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Computation of the chiral condensate using $N_f = 2$ and $N_f = 2+1+1$ dynamical flavors of twisted mass fermions

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We apply the spectral projector method, recently introduced by Giusti and Lüscher, to compute the chiral condensate using $N_f = 2$ and $N_f = 2+1+1$ dynamical flavors of maximally twisted mass fermions. We present our results for several quark masses at three different lattice spacings which allows us to perform the chiral and continuum extrapolations. In addition we report our analysis on the $O(a)$ improvement of the chiral condensate for twisted mass fermions. We also study the effect of the dynamical strange and charm quarks by comparing our results for $N_f = 2$ and $N_f = 2+1+1$ dynamical flavors.

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1. Introduction

The Banks-Casher relation [1] connects the low lying spectrum of the Dirac operator with the spontaneous chiral symmetry breaking in the following way

$$\lim_{\lambda\to 0}\lim_{m\to 0}\lim_{V\to\infty}\rho(\lambda,m) = \frac{\Sigma}{\pi}. \tag{1.1}$$

Eq. (1.1) relates the chiral condensate Σ to the spectral density $\rho(\lambda,m)$. The recently introduced method based on spectral projectors [2] offers a new strategy to compute spectral observables, such as the chiral condensate, in an affordable way. Moreover it allows us, via the connection to density chains, to compute this quantity using a representation which is free of short distance singularities and therefore leads to the correct continuum limit.

The integrated spectral density or mode number $\nu(M,m)$ is defined as the number of eigenvalues λ of the hermitian Dirac operator $D^\dagger D$ below a certain threshold value M^2. To study the renormalization and $O(a)$ cutoff effects properties of the mode number it is advantageous to consider the spectral sums $\sigma_k(\mu,m)$ which are directly related through the following expression

$$\sigma_k(\mu,m) = \int_0^\infty dM\ \nu(M,m)\frac{2kM}{(M^2+\mu^2)^{k+1}}. \tag{1.2}$$

In particular, it is convenient to write the spectral sums σ_k in terms of density chains like

$$\sigma_3(\mu,m) = -a^{24}\sum_{x_1,\ldots,x_5}\left\langle P^+_{12}(x_1)P^-_{23}(x_2)P^+_{34}(x_3)P^-_{45}(x_4)P^+_{56}(x_5)P^-_{61}(0)\right\rangle, \tag{1.3}$$

where $P^\pm_{ab} = \overline{\psi}_a\gamma_5\tau^\pm\psi_b$ are charged pseudoscalar densities, $\tau^\pm$ are defined in flavor space and μ is the twisted mass. $\overline{\psi}_a = (\overline{u_a},\overline{d_a})$ represent isospin doublets of twisted mass valence fermions. The index $a = 1,\ldots,k$ indicates the doublet; in this particular example we add 6 doublets to the theory, which is the minimum number of flavors that still guarantees the renormalizability as it was shown in [2].

Nevertheless, in the end, the mode number contains the same information as the spectral density and therefore it is directly linked to the chiral condensate as proposed in Ref. [2]

$$\Sigma_R = \frac{\pi}{2V}\sqrt{1-\left(\frac{\mu_R}{M_R}\right)}\frac{\partial}{\partial M_R}\nu_R(M_R,\mu_R). \tag{1.4}$$

Notice that $\nu_R(M_R,m_R) = \nu(M,m)$, the mode number is a renormalization group invariant [2].

It goes beyond the scope of this proceedings to explain in detail the spectral projectors method and its implementation to compute the chiral condensate among other observables. Thus, for further details we refer to the original article Ref.[2] and also to Ref. [3] for the implemention of the setup used here. For a computation of the topological susceptibility in the same set up using spectral projectors, following the method described in Ref.[4], see Ref. [5]. In this contribution we present the continuum limit results obtained for the chirally extrapolated condensate Σ for $N_f = 2$ and $N_f = 2+1+1$ dynamical flavors of maximally twisted mass fermions [6, 7, 8, 9] using the spectral projectors method. In addition we discuss the $O(a)$ improvement of this quantity, since it is, in principle, possible that contact terms that arise in the Symanzik expansion introduce $O(a)$ cut-off effects and spoil automatic $O(a)$ improvement.

