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Quark number susceptibilities at high temperatures

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Second and fourth order quark number susceptibilities are calculated in 2+1 flavor QCD in the high temperature region using two improved staggered fermion formulations p4 and HISQ. The calculations are performed at several lattice spacing and we show that in the continuum limit the two formulations give consistent results.

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1. Introduction

At high temperatures strongly interacting matter undergoes a deconfinement transition to a new state, where thermodynamics can be described in terms of quark and gluonic degrees of freedom. Studying the properties of this new state of matter is subject of a large effort in lattice QCD, see Refs. [1, 2]. This is in part due to the fact that non-perturbative effects could be important even at very high temperatures due to infrared problems [3]. Therefore, it is important to clarify using lattice QCD calculations at what temperatures the deconfined medium can be described as weakly interacting quark-gluon gas. This is especially important in the light of recent experimental findings showing that the matter created in ultra-relativistic heavy ion collisions behaves like a strongly coupled liquid [4].

Fluctuations of conserved charges are known to be sensitive probes of deconfinement and suitable for testing the weakly (or strongly) interacting nature of the deconfined medium. They are defined as the derivatives of the pressure with respect to the corresponding chemical potentials. Fluctuations of conserved charges are expected to be exponentially small in the low temperature region where the conserved charges are carried by massive hadrons. However, they are not suppressed at high temperatures, where the dominant degrees of freedom are light quarks. Therefore, fluctuations of conserved charges are good probes of deconfinement.

In 2+1 flavor QCD there are three chemical potentials corresponding to baryon number, electric charge and strangeness, or equivalently to u,d and s quark chemical potentials. Since at high temperatures the dominant degrees of freedom are quarks and gluons, the quark number basis provides a natural way to study the fluctuations. In this paper second and fourth order quark number fluctuations are studied, which are also known as quark number susceptibilities, and are defined as

$$\chi_{2n}^{q} = \left. \frac{\partial^{2n}(p/T^4)}{\partial(\mu_q/T)^{2n}} \right|_{\mu_q=0}, \quad q = u,d,s,\ n = 1,2. \tag{1.1}$$

Though in all calculations the u and d quark masses are degenerate, the corresponding chemical potential are always assumed to be different, i.e. we consider single flavor susceptibilities. This proceeding contribution addresses the high temperature behavior of the quark number susceptibilities. To better control the cutoff effects and the continuum extrapolation two different improved staggered quark formulations have been used, namely the so-called p4 action [5] and the Highly Improved Staggered Quark (HISQ) action [6]. The analysis of the quark number susceptibilities performed by the speaker and his collaborators using these two actions has been published in Ref. [7]. This paper also contains all the details of the lattice calculations, including lattice volumes, statistics and scale setting.

2. Results

In this section we present numerical results for χ_2^q and χ_4^q and discuss the cutoff effects in quark number susceptibilities, as well as the details of the continuum extrapolations. The main result is summarized in Fig. 2.

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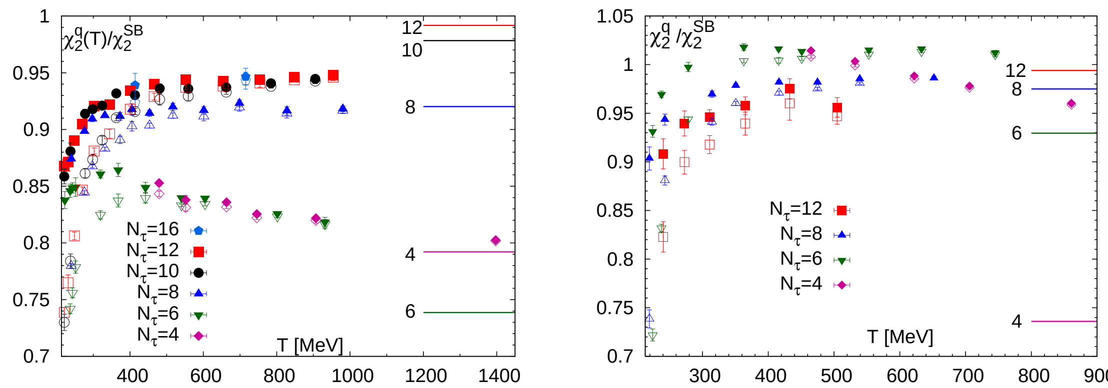

Figure 1: Second order quark number susceptibilities calculated on different lattices with HISQ action (left) and p4 action (right) as function of the temperatures. The filled symbols correspond to light quark number susceptibilities, while open symbols correspond to strange quark number susceptibilities. The horizontal lines refer to the values of the quark number susceptibilities in the free theory corresponding to different temporal extent N_τ. The free theory result for $N_\tau = 4$ and p4 action has been shifted upwards by 0.2 for better visibility. Note that $\chi_2^{SB} = 1$.

