2+1 flavour thermal studies on an anisotropic lattice

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The FASTSUM collaboration has initiated a detailed study of thermal QCD using 2+1 flavours of improved Wilson quarks on anisotropic lattices. Spatial volumes of $(3\text{fm})^3$ and $(4\text{fm})^3$ are used at fixed cut-off with temperatures ranging from 120 to 350 MeV (corresponding to temporal lattice extents of 48 to 16 lattice units). Results presented here include the deconfinement temperature and a study of the restoration of chiral symmetry, together with a brief summary of our collaboration's other results from these ensembles.

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1. Introduction

Particle physics data are famously collated and summarised in the Particle Data Book [1]. However, it is interesting to note that there are no entries on the deconfined phase of QCD – a symptom of the difficulty of studying (experimentally and theoretically) this new phase. The FASTSUM Collaboration has studied QCD at non-zero temperature for a number of years using dynamical quarks on *anisotropic* lattices where the temporal lattice spacing, a_τ, is less than the spatial one, a_s. Since the temperature $T = 1/(a_\tau N_\tau)$, where N_τ is the number of lattice points in the temporal direction, this gives the distinct advantage that more points are sampled in a euclidean correlator for a given temperature compared to the isotropic case.

Our research programme began with two-flavour dynamical "1st generation" ensembles from which we studied a number of phenomenological quantities, such as spectral features in charmonium and bottomonium at zero and non-zero momenta, and the inter-quark potential in charmonium [2]. We have now improved the accuracy of our results by producing our "2nd generation" lattices which have 2+1 dynamical flavours, a larger volume, improved discretisation and more realistic dynamical quark masses, see Table 1.

In this talk, I give an overview of our 2nd generation ensembles including our estimate of the deconfinement temperature, T_c, obtained from the Polyakov loop. I discuss the partial restoration of chiral symmetry in the light meson spectrum and briefly review four results obtained from these lattices which are covered fully in other talks [3, 4, 5, 6].

2. Lattice details

Our 2nd generation ensembles use the Hadron Spectrum Collaboration's (HSC) Symanzik-improved gauge action [8], with

$$S_G = \frac{\beta}{\gamma_g} \left\{ \sum_{x,s\neq s'} \left[\frac{5}{6u_s^4} \mathscr{P}_{ss'}(x) - \frac{1}{12u_s^6} \mathscr{R}_{ss'}(x) \right] + \sum_{x,s} \gamma_g^2 \left[\frac{4}{3u_s^2 u_\tau^2} \mathscr{P}_{s\tau}(x) - \frac{1}{12u_s^4 u_\tau^2} \mathscr{R}_{s\tau}(x) \right] \right\}, \quad (2.1)$$

where $\mathscr{P}$ and $\mathscr{R}$ are the usual 1×1 plaquette and 2×1 rectangular Wilson loops, $u_{s(\tau)}$ are the spatial (temporal) tadpole factors of the bare links, $\gamma_{g(f)}$ are the bare gauge (fermion) anisotropies and, as usual, $\beta = 2N_c/g^2$ and $N_c = 3$ is the number of colours. The means of the stout-smeared links are $\tilde{u}_\mu$ (with $\tilde{u}_\tau = 1$).

We use a tadpole-improved clover fermion action and stout-smeared links [9] using the same parameters as the Hadron Spectrum Collaboration [8],

$$
\begin{aligned}
S_F = \sum_x \overline{\psi}(x) \frac{1}{\tilde{u}_\tau} &\left\{ \tilde{u}_\tau m_0 + \gamma_\tau \nabla_\tau + \nabla_\tau^2 + \frac{1}{\gamma_f} \sum_s [\gamma_s \nabla_s + \nabla_s^2] \right. \\
&\left. - \frac{1}{4} \left(\frac{\gamma_g}{\gamma_f} + \frac{1}{\xi} \right) \frac{1}{\tilde{u}_\tau \tilde{u}_s^2} \sum_s \sigma_{\tau s} F_{\tau s} + \frac{1}{2\gamma_f} \frac{1}{\tilde{u}_s^3} \sum_{s<s'} \sigma_{ss'} F_{ss'} \right\} \psi(x).
\end{aligned}
\quad (2.2)
$$

The first line is the usual Wilson action and the second line is the clover piece with τ and s referring to temporal and spatial directions. The ∇_μ are covariant finite differences and $\xi = a_s/a_\tau$ is the renormalised anisotropy. $\gamma_{s(\tau)}$ are the spatial (temporal) Dirac matrices and $\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$.