2. $O(a)$ improvement of the chiral condensate

The representation of the mode number and the spectral density of the Wilson operator through density chains correlator as in Eq. (1.3) allows to discuss the renormalization and improvement properties of such quantities. This is particularly important when computing the mode number using Wilson twisted mass fermions at maximal twist. The maximal twist condition guarantees automatic $O(a)$ improvement of all physical quantities [10]. In fact one can show that the observable introduced in Eq. (1.3) is even under $\mathscr{R}_5^{1,2}$ transformations given by $\chi_i(x) \to i\gamma_5\tau^{1,2}\chi_i(x)$, $\quad \overline{\chi_i}(x) \to \overline{\chi_i}(x)i\gamma_5\tau^{1,2}$, where χ_i refers to the valence and sea twisted mass quarks. Consequently the automatic $O(a)$ improvement obtained by tuning to maximal twist should apply.

Density chains correlators are affected by short distance singularities and the integration over the whole space-time of such singularities generates additional $O(a)$ terms that could spoil the property of automatic $O(a)$ improvement. In this section we argue that those terms vanish at maximal twist. The details of the proof will be discussed in a forthcoming publication [11].

The short distance singularities on the r.h.s of Eq. (1.3) could correspond to additional $O(a)$ terms in the represenation of the lattice correlator in the Symanzik effective theory. More specifically these $O(a)$ are produced by the short distance expansion of two consecutive densities, since the short distance behavior of three or more densities leads to $O(a^2)$ or higher order terms.

We can study the short distance singularities of a product of two operators through the operator product expansion (OPE). For a generic twist angle products like $P^+_{ab}(x)P^-_{bc}(0)$ will have an OPE for $x \to 0$ containing scalar densities.

Taking into account the presence of the contact terms and the standard Symanzik expansion we can write for the renormalized observable introduced in Eq. (1.3)

$$\begin{aligned}\sigma_{3,R}(\mu_R, m_R) \;=\; &-\int d^4x_1 d^4x_2 d^4x_3 d^4x_4 d^4x_5 \left\langle P^+_{12}(x_1)P^-_{23}(x_2)P^+_{34}(x_3)P^-_{45}(x_4)P^+_{56}(x_5)P^-_{61}(0)\right\rangle_0 \\ &+a\ \text{S.T.} + a\ \text{C.T.}\end{aligned} \tag{2.1}$$

In Eq. (2.1) $\langle\rangle_0$ represents the continuum expectation values. The term labelled with S.T. corresponds to the standard terms appearing in the Symanzik expansion and the one labelled with C.T. corresponds to the $O(a)$ terms arising from the short distance singularities in the product of two densities. If we tune our lattice action parameters to achieve maximal twist one can use the standard arguments leading to automatic $O(a)$ improvement to show that the S.T. vanish.

For the discussion of the C.T we keep generic values for the twisted and untwisted quark masses. An example of the C.T. is given by

$$\begin{aligned}&\int d^4x_2 d^4x_3 d^4x_4 d^4x_5 \left\langle S^{\uparrow}_{13}(x_2)P^+_{34}(x_3)P^-_{45}(x_4)P^+_{56}(x_5)P^-_{61}(0)\right\rangle_0 + \\ &+\int d^4x_2 d^4x_3 d^4x_4 d^4x_5 \left\langle P^-_{23}(x_2)P^+_{34}(x_3)P^-_{45}(x_4)P^+_{56}(x_5)S^{\downarrow}_{62}(0)\right\rangle_0 ,\end{aligned} \tag{2.2}$$

where $S^{\uparrow,\downarrow}_{ac} = \overline{\psi}_a \frac{1}{2}(\mathbb{1} \pm \tau^3)\psi_c$.