2.1 Numerical results on second order quark number susceptibilities

We start the discussion of the numerical results with the second order light and strange quark number susceptibilities. In Fig. 1 we show the numerical results for HISQ and p4 actions in the high temperature region, $T > 200$ MeV. In the continuum limit the quark number susceptibilities approach unity at very high temperatures. The strange quark number susceptibilities approach the high temperature limit slower than the light quark number susceptibilities. The difference between the light and strange quark number susceptibilities becomes small above temperatures of 400 MeV and negligible for $T > 600$ MeV. The difference between the light and strange quark number susceptibilities can be understood in terms of the strange quark mass for $T > 250$ MeV, but at lower temperatures it is significantly larger than the expected suppression due to the relatively large strange quark mass, $m_s \simeq 90$ MeV, see the discussion in Ref. [8]. The cutoff effects are relatively small for $T < 300$ MeV but become significant above that temperature.

In the case of the HISQ action there is a qualitative change in the behavior of the cutoff effects for $T > 300$ MeV, i.e., the ordering of quark number susceptibilities calculated at different N_τ qualitatively starts to follow expectations based on the systematics seen for the free theory. The continuum limit seems to be approached from below. The free theory result, shown as horizontal lines in Fig. 1 shows larger cutoff dependence than the numerical data. This is expected to some extent; from the analysis of the high temperature limit of pure gauge theories [9] it is known that cutoff effects in the interacting theory typically are about a factor two smaller than in the free theory. Interestingly enough though, at the highest temperature the $N_\tau = 4$ data point is very close to the lattice ideal gas value.

For the p4 action the pattern of the cutoff dependence observed in the free theory is not seen in the interacting case for the entire temperature range explored by us. For improved actions the quark number susceptibilities in the free theory approaches the continuum limit from below. The p4 lattice data, on the other hand, seem to approach the continuum limit from above. This implies that

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cutoff effects proportional to α_s and higher orders in the coupling constant are significant, contrary to the case of the HISQ action. The main difference between the p4 and HISQ action that could be responsible for the difference in the cutoff behavior is the use of smeared gauge fields in the later. The use of smeared gauge fields is known to reduce cutoff effects in higher order perturbative corrections in lattice calculations [10].

2.2 Continuum extrapolation of second order quark number susceptibilities

Using the lattice results for quark number susceptibilities at different N_τ we perform continuum extrapolations. First, for each N_τ we interpolate the lattice results using smooth splines. The errors on the interpolated values were estimated using the bootstrap method. We use the R package for this analysis [11]. Using the interpolations we obtain the values of the quark number susceptibilities at the same set of temperatures. Finally we perform continuum extrapolations at this set of temperatures. The $N_\tau = 4$ data have not been used in the continuum extrapolations.

In the case of the HISQ action we consider the temperature interval from 225 MeV to 950 MeV for each N_τ, with the step of 25 MeV for $T \leq 400$ MeV, and the step 50 MeV for larger temperatures. Our extrapolations for HISQ are motivated by the leading order N_τ dependence of quark number susceptibilities in the free theory. Namely, we perform continuum extrapolations using the following form

$$\chi_2^q(N_\tau) = a + b/N_\tau^4 + c/N_\tau^6. \tag{2.1}$$

We also perform extrapolations using the simpler $a + b/N_\tau^4$ form and data for $N_\tau \geq 8$ only. The two fits give identical results within the estimated errors. Furthermore, we use the complete tree-level result for the N_τ dependence of the quark number susceptibility to perform the continuum extrapolation. I.e., we fitted the data for each temperature with $a + c\cdot(\chi_2^{q,free}(N_\tau) - \chi_2^{SB})$. This gives extrapolated values for χ_2^q that are systematically lower than the above fits, but still agree within errors. For the coefficient c we typically find values around 0.6, i.e. cutoff effects in the interacting theory are 40% smaller than in the free field limit. Since beyond tree level there are also discretization errors proportional to $1/N_\tau^2$ we also performed extrapolations using $a + c\cdot(\chi_2^{q,free}(N_\tau) - \chi_2^{SB}) + d/N_\tau^2$. These extrapolations give results that agree within errors with the extrapolations obtained using the Ansatz $a + b/N_\tau^4 + c/N_\tau^6$, though they are systematically higher. The coefficient d turns out to be negative and at the highest temperatures it is compatible with zero. Thus it is possible that the $1/N_\tau^2$ term just mimics cutoff effects proportional to $1/N_\tau^n,\ n \geq 4$ at higher order in the weak coupling expansion. It turns out that the differences between the mean values of χ_2^q obtained by using the above two fits and the fit that uses Eq. (2.1) are smaller or equal to the statistical errors of the 3-parameter fit given by Eq. (2.1). In other words, the systematic errors estimated as the differences of the above three fits is smaller or of the same size as the statistical errors of that fit. Therefore, we use the extrapolation based on Eq. (2.1) and its statistical errors as our final continuum result for HISQ.

