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Finite-Temperature Behavior of Glueballs in Lattice Gauge Theories

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We propose a new method to compute glueball masses in finite temperature Lattice Gauge Theories which at low temperature is fully compatible with the known zero temperature results and as the temperature increases leads to a glueball spectrum which vanishes at the deconfinement transition. We show that this definition is consistent with the Isgur-Paton model and with the expected contribution of the glueball spectrum to various thermodynamic quantities at finite temperature. We test our proposal with a set of high precision numerical simulations in the 3d gauge Ising model and find a good agreement with our predictions.

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1. Introduction

The glueball masses in zero temperature pure lattice gauge theories are, by now, known with rather satisfactory precision [2, 4, 3]. The situation at finite temperature is very different and a good understanding of the finite temperature behavior of glueballs is still lacking. The standard approach [5, 6] is to measure the correlator of the glueball operator along the compactified "time" direction. The spectrum extracted by this method turns out to be almost constant as the temperature is raised, and it also survives the deconfinement transition. We believe that this picture is not fully satisfactory, the reasons are twofold. The first is that a very successful phenomenological model of glueballs, the Isgur-Paton model [7] and its recent generalizations [8], gives very different predictions if trusted at finite temperature. In this model glueballs are considered as "closed flux tubes", they are kept together by the same string tension of the interquark potential and, thus, the glueballs should disappear from the spectrum at the deconfinement point when the string tension vanishes. This model also suggests the temperature dependence of glueball masses, in fact it predicts the zero temperature masses as adimensional ratios $\frac{m_i(0)}{\sqrt{\sigma(0)}}$ (where $\sigma(0)$ is the zero temperature string tension). At finite temperature we expect the same ratios, but with $\sigma(0)$ substituted by the finite temperature tension $\sigma(T)$. In other words we expect

$$m_i(T) = \frac{m_i(0)}{\sigma(0)}\sqrt{\sigma(T)}, \tag{1.1}$$

the masses should decrease as the temperature approaches the deconfinement transition at $T = T_c$. The latter is in complete disagreement with the almost constant spectrum proposed in [5, 6]. Another clue against a constant spectrum comes from pure gauge thermodynamics. Recently very precise measurements of various thermodynamic observables have been extracted from lattice simulations of pure gauge theories in $d = 3+1$ [9, 10] and $d = 2+1$ [11, 12] dimensions. In the deconfined phase the thermodynamics seems to be due to a gas of free gluons (the thermodynamic observables scale as N^2) with little space left to extra degrees of freedom. Therefore the presence of glueball-like excitations seems very unlikely in the deconfined phase. This analysis suggested that the current method is probably not appropriated to measure glueball masses at finite temperature. In [1] we proposed another method which turns out to be compatible with the observations just discussed.

2. An alternative method

To ensure the correct finite temperature behavior we build an observable with the correct quantum numbers using only Polyakov loops. The simplest proposal is to choose a pair $PP^\dagger$ of nearby Polyakov loops,

$$M(x) = P(x)P^\dagger(x+a), \tag{2.1}$$

where a is the lattice spacing. The space-like version of this observable (usually denoted as "torelon pair") was used in [3, 4] as part of the operator basis to obtain the $T = 0$ glueball spectrum. In the proposed set up, this interpretation is new. The glueball mass will be extracted from the connected correlator of two M operators see fig.(1)

$$G(R,T) \equiv \langle M(0)M(x)\rangle - \langle M\rangle^2 \approx_{R\to\infty} e^{-m_0(T)R}. \tag{2.2}$$

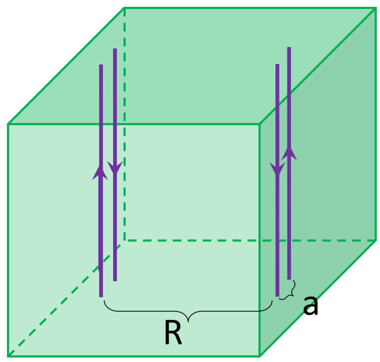

Figure 1: The glueball correlator discussed in the text.

The nice feature of our proposal is that it has a natural interpretation in terms of the effective string model of pure gauge theories. It is the four point correlator of four closed effective strings (see fig. 2). The external legs correspond to the four Polyakov loops (which are described as closed strings due to the compactification of the time direction), while the glueballs are the excitations of the closed string joining together the four legs. As mentioned in the introduction this proposal is strongly based on our intuition of the glueball dynamics coming from the Isgur-Paton model. There is however an apparent problem with this observable, in fact at T very close to the deconfinement temperature T_c, due to the dimensional reduction a mass extracted from an observable of this type should scale as $m_s \propto \frac{\sigma(T)}{T}$. Which is very different from eq.(1.1), we will see how this problem is solved in a concrete example.

3. A test in the 3d gauge Ising model

As a test for our proposal we computed the lightest glueball mass in the 3D gauge Ising model. The Ising model is a perfect laboratory to test our observable since for this model very precise estimates of the zero temperature lightest glueball mass exist [13].

$$m_0 = 3.15(5)\sqrt{\sigma(0)} \tag{3.1}$$

against which we can compare our results in the low temperature regime. Moreover as anticipated above, using dimensional reduction [14] and the fact that the Ising model deconfinement transition is in the same universality class of the $2D$ Ising magnetization transition, the behavior of the correlator $G(R,T)$ at $T \sim T_c$ can be predicted from the exact solution of the Ising model. We expect an

Figure 2: Effective string description of our proposal.