We use the same parameters as the HSC employed in their studies [10] corresponding to an anisotropy, $\xi = 3.5$. We generate ensembles with two volumes, 24^3 and 32^3, enabling us to study finite volume effects. We also make use of the $T = 0$ (i.e. $N_\tau = 128$) configurations kindly made available to us from HSC. Table 1 gives a full list of our parameters. The generation of the ensembles were performed using the Chroma software suite [11] with Bagel routines [12].

3. Determination of the deconfining temperature

The Polyakov loop, L, can be used to determine T_c as follows [13]. We note that L is related to the free energy, F, of a static quark, via:

$$L(T) = e^{-F(T)/T}.$$

$$(3.1)$$

However, F is only defined up to an additive renormalisation constant, $\Delta F = f(\beta, m_0)$. We can impose a renormalisation condition at a renormalisation temperature, T_R, by requiring

$$L_R(T_R) \equiv c,$$

$$(3.2)$$

for some suitable choice of T_R and c. This means a multiplicative renormalisation constant, Z_L, can be fixed as follows

$$L_R(T) = e^{-F_R(T)/T} = e^{-(F_0(T)+\Delta F)/T} = L_0(T)e^{-\Delta F/T} = L_0(T)Z_L^{N_\tau}.$$

$$(3.3)$$

In Fig. 1, we plot the Polyakov loop with three different renormalisation schemes corresponding to different choices of T_R and the constant in Eq. (3.2), as listed in the figure caption. By fitting the data to cubic splines we obtain the point of inflection $a_\tau T_c = 0.0329(7)$ where the error reflects the spread from the three renormalisation schemes. This statistical uncertainty is given by the thickness of the three interpolating curves and can be seen to be negligible in this context. The result is then $N_\tau^{\mathrm{crit}} = 30.4(7)$ or $T_c = 185(4)$ MeV.

4. First results

The deconfinement transition is expected to occur in the same T range as the chiral symmetry restoration. For this reason it is interesting to study the chiral partners in the light meson sector to find evidence of this effect. In Fig. 2 we show the pseudoscalar and scalar meson correlator for $T/T_c = 0.63$ and 1.90. As can be seen, in the high temperature case, these two channels are closer together than at low temperature illustrating the partial restoration of chiral symmetry in these quantities.

We have commenced studying several quantities on our 2nd generation ensembles. Results on the following quantities have been reported in this conference and elsewhere.

- **Susceptibility** [3]. We study the electric charge susceptibility which is of interest experimentally to quantify fluctuations in heavy-ion collision experiments and for the determination of the electric charge diffusion coefficient.

	1st Generation					2nd Generation				
Flavours	2					2+1				
Volume(s)	$(\sim 2\text{fm})^3$					$(\sim 3\text{fm})^3$ & $(\sim 4\text{fm})^3$				
a_τ [fm]	0.0268(1)					0.03506(23)				
a_s [fm]	0.162(4)					0.1227(8)				
$\xi = a_s/a_\tau$	6					3.5				
M_π/M_ρ	~ 0.54					~ 0.45				
$N_\tau^{\text{crit}} = (a_\tau T_c)^{-1}$	33.5					30.4(7)				
Gauge Action	Symanzik Improved					Symanzik Improved				
Fermion Action	Stout Link, Fine-Wilson, Coarse-Hamber-Wu					Tadpole Improved Clover				
	N_s	N_τ	T (MeV)	T/T_c	N_{cfg}	N_s	N_τ	T (MeV)	T/T_c	N_{cfg}
	12	16	459	2.09	1000	24	16	352	1.90	1000
	12	18	408	1.86	1000	32	16	352	1.90	1000
	12	20	368	1.68	1000	24	20	281	1.52	1000
	12	24	306	1.40	500	24	24	235	1.27	1000
	12	28	263	1.20	1000	32	24	235	1.27	500
	12	32	230	1.05	1000	24	28	201	1.09	1000
	12	80	92	0.42	250	32	28	201	1.09	500
						24	32	176	0.95	1000
						32	32	176	0.95	500
						24	36	156	0.84	500
						24	40	141	0.76	500
						32	48	117	0.63	250
						16	128	44	0.24	500
						24	128	44	0.24	550

