

More effects of Dirac low-mode removal

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In previous studies we have shown that hadrons, except for a pion, survive the removal of the lowest lying Dirac eigenmodes from the valence quark propagators. The low-modes are tied to the dynamical breaking of chiral symmetry and we found chiral symmetry to be restored by means of matching masses of chiral partners, like, e.g., the vector and axial vector currents. Here we investigate the influence of removing the lowest part of the Dirac spectrum on the locality of the Dirac operator. Moreover, we analyze the influence of low-mode truncation on the quark momenta and thereupon on the hadron spectrum and, finally, introduce a reweighting scheme to extend the truncation to the sea quark sector.

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1. Motivation

In previous work [1–4] we studied the effects of artificially removing the chiral condensate from the valence quark sector. We excluded a variable number of the lowest eigenmodes of the hermitian Dirac operator $D_5 \equiv \gamma_5 D$ from the valence quark propagators and subsequently investigated the hadron spectrum within this approximation. While the exponential decay in the pion channel got lost, the correlators of all other mesons and baryons we studied retained an exponentially decaying signal with significantly improved signal-to-noise ratios. Therefore we were able to extract masses of the hadron states within our approximation. In Fig. 1 we show the masses of the vector and axial vector mesons as a function of the truncation level.

We observe degenerate masses of the would-be chiral partners ρ and a_1 after having subtracted modes up to ~ 30 MeV (twice the bare quark mass in our setup) of the spectrum of D_5 which indicates the restoration of the dynamically broken chiral symmetry. While the matching of the two masses in Fig. 1 is expected, the increasing behaviour is rather counter intuitive and will be explored here.

Another indication of a restored chiral symmetry is the loss of dynamically generated mass in the quark propagator. In [3] it has been shown that when removing the low-lying part of the Dirac spectrum, the quark mass function (in Landau gauge) indeed becomes flat. When comparing the amount of dynamically generated mass as a function of the truncation level with the splitting of the would-be chiral partners ρ and a_1 , see Fig. 2, we obtain different truncation levels for the restoration of the chiral symmetry. This discrepancy will be addressed here as well.

The remainder is organised as follows: in Sec. 2 we investigate the violation of locality within our approximation and in Sec. 3 the issue of increasing hadron masses under Dirac low-mode truncation is studied. In Sec. 4 the possible extension of the approximation to the sea quark sector is explored before concluding in Sec. 5.

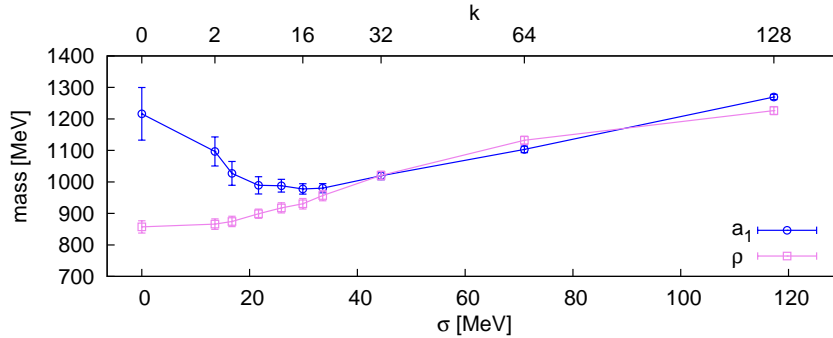


Figure 1: The masses of the vector and axial vector mesons, from left to right removing increasingly more and more Dirac low-modes.

2. Locality properties of the truncated CI Dirac operator

Locality of the Dirac operator is a vital property for a quantum field theory since it ensures the causality of the theory. The original CI Dirac operator is, like the Wilson operator, *ultra local* by

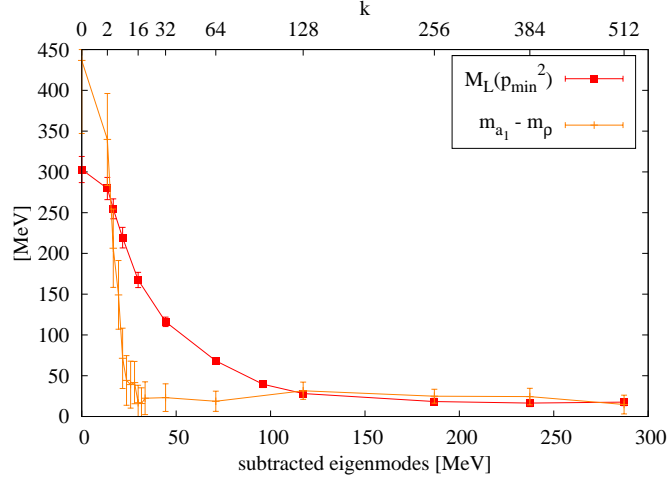


Figure 2: A comparison of the amount of dynamically generated mass in the quark mass function and the splitting of ρ and a_1 as a function of the truncation level.

definition. The Neuberger overlap operator violates locality at finite lattice spacing [5]. The authors of [6] have shown, though, that the nonlocal contributions of the overlap operator fall exponentially with the distance r/a to the source point and thus locality will eventually be restored in the naive continuum limit. It is not clear a priori to what extent the exclusion of the lowest lying part of the Dirac eigenspectrum violates the locality of a Dirac operator like the CI operator. The latter will be analyzed here.

