{dof} = 0.21$).

3.2 Chiral condensate

In this subsection, we analyze the chiral condensate whether this quantity shows the SχSB behavior. We perform a direct measurement $\langle \bar\psi\psi \rangle = \mathrm{Tr}[D_{HISQ}^{-1}(x,x)]/4$ and compare it with the quantity $\Sigma \equiv \frac{F_\pi^2 M_\pi^2}{4m_f}$, through GMOR relation. As seen in Fig. 3, both the results in the chiral limit are non-zero, and are consistent with each other in the small m_f region $0.015 \le m_f \le 0.04$: $\langle \bar\psi\psi \rangle|_{m_f \to 0} = 0.00052(5)$, $\Sigma|_{m_f \to 0} = 0.00059(13)$. We also estimate the chiral condensate in the chiral limit by

[2]The ChPT analysis in this case is self-consistent. The expansion parameter, $\mathscr{X} = N_f \left(\frac{M_\pi}{4\pi F/\sqrt{2}}\right)^2$ in Table 1, is $\mathscr{X} = O(1)$ in our $m_f^{\rm min}$, which is in contrast to the $N_f = 12$ case[6] where $\mathscr{X} \simeq 40$ at $m_f^{\rm min}$.

[3]The left panel of Fig. 2 also shows the $L = 12$ data of F_π, denoted as "region-A" in which F_π linearly goes to zero towards the chiral limit. Thus we don't include such data into the analysis.

multiplying F with the extrapolated value of M_π^2/m_f, as $F^2 \cdot \left(\frac{M_\pi^2}{4m_f}\right)\Big|_{m_f \to 0} = 0.00050(3)$, which is consistent with $\langle\bar{\psi}\psi\rangle$ and Σ in the small m_f defined by the analysis of F_π above. From the analyses up to chiral log of all the observables, F_π, M_π, M_ρ and $\langle\bar{\psi}\psi\rangle$ the chiral property of $N_f = 8$ QCD is consistent with that of SχSB in the small m_f region ($0.015 \leq m_f \leq 0.04$).

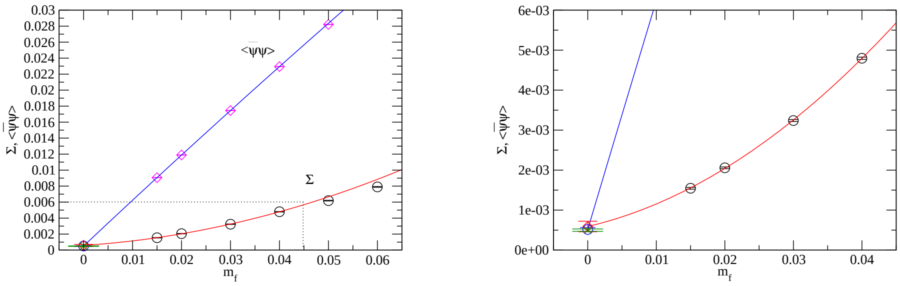

Figure 3: Σ and $\langle\bar{\psi}\psi\rangle$ (left panel) as a function of m_f. The small region, $0 \leq m_f \leq 0.045$ and $0 \leq \Sigma, \langle\bar{\psi}\psi\rangle \leq 0.0006$, in the left panel is enlarged to the right panel. The quadratic fit curves by using the data in $m_f \leq 0.04$ are shown. The green symbol is the value of $F^2 \cdot (\frac{M_\pi^2}{4m_f})|_{m_f \to 0} = 0.00050(3)$.

4. Study of remnants of conformality

We find the two regions of m_f having qualitatively different properties: $m_f = 0.015 - 0.04$ and $m_f = 0.05 - 0.16$ by the analysis of ChPT shown in the previous section. Furthermore, based on the scenario of WTC in Fig. 1, if this theory is near the conformal phase boundary, it is expected that some remnants of the conformal symmetry appear in physical quantities. Figure 2 shows the results of ChPT behavior in $0.015 \leq m_f \leq 0.04$. On the other hand, it is remarkable that in Fig. 2 the fit results in the mass range, $m_f \gtrsim 0.05$, are consistent with the HS behavior, $F_\pi(M_\rho) = C_1 m_f^{1/(1+\gamma)}$. This suggests that, although $N_f = 8$ QCD is in the SχSB phase, there exists a remnant of the conformality in the m_f region away from the chiral limit. Therefore, we will carry out further in depth analysis, which employs the FSHS test with the mass correction term [13], to investigate whether the remnant of the conformality really persists.

4.1 Finite size Hyper-Scaling (FSHS) fits with the correction term

If the system is in the conformal window, the data on a finite volume is in good agreement with the FSHS having a universal value of $\gamma = \gamma_*$ at IRFP for observables as $\xi_H = \mathscr{F}_H(L m_f^{\frac{1}{1+\gamma_*}})$. In general our data of $N_f = 8$ cannot satisfy the FSHS with universal γ in the whole range of m_f [7], because we showed that the theory is in the SχSB phase[4]. However, because of the power behavior in the middle range of the fermion mass as mentioned in Fig. 2, we carry out the FSHS test with the mass correction term [13] in our data to find a remnant of the conformality.

