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First order thermal phase transition with 126 GeV Higgs mass

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We study the strength of the electroweak phase transition in models with two light Higgs doublets and a light $\mathrm{SU}(3)_c$ triplet by means of lattice simulations in a dimensionally reduced effective theory. In the parameter region considered the transition on the lattice is significantly stronger than indicated by a 2-loop perturbative analysis. Within some ultraviolet uncertainties, the finding applies to MSSM with a Higgs mass $m_h \approx 126\,\mathrm{GeV}$ and shows that the parameter region useful for electroweak baryogenesis is enlarged. In particular (even though only dedicated analyses can quantify the issue), the tension between LHC constraints after the 7 TeV and 8 TeV runs and frameworks where the electroweak phase transition is driven by light stops, seems to be relaxed.

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1. Introduction

Electroweak baryogenesis (EWBG) [1] is an attractive scenario for the generation of the baryon number of the Universe. It requires the existence of a relatively strong first order thermal electroweak phase transition. As the Universe cools down from temperatures above the electroweak scale $T \approx T_{\rm EW} \sim 100\,$GeV, the metastability associated with first order transitions leads to thermal non-equilibrium, which is one of the necessary Sakharov conditions for successful baryogenesis. The other conditions (C, CP and baryon number violation) exist in the Standard Model (SM) and its simple extensions. Therefore, it is well motivated to study the thermodynamics of the electroweak "symmetry breaking" phase transition in these theories.

It was established already more than 15 years ago, however, that actually the SM does not have a strong first order phase transition. Indeed, in a series of lattice simulations [2]–[5] it was unambiguously shown that the transition is a smooth cross-over at Higgs masses larger than about 72 GeV. This value was well below the LEP limits of the time. Thus, it was necessary to look beyond the SM for possibilities for EWBG.

A strong first order phase transition is possible in several extensions of the SM, in particular in the Minimal Supersymmetric Standard Model (MSSM) if the right-handed stop, the scalar partner of the top quark, is sufficiently light and the left-handed stop is heavy [6]–[15]. Although tightly constrained, perturbative analyses of phase transition properties as well as of current LHC bounds indicate that the parameter space for MSSM EWBG may still be open [16]–[21].

However, the accuracy of perturbative analyses of phase transition properties is limited by the infrared singularities inherent to thermal field theory [22]: the physics of the momentum scales $p \sim g^2T/\pi$ is non-perturbative, even at weak coupling $g \ll \pi$. For reliable results it is thus necessary to use numerical lattice simulations. A striking example of these problems is provided by the phase transition in the SM: perturbation theory predicts that the transition becomes weaker but remains of first order as the Higgs mass increases, in contrast to the lattice results mentioned above which indicate that the transition ceases to exist for $m_H > 72\,$GeV.

Three-dimensional (3d) dimensionally reduced *effective theories* provide for a particularly convenient and successful tool for studying the thermodynamics of weakly coupled theories on the lattice. The effective theory is derived from the original four-dimensional (4d) theory using perturbative methods, in a computation which suffers from no infrared problems. The infrared sector of the original theory is fully transferred to the effective theory where it can be studied by lattice simulations (just in three dimensions and thus less demanding than in four, particularly recalling that both light and heavy chiral fermions are present in the SM and its extensions). Most of the above results concerning the SM phase transition were obtained using the effective theory approach [2]–[4]. It has also been successfully applied to high-temperature QCD (cf. e.g. refs. [23, 24]), although in that case the accuracy is limited by the larger gauge coupling.

The effective theory approach was used to study the phase transition in the MSSM in a series of papers in 1998–2001 [25]–[27]. The parameters used in those computations were chosen following the ever increasing LEP bounds, eventually extending up to $m_H \sim 115\,$GeV. The experimental discovery of the Higgs particle at 126 GeV thus motivates us to revisit the MSSM phase transition on the lattice. This was recently achieved in ref. [28], whose results we review here.

The results obtained should, however, be more generic than just for the specific case of MSSM.

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Indeed, we expect that *(i)* the electroweak phase transition is stronger than perturbatively estimated in a broad neighbourhood of the effective-theory parameter point analyzed here, and that *(ii)* several 4d theories with phase transitions driven by light colored (SU(3)$_c$ triplet) scalars can fall into this neighbourhood. Trivial examples are non-minimal supersymmetric models with a light scalar sector consisting of two Higgs SU(2)$_L$ doublets and a right-handed stop.

