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Electric charge catalysis by magnetic fields and a nontrivial holonomy

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We discuss an effect of charge generation by a combination of magnetic and pseudo-magnetic fields at finite iso-chemical potential. The phenomenon occurs due to the non-matching Landau level degeneracies in the particle and antiparticle sectors, producing charged ground states. The same phenomenon is observed in caloron background with nontrivial holonomy at finite magnetic field. We argue that the effect may result in a reduction of charge fluctuations at finite temperature, as well as in the suppression of longitudinal chromo-magnetic field at finite density and magnetic field.

31st International Symposium on Lattice Field Theory - LATTICE 2013
July 29 - August 3, 2013
Mainz, Germany

*Speaker.

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1. Introduction

In the recent years there has been growing interest to understand the effects of magnetic field on chiral fermions (see [1] and references therein). Amongst the most pronounced are the chiral magnetic effect [2, 3] and charge separation effect [4, 5].

In this work we illustrate a similar effect of the induction of electric charge, which is a unique effect of magnetic and pseudo-magnetic (for QCD chromomagnetic) fields and (iso-)chemical potential [6]. The effect has direct application in graphene where the role of pseudo-magnetic field is played by an in-plane strain. We also discuss the interplay of the effect of Polyakov loop at finite temperature in QCD and the suppression of charges due to its fluctuations.

2. Lattice results: Caloron in the magnetic field

In this section we describe lattice results of a KvBLL [7, 8] caloron configuration in the magnetic field. Calorons were shown to split into monopoles when a parameter called holonomy (i.e. Polyakov loop at spatial infinity) is taken to be nontrivial. In other words when

$$\frac{1}{2}\mathrm{tr}\,\mathscr{P}(|\boldsymbol{x}|\to\infty)=\frac{1}{2}\mathrm{tr}\,\mathscr{P}^{\infty}=\frac{1}{2}\mathrm{tr}\,Pe^{i\int_0^\beta A_0(|\boldsymbol{x}|\to\infty)d\tau}\neq 0 \tag{2.1}$$

where P denotes "path-ordering". These monopoles (and their bound states: bions) were shown to be of great importance in a large class of theories[1] such as deformed Yang-Mills (YM), QCD(adj) and (softly broken) $\mathcal{N}=1,2$ Super YM and Super QCD on $\mathbb{R}^3\times\mathbb{S}^1$ and remarkable connections were made to resurgence theory (see e.g. [9, 10, 11, 12] and references therein) They were also implemented to QCD and YM phenomenologically in [13, 14, 15, 16], and efforts of direct observation on the lattice have been made [17, 18, 19].

We first brifly describe these object and how they arise in YM on $\mathbb{R}^3\times\mathbb{S}^1$. Assuming a so-called stringy gauge, so that $A_0\to v\frac{\tau^3}{2}$, the self-dual monopole solution becomes, asymptotically[2]

$$A_\varphi\approx\frac{1}{r}\frac{\tau^3}{2}\tan(\theta/2)\qquad A_0\approx\left(v-\frac{1}{r}\right)\frac{\tau^3}{2}\,, \tag{2.2}$$

where ϕ is the polar angle coordinate and $\theta/2$. This asymptotic field describes the Dirac monopole, with a Dirac string lying on the negative z-axis, i.e. with the asymptotic magnetic field

$$B_i\approx\frac{r_i}{r^3}\frac{\tau^3}{2}\,. \tag{2.3}$$

The above configuration is easily shown to have the topological charge

$$Q_{BPS}=\frac{v\beta}{2\pi}\,. \tag{2.4}$$

[1]Theories such as softly broken SYM, and QCD(adj) in confined phase, are believed to be continuously connected to their non-supersymmetric counterpart.

[2]Because of self-duality $\boldsymbol{E}=\boldsymbol{B}$, these monopoles are sometimes referred to as dyons [13, 14, 20, 15]. This name is however somewhat misleading because these objects do not interact with the A_0 field like an electrically charged particle would (see discussions in [10, 21] as well as in the original reference [7]).