We can now use non-singlet axial Ward-Takahashi identities to rewrite Eq. (2.2) in a convenient form. For twisted mass fermions at a generic twist angle we have

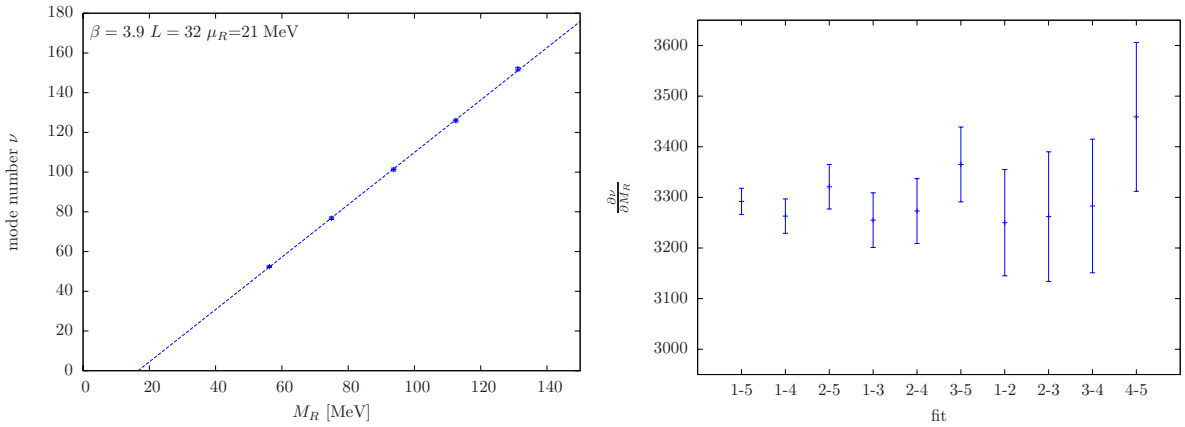

Figure 1: (left) Mode number ν as a function of the renormalized threshold parameter M_R for $N_f = 2$ at $\beta = 3.9$ and $a\mu = 0.004$. The line corresponds to a linear fit to all 5 points. (right) Result of the derivative $\partial\nu/\partial M$ for different ranges in the linear fit. The x-axis represents the points included in the linear fit, where 1 corresponds to the lowest and 5 the largest value of M respectively.

$$\begin{aligned} &\int d^4x_2 d^4x_3 d^4x_4 d^4x_5 \left\langle S^{\uparrow}_{13}(x_2) P^{+}_{34}(x_3) P^{-}_{45}(x_4) P^{+}_{56}(x_5) P^{-}_{61}(0) \right\rangle_0 + \\ &+ \int d^4x_2 d^4x_3 d^4x_4 d^4x_5 \left\langle P^{-}_{23}(x_2) P^{+}_{34}(x_3) P^{-}_{45}(x_4) P^{+}_{56}(x_5) S^{\downarrow}_{62}(0) \right\rangle_0 \\ &= 2m \int d^4x_2 d^4x_3 d^4x_4 d^4x_5 \int d^4x_1 \left\langle P^{+}_{12}(x_1) P^{-}_{23}(x_2) P^{+}_{34}(x_3) P^{-}_{45}(x_4) P^{+}_{56}(x_5) P^{-}_{61}(0) \right\rangle_0 , \end{aligned} \tag{2.3}$$

where m is the untwisted quark mass.

All the other terms stemming from the short distance singularities can be treated in an analogous manner.

For the sake of simplicity we have chosen to write a particular example for six flavors, however, a generalization of this derivation for a generic number of flavors is straightforward.

Our analysis indicates that despite the presence of additional $O(a)$ terms in density chain correlators, those terms turn out to be proportional to the untwisted quark mass m at a generic twist angle, thus they are bound to vanish at maximal twist.