2. Effective theory

Dimensionally reduced effective theories are by now well established as reliable tools to study weakly coupled field theories at high temperatures. Starting from the original 4d theory at finite temperature, the momentum modes $p \sim \pi T$ and $p \sim gT$ are integrated out in stages. The resulting theory contains only the non-perturbative soft sector $p \sim g^2 T/\pi$. Because all non-zero Matsubara modes have $p \sim \pi T$, all fermion modes are integrated out and the resulting theory lives in 3d (dimensional reduction). The parameters of the effective theory depend on the parameters of the 4d theory, which in turn are set by physical observables (e.g. pole masses).

We consider a 4d theory with weak SU(2)$_L$ and strong SU(3)$_c$ gauge fields, two colorless Higgs SU(2)$_L$ doublet scalars, H_1 and H_2, and an SU(3)$_c$ triplet scalar which is SU(2)$_L$ singlet. The masses of the scalars are at the electroweak scale. The 3d Lagrangian describing this theory has the most general form allowed by symmetries:

$$
\begin{aligned}
\mathscr{L}_{3\text{d}} = {} & \frac{1}{2}\,\mathrm{Tr}\, G_{ij}^2 + (D_i^s U)^\dagger (D_i^s U) + m_U^2\, U^\dagger U + \lambda_U\, (U^\dagger U)^2 \\
& + \gamma_1\, U^\dagger U H_1^\dagger H_1 + \gamma_2\, U^\dagger U H_2^\dagger H_2 + \Big[\gamma_{12}\, U^\dagger U H_1^\dagger H_2 + \text{H.c.}\Big] \\
& + \frac{1}{2}\,\mathrm{Tr}\, F_{ij}^2 + (D_i^w H_1)^\dagger (D_i^w H_1) + (D_i^w H_2)^\dagger (D_i^w H_2) \\
& + m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 + \Big[m_{12}^2 H_1^\dagger H_2 + \text{H.c.}\Big] \\
& + \lambda_1\, (H_1^\dagger H_1)^2 + \lambda_2\, (H_2^\dagger H_2)^2 + \lambda_3\, H_1^\dagger H_1 H_2^\dagger H_2 + \lambda_4\, H_1^\dagger H_2 H_2^\dagger H_1 \\
& + \Big[\lambda_5\, (H_1^\dagger H_2)^2 + \lambda_6\, H_1^\dagger H_1 H_1^\dagger H_2 + \lambda_7\, H_2^\dagger H_2 H_1^\dagger H_2 + \text{H.c.}\Big]\,.
\end{aligned}
$$

Here G_{ij} and F_{ij} are the SU(3)$_c$ and SU(2)$_L$ field strength tensors, respectively, and the parameters g_w^2, g_s^2, m_i^2, λ_j and γ_i have a well-defined dependence on the original 4d parameters (including the temperature T).

Had we added further particles in the 4d theory that are Boltzmann suppressed during the transition, or extra light fermions that do not become strongly coupled in either of the phases, then the 3d Lagrangian would still be the same and only the relations between 3d and 4d parameters would change. An example of a theory leading to the above $\mathscr{L}_{3\text{d}}$ is the MSSM when both Higgses and right-handed stops are light (to a good approximation, further MSSM scalars could be light too if weakly coupled to the light stop-Higgs sector). In such a case the 3d and 4d parameters are related as described in refs. [28, 29]. We focus on this model in the following.

We fix the parameters so that the masses of the lightest CP-even Higgs and right-handed stop are $m_H \approx 126\,\text{GeV}$ and $m_{\tilde{t}_R} \approx 155\,\text{GeV}$, respectively. Other squarks are heavy, $m_Q \gtrsim 7\,\text{TeV}$. For the remaining parameters we refer to ref. [28]. The large scale m_Q may induce large logarithms that have to be resummed for precise relations between 4d parameters and physical observables [31].

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$\beta_w = 4/(g_w^2 T^* a)$	volumes
8	12^3, 16^3
10	16^3
12	16^3, 20^3, 32^3, $12^2\times 36$, $20^2\times 40$
14	24^3, $14^2\times 42$, $24^2\times 48$
16	24^3, $16^2\times 48$, $20^2\times 60$, $24^2\times 72$
20	32^3, $20^2\times 60$, $26^2\times 72$, $32^2\times 64$
24	24^3, 32^3, 48^3, $24^2\times 78$, $30^2\times 72$
30	48^3

Table 1: Lattice spacings and volumes used in the simulations of ref. [28].

We do not perform this refinement here and uncertainties of several GeV may be present, especially in $m_{\tilde{t}_R}$. This ambiguity could be reduced with full 2-loop dimensional reduction, as was done for the SM [30]. To be reminded of the uncertainties we label the 4d parameters with an asterisk (*).