We project out a single column of the hermitian Dirac operator

$$\psi(x)^{[x_0, \alpha_0, a_0]} = \sum_y D_5(x, y) \eta(y)^{[x_0, \alpha_0, a_0]}, \quad (2.1)$$

where we use matrix/vector notation in color and Dirac space and the multiindex $[x_0, \alpha_0, a_0]$ labels a point-source

$$\eta_a^\alpha(y)^{[x_0, \alpha_0, a_0]} = \delta(y - x_0) \delta_{\alpha\alpha_0} \delta_{aa_0}. \quad (2.2)$$

In practice we set $x_0 = 0$ without loss of generality. Eq. (2.1) defines a set of 12 columns of the Dirac matrix, one for each combination of color a_0 and Dirac α_0 indices.

Next, we define a function that serves as an upper bound of the contributions from $\psi(x)$ as a function of the distance r to the source while taking the periodic boundary conditions of the lattice into account:

$$f(r) = \max_{x, \alpha_0, a_0} \{ \|\psi(x)\| \mid |x| = r \}, \quad (2.3)$$

where $\|\cdot\|$ is the usual vector norm over the internal color and Dirac structures of $\psi(x)$. Then we can analyze the expectation value $\langle f(r) \rangle$, which serves as a measure for the violation of locality.

We study (2.3) for the low-mode truncated Dirac operator D_5 , i.e., we consider columns of the truncated operator

$$\psi(x)_{\text{red}(k)}^{[x_0, \alpha_0, a_0]} = \sum_y D_5(x, y) \eta(y)^{[x_0, \alpha_0, a_0]} - \sum_{i=0}^k \mu_i v_i(x) \sum_y v_i(y)^\dagger \eta(y)^{[x_0, \alpha_0, a_0]} \quad (2.4)$$

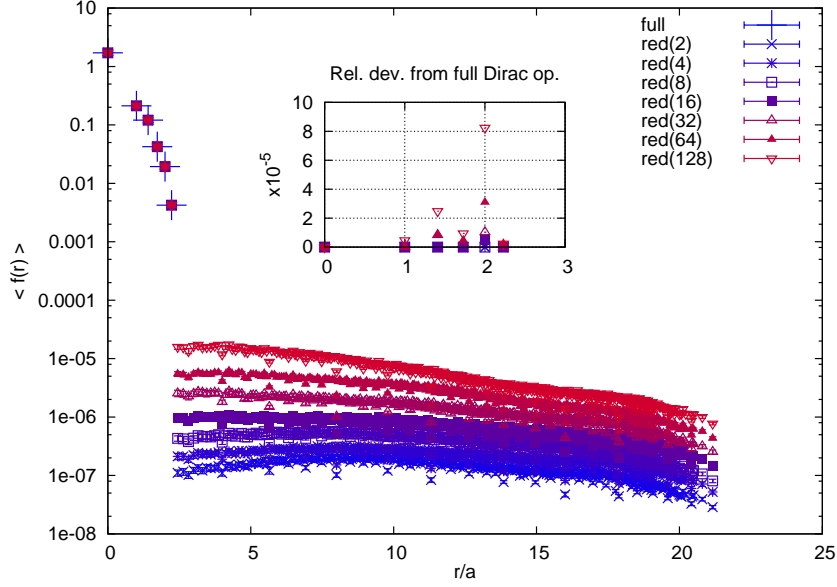


Figure 3: The expectation value $\langle f(r) \rangle$ for the truncated CI Dirac operator. Small nonlocal contributions are present.

where the μ_i are the eigenvalues of D_5 and v_i the corresponding eigenvectors. From these we calculate $\langle f(r) \rangle$ for truncation steps in powers of 2 from $k = 2, \dots, 128$, see Fig. 3. First, we observe that the deviation from the nonzero contributions of the full operator are of the order of 10^{-5} , as can be seen from the inner plot of the figure, and thus are very small. Moreover, the truncated Dirac operator collects some nonlocal contributions of similar order, $10^{-8} - 10^{-5}$, from distant points on the lattice.