3. Chiral and continuum extrapolation of the chiral condensate

In order to extract the chiral condensate through Eq. (1.4) we computed the mode number using spectral projectors at several values of the threshold parameter M in a range from around 60 to 120 MeV. In this region a linear behavior is expected, which allows us to compute the derivative which appears in Eq. (1.4) from a simple linear fit. Fig. 1(left) shows the observed linear behavior for a particular ensemble.

To reliably estimate the correct range of M, where the mode number behaves linearly, represents an important source of systematic error in this calculation. We performed a comprehensive analysis of the systematic contributions to the error and concluded that the error coming from the fitting range, is in fact the most prominent [3]. Recent studies [12] found large deviations from

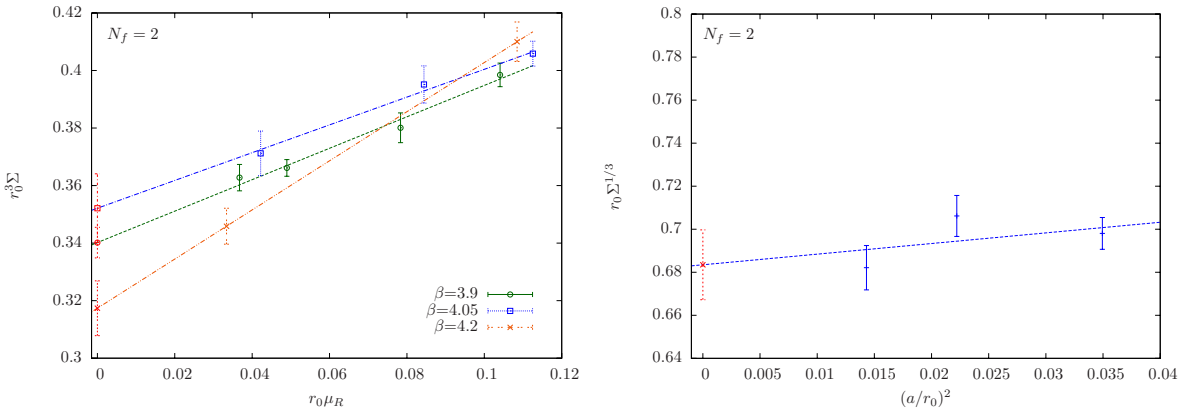

Figure 2: Chiral extrapolation (left) of $r_0^3\Sigma$ for $N_f = 2$ at three different values of the lattice spacing. Continuum limit (right) of chirally extrapolated $r_0\Sigma^{1/3}$ for $N_f = 2$.

the linear behavior in their data and concluded that NNLO effects could play an important role in the chiral extrapolation of the condensate. In our case, however, with the available data, we do not observe statistically significant deviations which shows that such effects are very mild in our setup. To show this mild effect in Fig. 1(right) the result of the derivative which appears in Eq. (1.4) for different ranges is shown, i.e. including different points showed in Fig. 1. One can see an agreement in the result for the four lowest values of M as expected. The value of the mode number corresponding to the largest M seems to slightly deviate from the linear behavior. We consider such deviation in our final systematic error. However, results from different fitting ranges are still statistically compatible.

All the results presented in this section were computed using ensembles generated by the ETM collaboration. For further details we refer to the extended reference [3] where all the relevant details are discussed.

3.1 Results for $N_f = 2$

In this section we summarize the results of the chiral condensate for $N_f = 2$ dynamical fermions of maximally twisted mass fermions. The details of the simulations can be found in Refs. [13, 14].

Fig. 2 (left) shows the chiral extrapolation of the dimensionless ratio $r_0\Sigma$ for three different lattice spacings which correspond to $a = 0.085$, 0.067 and 0.054 fm respectively [15]. The range of renormalized quark masses is from 15 to 45 MeV. We perform a linear extrapolation to the chiral limit as suggested in Ref. [2], since the NLO effects are negligible for the mentioned ranges of quark masses and M, as we have explicitly tested.