It should be stressed, however, that the ambiguity affects only 4d parameters. The theory we analyze by 3d perturbation theory and by 3d lattice simulations, although it might correspond to slightly different 4d observables than anticipated, is exactly the same. The comparison of perturbation theory and lattice results is therefore unambiguous within the effective theory.

3. Simulations and results

The lattice discretization of the theory is described in ref. [27]. Because the effective theory is super-renormalizable and all counterterms are known, the lattice parameters do not require any tuning and in the continuum limit the results can be directly compared with perturbative ones.

The lattice spacing a is parameterized through the SU(2)$_L$ gauge coupling: $\beta_w = 4/(g_w^2 T^* a)$. The simulation volumes are listed in table 1. Because the theory is fully bosonic, the simulations are inexpensive and we can access a range of almost four in lattice spacings. The update algorithm is a combination of heat bath and over-relaxation updates.

In fig. 1(left) we show a temperature scan of the continuum-extrapolated H_2 and stop condensates. (H_1 is much heavier and in practice inert in the transition.) The first-order nature is evident, as is the fact that the stop field responds to the jump in the Higgs condensate.

Most of our simulations are performed at the critical temperature, employing multicanonical techniques as described in ref. [27]. Some of the resulting probability distributions of $\langle H_2^\dagger H_2\rangle$ are shown in fig. 1(right). These distributions enable precise measurements of the critical temperature (equal area of the peaks), Higgs condensate discontinuity $v(T_c^*)$, the latent heat of the transition and the tension of the interface between the symmetric and broken phases. The continuum extrapolations of the critical temperature and the Higgs condensate are shown in fig. 2, using a linear + quadratic fit. At $\beta_w = 4/(g_w^2 a T^*) \geq 14$ the cutoff effects are small and a robust continuum limit can be obtained. At coarser lattice spacings the cutoff effects are sizable, which can be attributed to the rather heavy mass of the H_1 field.

The numerical results are compared with 2-loop perturbative ones in table 2. The transition is significantly stronger than indicated by the perturbative analysis. This is also evidenced by the

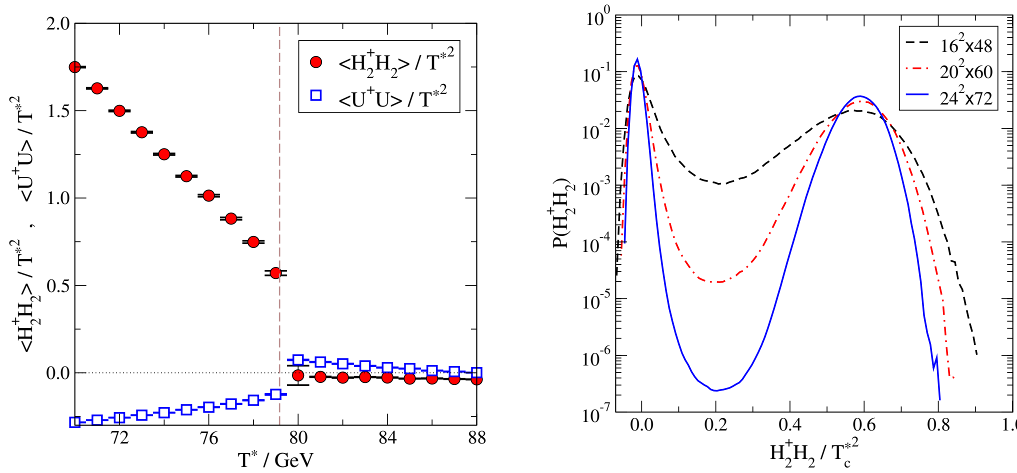

Figure 1: Left: continuum-extrapolated expectation values of $\langle H_2^\dagger H_2\rangle$ and $\langle U^\dagger U\rangle$, both renormalized in the $\overline{\text{MS}}$ scheme, as functions of the temperature. Right: the probability distribution of $\langle H_2^\dagger H_2\rangle$ at T_c^* using different volumes. Both plots are from ref. [28].

	lattice	2-loop
Transition temperature T_c^*/GeV	79.17(10)	84.4
Higgs discontinuity v/T_c^*	1.117(5)	0.9
Latent heat $L/(T_c^*)^4$	0.443(4)	0.26
Surface tension $\sigma/(T_c^*)^3$	0.035(5)	0.025

Table 2: Comparison of lattice and perturbative results, both for the same 3d effective parametrization [28].

Higgs condensate discontinuity shown in fig. 3: the transition becomes stronger as we go from 1 to 2 loops and then to lattice, and the transition temperature is simultaneously decreased. This is in line with the previous experiences at $m_H \lesssim 115$ GeV [27].