However, it is important to keep in mind that A_0 is really a compact field, and its length is not gauge invariant. Let us now consider the same solution as before, but with parameter v replaced by $2\pi/\beta - v$. If we perform an anti-periodic gauge transformation $U(\tau) = e^{i\frac{\tau}{\beta}\tau^3}$ and a global gauge rotation taking $\tau^3 \to -\tau^3$ the asymptotic field becomes $A_0 \sim (v + \frac{1}{r})\frac{\tau^3}{2}$ and the magnetic field has the opposite direction (i.e. is an anti-monopole) $B_i \approx -\frac{r_i}{r^3}\frac{\tau^3}{2}$. However, the topological charge of this monopole is

$$Q_{KK} = \frac{2\pi - v\beta}{2\pi} = 1 - Q_{BPS} \,. \tag{2.5}$$

The KK stands for Kaluza-Klein, as the monopole has a "gauge twist" in the compact time direction. Notice that this gauge twist does not affect the asymptotic fields, but it affects the core where the solution becomes non-abelian.

The nontrivial fact [7, 8] is that together these two solutions combine into a caloron (instanton at finite temperature), which is consistent with their topological charges adding to unity. This solution is known exactly, however its expressions are lengthy and we refer the reader to the original references [7, 8].

Here we wish to study the caloron solution at finite magnetic field. In particular the interesting observables are charge and charge density in this background. The charge is given by

$$\langle Q \rangle = \langle J_0 \rangle = \int d^3x \, \left\langle \bar{\Psi}(x)\gamma_0\Psi(x) \right\rangle = \mathrm{tr}\,(\mathscr{D}^{-1}\gamma_0) \,, \tag{2.6}$$

where $\mathscr{D}^{-1}(x-y) = \left\langle \bar{\Psi}(x)\Psi(y)) \right\rangle$ is the Dirac propagator, and tr denotes the trace over all matrix indices and space. The integrand in the above expression is the charge density.

Since the Dirac propagator is anti-hermitian, at zero chemical potential the charge density must be either zero or purely imaginary. Since imaginary charge is absurd, one might conclude that the charge density must vanish identically if no chemical potential is present. This reasoning is wrong, as it assumes the gauge background to be physical. In fact the gauge background has an unphysical component. Namely the A_0 gauge field does not represent a physical field, but instead is an auxiliary field imposing Gauss constraint (see e.g. [22]). For any fixed configuration however A_0 looks like an imaginary color chemical potential. The gauge invariance is not violated, as the background which we discuss is a background of a spontaneously broken gauge symmetry, where the color direction is selected by the asymptotic value of the Polyakov loop.

We postpone the meaning of this imaginary charge until section 4, and for now we simply present the results of the lattice computation of the charge density and total charge in the background of the caloron and magnetic field. The results are presented in Fig. 1 (for details of the Lattice computation see [6]). The results are for the magnetic field aligned with the symmetry axis of the caloron (i.e. line connecting the two constituent monopoles). There is a pronounced charge accumulation in between the two monopole-constituents. There are also two bumps around the monopoles, with opposite signs, which tend to cancel the overall charge. The accumulation of charge, therefore, comes from the center. To understand this, let us first discuss a simpler example in two dimensions.

3. Charge catalysis in 2D

In this section we describe a simple setup of massless fermions in 2 space dimensions coupled