For the p4 action the continuum limit is approached from above contrary to the free field expectations. This implies that the dominant cutoff effects come from higher orders in the weak coupling expansion and scale like $1/N_\tau^2$. Therefore we perform the continuum extrapolation using the simple constant plus $1/N_\tau^2$ form. We also tried to add an $1/N_\tau^4$ term in the fit when doing the continuum extrapolations. It turns out that the inclusion of such a term did not change the result

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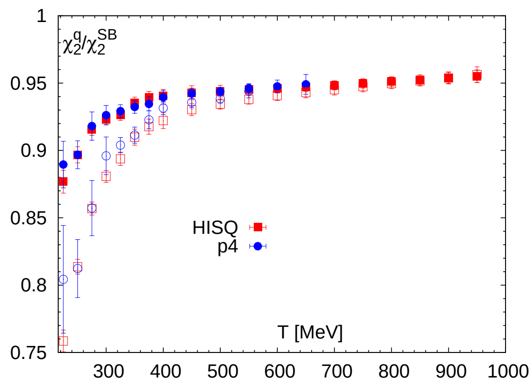

Figure 2: Second order light (filled symbols) and strange (open symbols) quark number susceptibilities as function of the temperature in the continuum limit. Note that $\chi_2^{SB} = 1$.

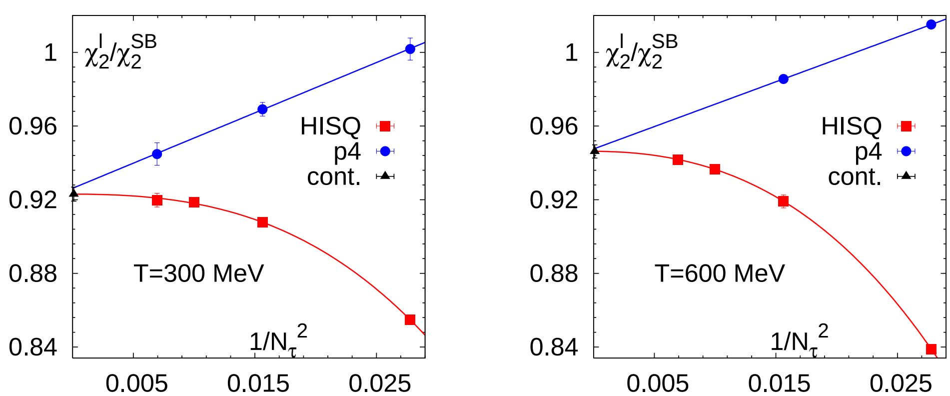

Figure 3: Contimuum extrapolatuions for HISQ and p4 actions at two temperatures.

within the errors. Moreover, the coefficient of the $1/N_\tau^4$ term was 4-10 times smaller than in the free theory. This confirms our assumption that the dominant cutoff effects in the case of the p4 action go like $1/N_\tau^2$.

The continuum extrapolated quark number susceptibilities are shown in Fig. 2 for both p4 and HISQ actions. Overall the p4 results and HISQ results agree quite well. Note that the agreement between p4 results and HISQ results is particularly good for $T > 400$ MeV. This is remarkable in view of the different nature of cutoff effects for the HISQ and p4 actions. To better illustrate this point in Fig. 3 we show the N_τ dependence of χ_2^l for p4 and HISQ action together with the continuum extrapolations based on $1/N_\tau^2$ form and Eq. (2.1), respectively, for two temperatures $T = 400$ MeV and $T = 500$ MeV. In the figure we also show the N_τ dependence of the HISQ results for $T = 700$ MeV together with the fit based on Eq. (2.1). Note that the $N_\tau = 16$ HISQ data point was not included in the continuum extrapolations but happens to lie on the fitted curve.