4. Conclusions

By means of dimensional reduction and 3d lattice simulations, we have studied the finite temperature phase transition in models with two Higgs SU(2)$_L$ doublets and a SU(3)$_c$ triplet as light scalar degrees of freedom. In the parameter region considered, the transition on the lattice is stronger than indicated by 2-loop perturbation theory. In particular, it is strong enough for EWBG: as confirmed by real-time simulations within the SM [32]–[34] the sphaleron rate is strongly correlated with v/T and for the value quoted in table 2 so suppressed that (if no sizeable magnetic background is present [35, 36]) it does not erase any baryon asymmetry generated. A strong transition also implies significant supercooling and non-trivial dynamics, leading to a gravitational wave signal [37], although probably not large enough to be observable in the foreseeable future.

Our result applies to any model with the above light scalar fields participating in the transition, but in particular it covers many supersymmetric frameworks. Within some ultraviolet uncertainties, it shows that there seems to be room for EWBG in MSSM with $m_H \approx 126$ GeV and $m_{\tilde{t}_R} \lesssim 155$ GeV. If the latter bound were confirmed by more precise analyses, MSSM baryogenesis would be less

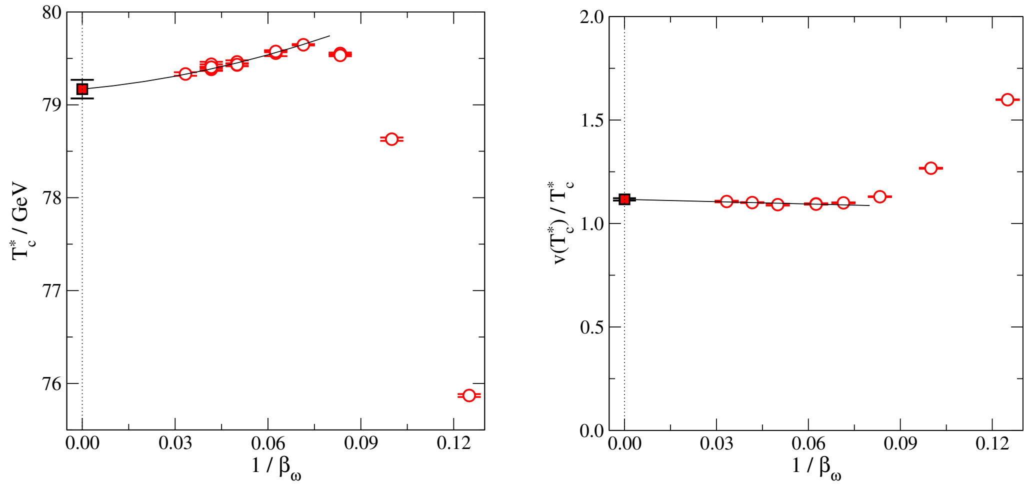

Figure 2: Continuum extrapolations for the critical temperature (left) and the Higgs condensate (right), the latter defined on the lattice as $v^2 \equiv 2\Delta\langle\sum_{i=1}^{2} H_i^\dagger H_i\rangle$. Both plots are from ref. [28].

in tension with LHC data than previously estimated, with $m_{\tilde{t}_R} \lesssim 110\,\text{GeV}$ [17]. Indeed, constraints from Higgs searches would relax as stop enhancement in Higgs gluon fusion would reduce, and loopholes in LHC stop analyses [17] might open up.

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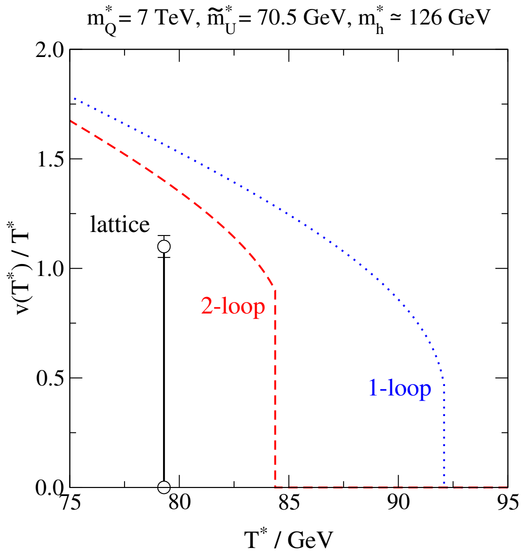

Figure 3: Comparison of 1-loop, 2-loop and lattice results for the properties of the phase transition (from ref. [28]). To be precise the perturbative $v(T^*)$ refers to that in the Landau gauge.

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