Since $N_f = 8$ theory is in SχSB phase, FSHS cannot become accurate by approaching to the chiral limit, which is in contrast to the $N_f = 12$ case [6]. Since it might be possible to obtain a

[4] The exponents for F_π and M_ρ in the power function fit shown in Fig. 2 correspond to $\gamma = 0.94$ and 0.81 respectively, being non-universal value in the naïve HS relation.

common value of the γ, FSHS is only expected for larger mass region, where mass corrections may not be negligible [13], $\xi_H = C_0^H + C_1^H X + C_2^H L m_f^\alpha$ for $m_f \geq 0.05$ and $\xi_\pi \geq 8$. We perform simultaneous fit [7] in this region[5] of m_f and ξ_π using M_π, F_π, and M_ρ with a common γ. We carry out simultaneous FSHS fits with the correction term, since a universal γ would be expected if the theory is very close to the conformal phase boundary even in the SχSB phase. The example with $\alpha = 1$ of this analysis is shown in Fig. 4, as a typical result of the simultaneous fit. Under the assumption that all the observables give a universal γ, we estimate $\gamma = 0.78$–0.93 with $\chi^2/\text{dof} = O(0.1)$. These estimated values of γ would be identified as the mass anomalous dimension in the walking regime.

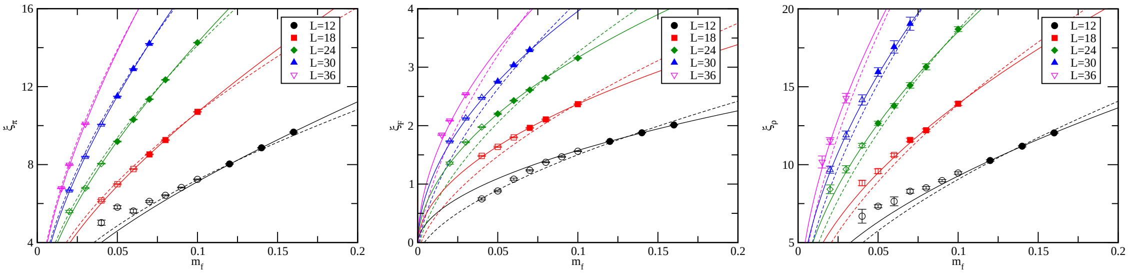

Figure 4: Simultaneous FSHS fit in ξ_π(left), ξ_F(center) and ξ_ρ(right) with $\alpha = 1$. The filled symbols are included in the fit, but the open symbols are omitted. The fitted region is $m_f \geq 0.05$ and $\xi_\pi \geq 8$. The solid curve is the fit result. For a comparison, the simultaneous fit result without correction terms is also plotted by the dashed curve, whose $\chi^2/\text{dof} = 83$.

5. Summary and Discussion

In search for a candidate for the Walking Technicolor, we have investigated meson spectrum of $N_f = 8$ QCD by the lattice simulations based on the HISQ action for $\beta = 6/g^2 = 3.8$ [7]. We found that the data of F_π, M_π, M_ρ and $\langle\overline{\psi}\psi\rangle$ are consistent with the SχSB well described by the ChPT in the small m_f region, $0.015 \leq m_f \leq 0.04$, (Case 1 in Fig. 1 for a schematic view), suggesting that $F = 0.031(1)(^{+2}_{-10})$, $\langle\overline{\psi}\psi\rangle|_{m_f\to 0} = 0.00052(5)(^{+8}_{-29})$, and $\frac{M_\rho}{F/\sqrt{2}} = 7.7(1.5)(^{+3.8}_{-0.4})$ in the chiral limit extrapolation with the systematic error. Remarkably enough, in contrast to the data near the chiral limit ($m_f \leq 0.04$) indicating the SχSB, those for the relatively large fermion bare mass $m_f \geq 0.05$ away from the chiral limit actually exhibited a FSHS with the scaling exponent $\gamma(M_\pi) \simeq 0.57$, $\gamma(F_\pi) \simeq 0.93$ and $\gamma(M_\rho) \simeq 0.80$. This, non-universal value of γ, implies that there exists a remnant of the IR conformality where the SχSB effects are negligible (Case 2 in Fig. 1). Therefore, there could exist large mass corrections on the FSHS. If we include possible mass corrections on FSHS, we obtained $0.78 \lesssim \gamma \lesssim 0.93$ in simultaneous FSHS fits for $m_f \geq 0.05$ and $\xi_\pi \geq 8$. Summarizing all our analyses we may infer that a typical technicolor ("one-family model") as modeled by the $N_f = 8$ QCD can be a walking technicolor theory having an approximate scale invariance with large anomalous dimension $\gamma_m \sim 1$.

[5] It is noted that a simultaneous fit including the lighter mass with $m_f \geq 0.015$ in $\xi_\pi \geq 6.8$ fails with a large $\chi^2/\text{dof} = 3.5$ even if the mass correction is included. This is because the chiral property is dictated by SχSB and should not be consistent with universal HS near the chiral limit.

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Finally, we should comment on the possible light flavor-singlet scalar meson in $N_f = 8$ QCD. The walking technicolor predicts a light composite Higgs-like scalar boson (techni-dilaton) as a pseudo Nambu-Goldstone boson of the approximate scale invariance. We made the studies [14] of both flavor-singlet scalar and scalar glueballs for $N_f = 12$, in which we found a hint of a flavor-singlet scalar bound state lighter than π for $N_f = 12$. Then, if the $N_f = 8$ QCD behaves as a walking theory with approximate scale invariance, it would be expected that a light flavor-singlet scalar composite does exist. These studies are currently under way [15].

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