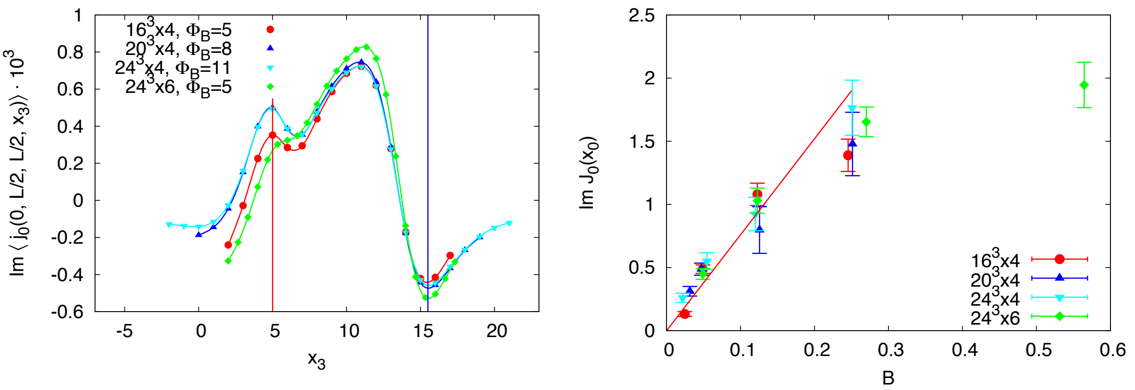

Figure 1: (Left) Lattice results of charge density on the symmetry axis of the caloron at maximally nontrivial holonomy (i.e. $\mathrm{tr}\,\mathscr{P}^\infty = 0$) using the Overlap Dirac Operator. The red and the blue lines indicate the positions of the constituent monopoles. (Right) the total (imaginary) charge as a function of magnetic field. Figures taken from [6].

to constant $U(1)\times U(1)$ magnetic field $\mathscr{B} = B + F\tau^3$ where we refer to $\tau^3 = \pm 1$ as the iso-spin index[3], and to B as magnetic, and F as pseudo-magnetic field. The spectrum is given by

$$E_n^{\pm,q} = \sqrt{2n|qB \pm qF|}\,, \qquad \text{for } \tau^3 = \pm 1 \tag{3.1}$$

where $q = \pm 1$ is the elementary charge. If we introduce iso-chemical potential μ_3, it is not difficult to see that at zero temperature and with $\mu_3 < \sqrt{2|B-F|}$ due to the unmaching degeneracies of populated particle and antiparticle states there is an overall induced charge in the system (see Fig. 2 left)

$$\langle Q \rangle = \frac{A}{2\pi}\left(|B+F| - |B-F|\right) = \frac{A}{\pi}\max(B,F) = \frac{\max(\Phi_B, \Phi_F)}{\pi} \tag{3.2}$$

where A is the area and $\Phi_{B,F}$ are the magnetic flux quanta.

If however we take μ_3 arbitrary, when μ_3 reaches the first excited state with degeneracy $\propto |B-F|$ the induced charge reduces in steps, and the zero-temperature expression becomes

$$\langle Q \rangle = |\Phi_B + \Phi_F| - |\Phi_B - \Phi_F| + 2\left\lfloor \frac{\mu_3^2}{2|B+F|} \right\rfloor |\Phi_B + \Phi_F| - 2\left\lfloor \frac{\mu_3^2}{2|B-F|} \right\rfloor |\Phi_B - \Phi_F|\,. \tag{3.3}$$

The factor of 2 in the last two terms is due to the spin degeneracy of the excited states and $\lfloor \dots \rfloor$ is the floor function. The expression above shows an oscillatory pattern as a function of chemical potential/magnetic field. Fig. 2 shows these oscillations at zero and finite temperature as a function of F/B at fixed μ. The effect is clearly washed away with temperature.

4. Charge catalysis in Yang-Mills

In this section we will generalize the charge catalysis in 2D to the one in 4D Yang-Mills theory. We will see that this will explain the numerical observation made in Sec. 2. In the case

[3]For applications of this setup to graphene see[6]