For the p4 action the cutoff effects at order $1/N_\tau^4$ and higher seem to be very small and the dominant cutoff effects are proportional to $1/N_\tau^2$ with a positive coefficient. For HISQ action there is no indication for such a term in the data and if it is put in the Ansatz for the extrapolations, the corresponding coefficient turns out to be negative. The cutoff dependence thus is well described

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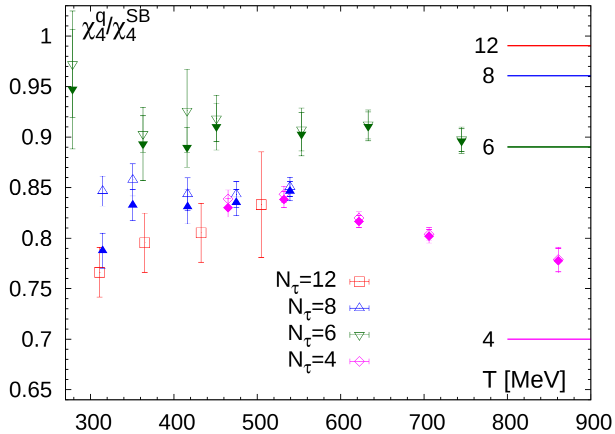

Figure 4: Fourth order quark number susceptibilities calculated with the p4 action and normalized by the corresponding Stefan-Boltzmann value. The horizontal lines correspond to the free field value of χ_4^q/χ_4^{SB} in the massless limit. The free field result for $N_\tau = 4$ has been shifted upwards by 0.44 for better visibility. The open symbols correspond to the strange quark, while the filled symbols correspond to the light quarks. Note that $\chi_4^{SB} = 6/\pi^2$.

by the free theory modified by a multiplicative factor. Remarkably, the continuum extrapolated χ_2^q results at high temperatures deviate from the massless ideal gas limit only by 5%. We also have compared our results with recent continuum extrapolated results obtained by using the stout action [12]. For χ_2^s our result agrees with the stout results within errors up to 350 MeV. For 350 MeV $< T <$ 400 MeV the stout results are lower by $(1.2-1.5)$ standard deviations. For χ_2^l our results agree with the stout results only up to 260 MeV. Above that temperature the stout results are lower than ours by two standard deviations.

2.3 Fourth order quark number susceptibilities

For p4 action we also calculated the strange and light fourth order quark number susceptibilities for $N_\tau = 6$ and 8. The calculation of the fourth order light quark number susceptibility is more demanding than the calculation of the corresponding strange quark number susceptibility. Therefore, for $N_\tau = 12$ we calculated the fourth order quark number susceptibility only in the strange quark sector. Our numerical results for χ_4^q normalized by the corresponding massless ideal gas (SB) value are shown in Fig. 4. The cutoff dependence of χ_4^q is qualitatively the same as for χ_2^q, namely the continuum limit is approached from above contrary to the free theory results shown in Fig. 4 as horizontal lines. As discussed above the difference between the light and strange quark number susceptibilities is expected to be small for $T > 400$ MeV and our numerical data clearly show this. In fact, with the exception of the $N_\tau = 8$ data point at the lowest temperature, the difference between the light and strange fourth order quark number susceptibilities is of the same order or smaller than the statistical errors. For χ_4^q the deviations from the ideal gas value for seem to be significantly larger than for χ_2^q at temperatures 350 MeV $< T <$ 500 MeV, and increase with increasing N_τ.

Given the large statistical errors of the $N_\tau = 12$ lattice data it is at present difficult to perform

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a reliable continuum extrapolation for χ_4^s. However, the N_τ dependence of χ_4^s for 350 MeV $< T <$ 500 MeV is compatible with $1/N_\tau^2$ behavior of the cutoff effects, and such an extrapolation would result in the value of $\chi_4^s/\chi_4^{q,SB}$ around 0.77, i.e. the deviations from the massless ideal gas limit for χ_4 could be almost four times larger than for the second order quark number susceptibilities. Looking at the data shown in Fig. 4 it is possible that the ordering of $N_\tau = 8$ and $N_\tau = 12$ data will change for $T > 550$ MeV, becoming qualitatively compatible with the free theory expectations. Therefore it would be very important to extend the lattice calculations of fourth order susceptibility to higher temperatures.

3. Conclusions

We have calculated second and fourth order quark number susceptibilities for $T > 200$ MeV in lattice QCD using two different improved staggered fermion formulations: the p4 and HISQ actions. We performed continuum extrapolations for the second order quark number susceptibilities.While the cutoff dependence of the quark number susceptibilities is quite different for the HISQ and p4 actions, we obtain consistent results in the continuum limit. This makes us confident that the continuum extrapolations are under control. The detailed study of the cutoff effects in the quark number susceptibilities at high temperatures provides valuable information for analyzing the cutoff dependence of the pressure and other thermodynamic quantities, where the numerical calculations are much more involved due to the need of proper vacuum subtractions. In particular, we find indications that the cutoff effects in the temperature interval 400 MeV $< T <$ 950 MeV are 40% smaller than in the free theory.

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