The continuum limit of the chirally extrapolated condensate is shown in Fig. 2 (right) and leads to the following result $r_0\Sigma^{1/3} = 0.685(16)(32)$, where the first error quoted combines the statistical error and the error obtained from the uncertainty in the estimation of Z_P and from r_0/a in quadrature. The systematic error corresponding to the uncertainty in the linear regime of the mode number is given as the second error quoted.

All the errors presented in this article were computed using the method described in Ref. [16], whereas in Ref. [3] the bootstrap with blocking method was applied. Thus the results slightly differ

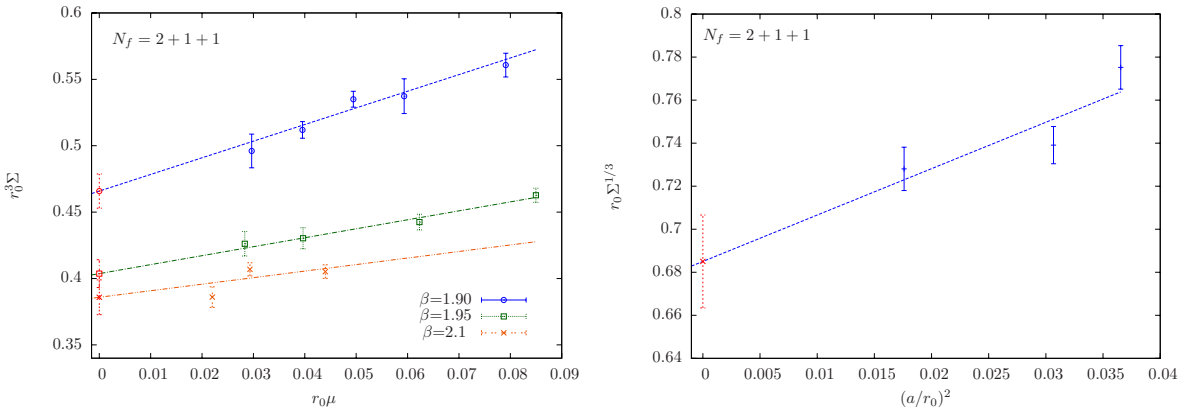

Figure 3: Chiral extrapolation (left) of $r_0\Sigma$ for $N_f = 2+1+1$ at three different values of the lattice spacing. Continuum limit (right) of chirally extrapolated $r_0\Sigma^{1/3}$ for $N_f = 2+1+1$.

although they remain perfectly compatible. In Ref. [3] the result is compared to others found in the literature and a very good agreement is observed.

3.2 Results for $N_f = 2+1+1$

In this section we summarize the results of the chiral condensate, described in detail in Ref. [3], for $N_f = 2+1+1$ dynamical fermions of maximally twisted mass fermions [13, 17, 18].

Again we perform a chiral extrapolation following the strategy presented in Ref. [2] for three different values of the lattice spacing $a = 0.086,\ 0.078$ and 0.061 fm respectively [18]. Fig. 3 (left) shows the results at different values of the renormalized quark mass in a range between 13 and 45 MeV. The line represents the extrapolation at leading order of χPT.

In Fig. 3 (right) the continuum extrapolation is plotted. Again we extrapolate in a^2 since, as we showed in the previous section, the chiral condensate is $O(a)$ improved for twisted mass fermions at maximal twist.

The final result for $N_f = 2+1+1$ is $r_0\Sigma^{1/3} = 0.683(19)(18)$. The errors quoted represent the same uncertainties as in the result presented for $N_f = 2$. Our value found for $N_f = 2+1+1$ flavours is compatible with the continuum limit value found for $N_f = 2$ twisted mass fermions and quoted in the previous section.

If we compare the errors of both, the result for $N_f = 2$ and for $N_f = 2+1+1$, one can see that the data are less sensitive to the variations in the fit interval for $N_f = 2+1+1$ than for $N_f = 2$. This is mostly due to the fact that slopes for the case of $N_f = 2+1+1$ flavors are smaller which contributes to decreasing the systematic error.

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