This investigation shows that for our specific setup the nonlocal contributions of the truncated Dirac operator are very small and the locality of the theory is conserved in good approximation.

3. Mass generation

In [3] we investigated the quark propagator in Landau gauge¹ under Dirac low-mode removal, with the goal of finding the effects on the quark wavefunction renormalization function and on the quark mass function, which exhibits the dynamical generation of mass due to the dynamical chiral symmetry breaking. The first observation was that removing the lowest Dirac modes causes the dynamical mass generation to cease, while the value of the bare mass (which is determined by the magnitude of the lowest Dirac eigenmodes) is not affected. The truncation level $k \approx 128$, at which the quark mass function appears flat, must coincide with the level of complete removal of the chiral condensate from the valence quark sector. Not only the mass function is affected by Dirac low-mode truncation, the quark wavefunction renormalization function has also been found to be strongly suppressed when subtracting more and more low-modes. Infrared suppression of the wavefunction renormalization function, which appears as an overall factor of the renormalized

¹The code package *cuLGT* [7, 8] for gauge fixing on graphic processing units has been used to fix the gauge.

quark propagator, can be interpreted as suppression and eventual extinction of low-momentum quarks in the spectrum. The latter is in accordance with the fact that the low-momentum states of quarks are directly connected to low Dirac eigenvalues as can be derived explicitly for free quarks.

Matching of the masses of most of the chiral partners is found at an earlier truncation level than suggested by the vanishing of the dynamically generated mass of quarks: degenerate states of, e.g., the vector and axial vector currents are observed at truncation level $k \approx 16$ while the chiral condensate decreases with the truncation level until it disappears completely only at $k \approx 128$ as shown in Fig. 2. This can be explained by taking the increased momenta of the quarks under Dirac low-mode removal into account. The early chiral restoration, as displayed by the degeneracy of states, must be an effective restoration that is a combination of two underlying phenomena: first, the dynamically generated mass of the quarks has shrunk to about sixty percent of its original value and second, the momenta of the quarks are increased such that the effective dynamical mass at that momentum value tends towards zero.

4. The sea quark sector

Throughout our work we used gauge configurations with two dynamical fermions and only low-mode truncated the valence quark sector. Here we elaborate how the sea quark sector can in principle be low-mode truncated *a posteriori* via a reweighting procedure of the configurations.

We recall that we calculate observables \mathcal{O} on the lattice via

$$\langle \mathcal{O}[U] \rangle = \frac{\int \mathcal{D}U e^{-S_G[U]} \det D_u \det D_d \mathcal{O}[U]}{\int \mathcal{D}U e^{-S_G[U]} \det D_u \det D_d}. \quad (4.1)$$

In the latter expression the fermionic degrees of freedom have been integrated out giving rise to the fermion determinant. In the following we assume the two light quark flavors to be degenerate.

The fermion determinant can be written as the product of the Dirac eigenvalues λ_i and therefore can be split into the product of a low-mode part and a reduced part

$$\det D_{\text{lm}(k)} = \prod_{i \leq k} \lambda_i, \quad \det D_{\text{red}(k)} = \prod_{i > k} \lambda_i. \quad (4.2)$$

Then we can formally include a weight w_k , which we define as

$$w_k \equiv (\det D_{\text{lm}(k)})^{-2}, \quad (4.3)$$

into (4.1) in order to cancel the low-mode contribution of the fermion determinant

$$\langle \mathcal{O}[U] \rangle_{w_k} = \frac{\int \mathcal{D}U e^{-S_G[U]} (\det D_{\text{red}(k)})^2 \mathcal{O}[U]}{\int \mathcal{D}U e^{-S_G[U]} (\det D_{\text{red}(k)})^2}. \quad (4.4)$$

Consequently, only the reduced part of the fermion determinant remains in the path integral to represent the sea quarks. Replacing the integration over the gauge fields in (4.4) with a Monte Carlo integration, this is equivalent to

$$\langle \mathcal{O}[U] \rangle_{w_k} \approx \frac{\sum_n \mathcal{O}[U_n] w_k[U_n]}{\sum_n w_k[U_n]} \quad (4.5)$$