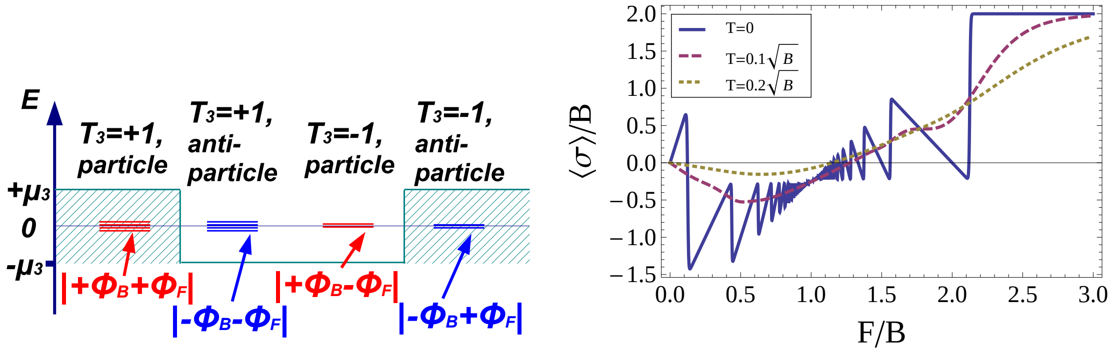

Figure 2: (Left) An illustration of charge catalysis by magnetic and pseudo-magnetic field at zero temperature and with $\mu_3 < \sqrt{2|B-F|}$. The shaded region shows the step function of Fermi-Dirac distribution at zero temperature. The red and blue labels show degeneracies of the Landau zero mode states. It is clear that the non-compensation of populated $\tau_3 = +1$ particle and $\tau_3 = -1$ antiparticle states induces a charge in the system (Right) Charge per area as a function of field F in units of B for $\mu_3 = 1.5\sqrt{B}$. Figures taken from [6]

of the caloron, the main contribution to the generated charge came from the region in between two monopoles. This is precisely the region where the quasi-abelian chromomagnetic field can be approximated to be uniform and along the symmetry axis of the caloron. We therefore take that the total magnetic field is given by

$$\mathscr{F}_{12} = B = B + F\tau^3 . \tag{4.1}$$

The one loop effective action (i.e. free energy) for non-interacting quarks, is given by

$$\log\det \not{D} = \log Z = \sum_n \sum_{q,\tau^3=\pm 1} g_n^q \log(1 + e^{(q\mu + q\tau^3 i\nu - E_n^{q,\tau^3})\beta}) + \cdots \tag{4.2}$$

where g_n is the degeneracy factor and dots represent the vacuum contribution which does not depend on chemical potential or on holonomy. The holonomy dependence appears as an imaginary chemical potential in the expression.

Since $E_n^{q,\tau^3} = \sqrt{2n|qB + q\tau^3 F + k_z|}$, taking temperature to be low enough so that we can ignore $n \neq 0$ contributions, and with $\sum_n g_n \to V \int \frac{dk_z}{2\pi} \frac{|qB+q\tau^3 F|}{2\pi}$ a simple calculation shows that

$$\langle Q \rangle = \frac{\partial}{\partial \mu} \log Z \bigg|_{\mu=0} = iV \frac{\nu}{(2\pi)^2} (|B+F| - |B-F|) , \qquad \nu \in (-2\pi T, 2\pi T) \tag{4.3}$$

where V is the volume of the system. Notice that the charge grows linearly with B until $B = F$ and then it remains constant, similar to the numerical observation[4] of Sec. 2.

However, as we mentioned, the ν is not a physical field, and it should be integrated over, so the total charge will always be zero. However the effect will show up in charge fluctuations. Although

[4]Note that one might naively conclude that the saturation happens when $B = F_c$ where F is the chromomagnetic field at the center of the caloron. This is in fact incorrect, as the chromomagnetic field is not the strongest in between the two monopoles. Nevertheless because of the good alignment of the chromomagnetic and magnetic field there it is visually most pronounced in this region.

naively the charge fluctuations will scale with V^2, the fluctuations in holonomy v^2 will scale with $\sim 1/V$. The final result for this simple model then becomes [20].

$$\langle Q^2 \rangle = \frac{VT \max(|F|,|B|)}{\pi^2} \left(1 - \frac{\min(F^2,B^2)}{\max(F^2,B^2)}\right) \tag{4.4}$$

where the second term in the parenthesis comes from the effect we described.

However the effect might be even more pronounced at zero temperature and finite quark chemical potential. In particular introduction of quark chemical potential at zero temperature would generate color charge if there is chromomagnetic field in the vacuum. Since color is confined[5], this scenario should be severely disfavored, reducing the fluctuations of the field-strength along the magnetic field, i.e. $\langle \mathrm{tr} F_{12}^2 \rangle$ drastically. However due to charge oscillations similar to the ones observed in 2D (see Fig. 2), this suppression would have an oscillatory character. Presumably this can be tested on the lattice in two-color QCD with even number of flavors, where there is no sign problem at finite density.

Acknowledgments

P. B. and F. B. are supported by the S. Kowalewskaja award from the Alexander von Humboldt foundation. T. S. is supproted by BayEFG.

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