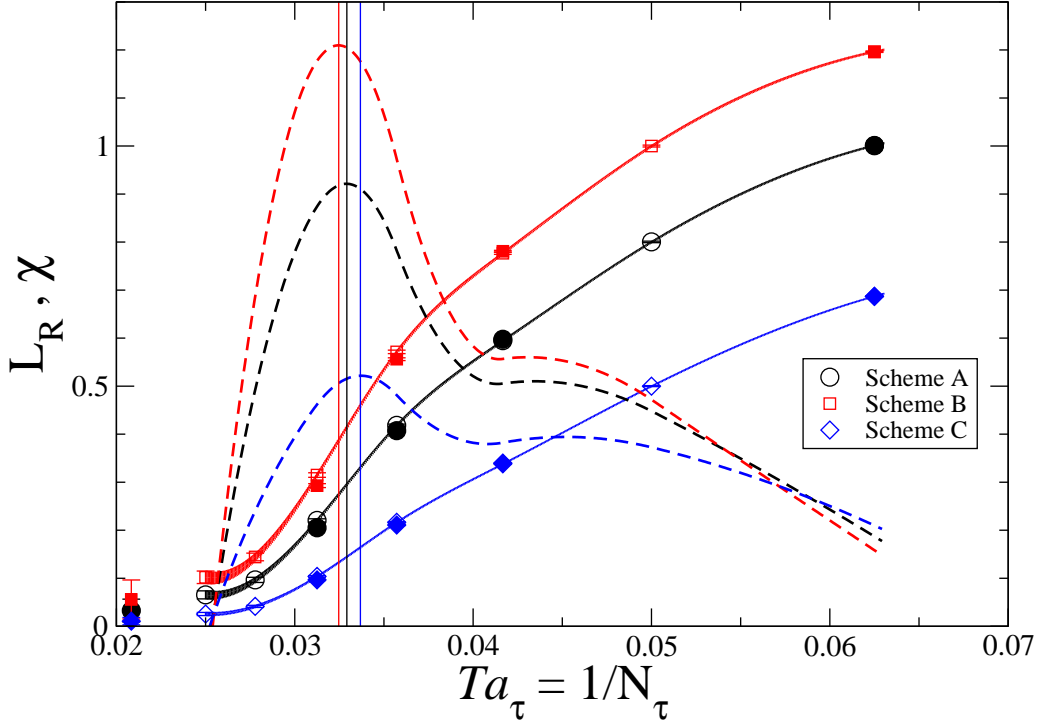**Table 1:** A list of the lattice parameters used for our 1st and 2nd generation ensembles.

Figure 1: The renormalised Polyakov loop, L_R, depicted by solid (32^3) and open (24^3) symbols. The solid curves are obtained by cubic splines and their temperature derivatives, χ, are depicted by dashed curves. Three renormalisation schemes are considered, Scheme A: $L_R(N_\tau = 16) = 1.0$, Scheme B: $L_R(N_\tau = 20) = 1.0$, Scheme C: $L_R(N_\tau = 20) = 0.5$.

- **Electrical conductivity** [4, 14]. The temperature dependence of the electrical conductivity has been calculated on our lattices, using the exactly conserved lattice current. We find that the conductivity divided by the temperature increases with temperature across the deconfinement transition. This is the first time this quantity has been computed as a function of temperature.

- **Inter-quark potential in charmonium** [5]. This is the first time this quantity has been calculated at high temperature with relativistic (rather than static) quarks. We find that its behaviour at low temperature agrees with the (confining) Cornell potential and that it becomes less confining as the temperature increases.