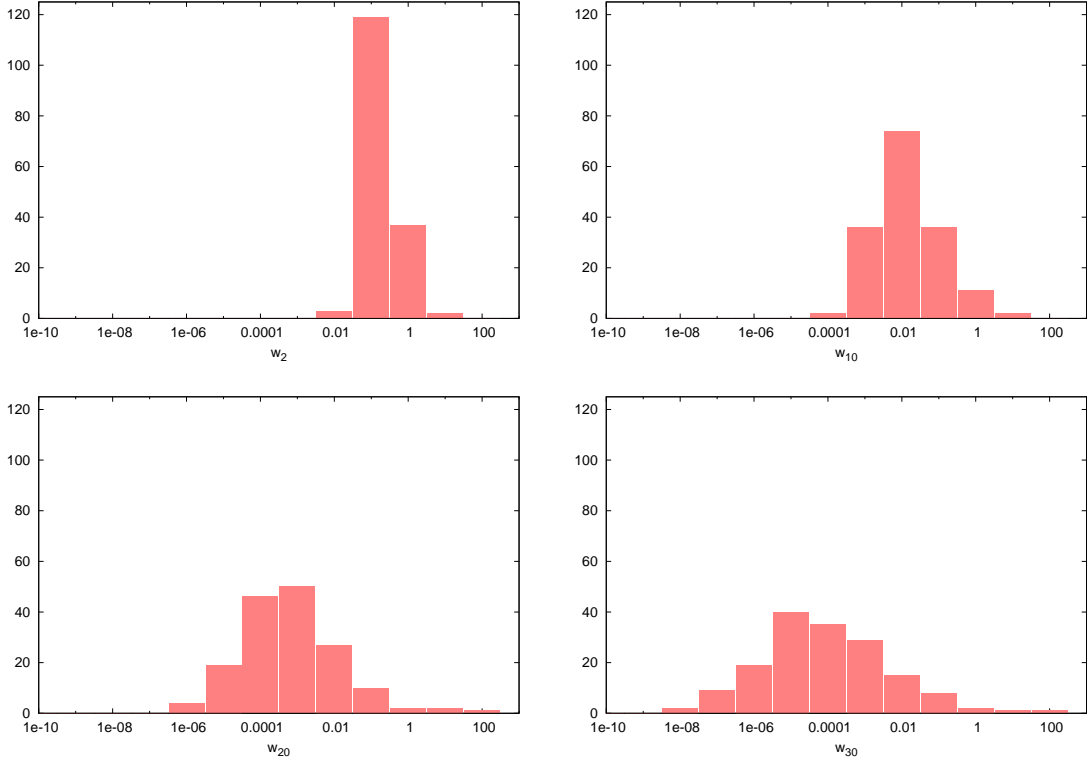


Figure 4: Histograms of the values \bar{w}_k for $k = 2$ (top left), $k = 10$ (top right), $k = 20$ (bottom left) and $k = 30$ (bottom right) from a set of 161 gauge field configurations.

where the finite number of gauge field configurations U_n , $n = 1, \dots, N$, have been generated with the standard weight factor $e^{-S_G[U]} (\det D)^2$.

We can bring (4.5) in a form more similar to the unweighted case by defining the ratio

$$\bar{w}_k[U_n] \equiv \frac{w_k[U_n]}{\sum_n w_k[U_n]} \cdot N. \quad (4.6)$$

We can rewrite (4.5) to obtain

$$\langle \mathcal{O}[U] \rangle_{w_k} \approx \frac{1}{N} \sum_n \mathcal{O}[U_n] \bar{w}_k[U_n], \quad (4.7)$$

which equals the unweighted case up to the factors $\bar{w}_k[U_n]$, which multiply the observable $\mathcal{O}[U_n]$ on each configuration that enters into the Monte Carlo integration.

In Fig. 4 we show the distribution of the values \bar{w}_k for $k = 2, 10, 20, 30$ from our set of 161 gauge field configurations. Unfortunately, this shows that for truncation levels $k \geq 20$, which are the level of interest for the restoration of the chiral symmetry (see Fig. 1), the Monte Carlo sum is highly dominated by very few gauge configurations. Therefore, we would need many more gauge field configurations in order to obtain the same statistical quality as before. There is a chance, however, that the distribution of the weight factors is narrower for overlap fermions due to the strict circular distribution of the eigenvalues of the overlap operator in the complex plane, opposed to the eigenvalues of the CI that are spread with respect to the circle.

5. Conclusions

We find that our procedure of leaving out the lowest-lying part of the Dirac eigenspectrum does not crucially violate the locality of the theory. We have shown in Sec. 2 that the nonlocal contributions to the truncated Dirac operator are of the order of one million times smaller than the local contributions, and moreover, fall exponentially with the distance at finite lattice spacing.

The effects of Dirac low-mode removal on the quarks are twofold: first, the dynamical mass generation of the quarks vanishes, signaling the restoration of the chiral symmetry. Secondly, the average momentum of the quarks is increased compared with the full theory which accounts for the larger hadron masses and is responsible for a partial effective chiral restoration at low truncation levels.

Lastly, we introduced a low-mode reweighting procedure to extend our approximation to the sea quark sector. The broad shape of the distribution of the weight factors renders the scheme impractical for CI fermions.

Acknowledgments

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