- **Bottomonium spectrum** [6]. We have used the NRQCD formulation to study spectral functions in bottomonium via the Maximum Entropy Method. We confirm our earlier result [2] that the S-wave (Υ and η_b) ground states survive to $T \sim 2T_c$ whereas excited states are suppressed, while the P-wave ($h_b, \chi_{b0,b1,b2}$) ground states dissociate close to T_c.

- **Charmonium spectrum** [7]. A study of charmonium spectral functions across the deconfining transition is also in progress.
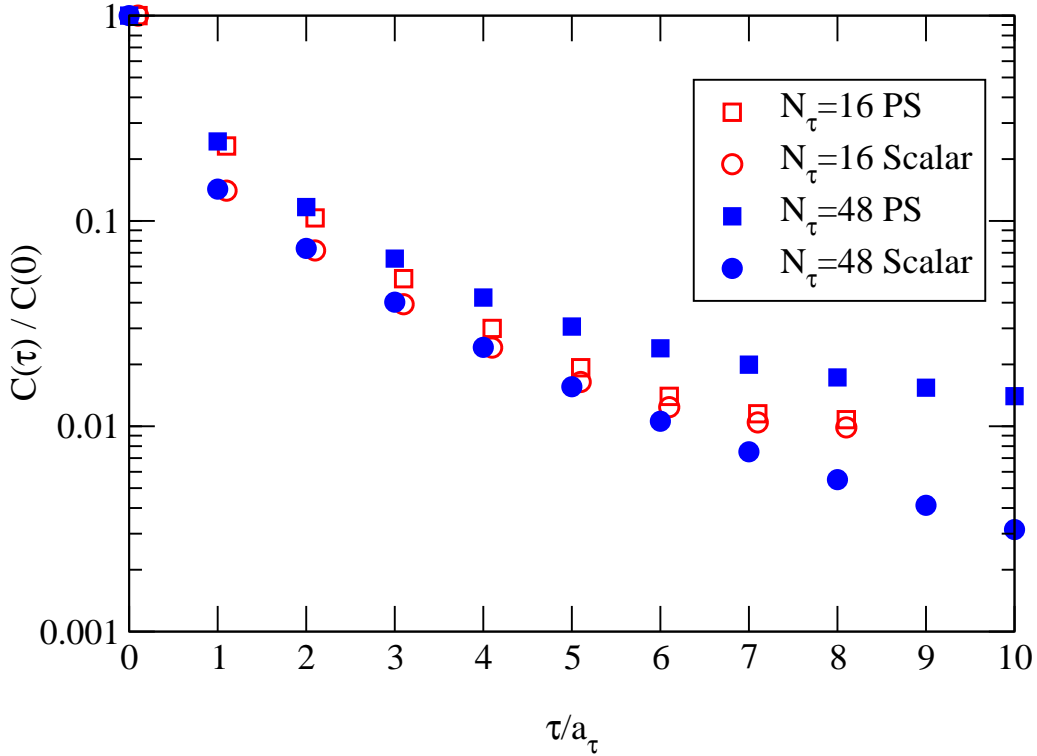
Figure 2: Correlation functions (normalised relative to $\tau = 0$) for the light scalar and pseudoscalar mesons at two different temperatures on either side of the deconfinement transition, showing partial restoration of chiral symmetry. The $N_\tau = 16$ points have been shifted horizontally for clarity.

5. Conclusions

This talk summarises our FASTSUM collaboration's latest finite-temperature studies using anisotropic lattices. We have improved upon our 1st generation 2-flavour ensembles by generating ensembles which have 2+1 flavours, larger volume, improved discretisation, and smaller dynamical quark masses. In this talk, the deconfining temperature was presented and the (partial) chiral symmetry restoration in the light meson sector was studied. Other work presented elsewhere in this conference was summarised: the susceptibility, electrical conductivity, interquark potential in charmonium and (NRQCD) bottomonium spectral functions.

Our future plans are to improve our ensembles further – we are currently tuning our "3rd generation" ensembles which have a smaller temporal lattice spacing and have plans for a "4th generation" run with smaller spatial lattice spacing. We will thus be able to move towards a continuum extrapolation of all our quantities, leading to truly quantitative finite-temperature results for spectral quantities.

Acknowledgements

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