exponential fall off with decaying constant

$$m_s \sim 2\frac{\sigma(T)}{T}. \tag{3.2}$$

To summarize we expect two different scaling behavior: $m_0 \propto \sqrt{\sigma(T)}$ which we identificate with the glueball mass, and for the other mass, predicted from dimensional reduction, $m_s \propto \frac{\sigma(T)}{T}$ at very high temperature. We performed three sets of simulations at different values of the gauge coupling β in order to test scaling corrections, the relevant details are reported in Tab.(1). For all the values

β	$\frac{1}{T_c}$	L_s	N_t	R
0.743543	5.67 a	90	7,8,9	$6 \leq R \leq 20$
0.751805	8 a	90	9,10,11,12,13,14,20,56,64	$8 \leq R \leq 22$
0.756427	12 a	120	20	$12 \leq R \leq 33$

Table 1: For each of the three β values we report the corresponding critical temperature T_c[17] and the values of L_s, N_t and R that we studied.

of β, R, T we extracted the correlator $G(R,T)$ and, in a separate simulation, the finite temperature string tension $\sigma(T)$ (see [16] for the method) so as to be able to construct the functions in eq(1.1) and eq.(3.2).

4. Results

The correlator $G(R,T)$ behaves differently at low and high temperature. At low temperature ($T \lesssim 0.6T_c$) the data were perfectly fitted by the functional form

$$G(R,T) = a_0(T)\frac{e^{-m_0(T)R}}{\sqrt{R}} \tag{4.1}$$

with a very good χ^2 in the whole range of R considered. The data were precise enough to confirm even the presence of the prefactor $\frac{1}{\sqrt{R}}$. At high T ($T \gtrsim 0.6T_c$) a reasonable fit was possible only discarding the smallest R values and using the function

$$G(R,T) = \frac{a_s(T)e^{-m_s(T)R}}{R^2}. \tag{4.2}$$

Note that the prefactor $\frac{1}{R^2}$ is predicted by the dimensional reduction and it is exactly what is expected for the energy-energy correlator in the $2D$ spin Ising model. To fit the whole range of R values it has been necessary to use a two exponentials fitting function

$$G(R,T) = a_0(T)\frac{e^{-m_0(T)R}}{\sqrt{R}} + a_s(T)\frac{e^{-m_s(T)R}}{R^2}. \tag{4.3}$$

This kind of fit is very delicate, to stabilize it we used the following procedure: we introduced in the fit a gaussian prior for m_s, centered at its value at large R and with width similar to its uncertainty, and we left as free parameters only a_0, a_s and m_0. Following this procedure we were able to fit the data in the whole range of R with a very good reduced χ^2. The results are reported in Table 2 and plotted in Fig. 3.

β	$\frac{T}{T_c}$	$\sigma(T)$	$\frac{m_s(T)T}{\sigma(T)}$	$\frac{m_s(T)}{\sqrt{\sigma(T)}}$	$\frac{m_0(T)}{\sqrt{\sigma(T)}}$
0.743543	0.8	0.00961	2.04(3)	1.40(2)	2.8(1)
0.743543	0.7	0.01315	1.9(1)	1.74(9)	3.0(3)
0.743543	0.62	0.01542	1.7(2)	1.9(2)	3.22(8)
0.751805	0.89	0.00268	2.1(2)	0.99(8)	3.0(3)
0.751805	0.8	0.00444	1.94(8)	1.29(5)	3.0(2)
0.751805	0.73	0.00566	1.76(5)	1.46(4)	3.1(4)
0.751805	0.67	0.00654			3.3(1)
0.751805	0.62	0.00720			3.23(3)
0.751805	0.57	0.00771			3.29(5)
0.751805	0.4	0.00922			3.25(4)
0.751805	0.14	0.01037			3.14(3)
0.751805	0.125	0.01040			3.21(2)
0.756427	0.6	0.00326			3.29(6)

Table 2: Values of $\sigma(T)$, $\frac{m_s(T)}{\sigma(T)}$, $\frac{m_s(T)}{\sqrt{\sigma(T)}}$ and $\frac{m_0(T)}{\sqrt{\sigma(T)}}$.

5. Conclusion

Our analysis suggests that the lowest glueball mass scales, at finite temperature, as $\sqrt{\sigma(T)}$ and, therefore, is a decreasing function of the temperature. Moreover as $\sigma(T_c) = 0$ it vanishes at the deconfinement transition. The Isgur-Paton model seems to be valid also at finite temperature and its predicted scaling behavior can be conciliated with the different scaling predicted by the Svetitsky-Yaffe conjecture [14], thanks to the appearance of a new mass $m_s \propto \frac{\sigma(T)}{T}$, which dominates the

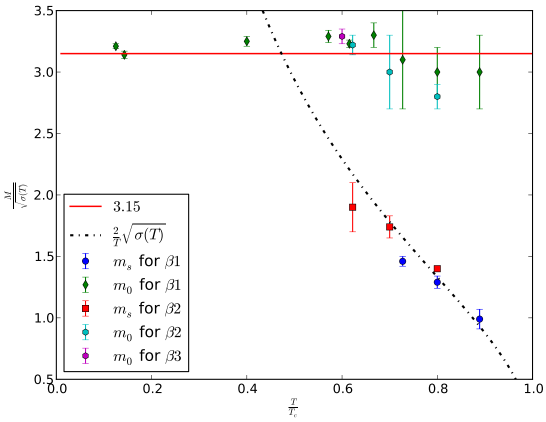

Figure 3: $\frac{m_0}{\sqrt{\sigma(T)}}$ and $\frac{m_s}{\sqrt{\sigma(T)}}$ plotted as a function of $\frac{T}{T_c}$ for $\beta_1 = 0.743543$, $\beta_2 = 0.751805$ and $\beta_3 = 0.756427$. The two curves correspond to the two expected scaling behaviours: $m_0(T) \sim 3.15\sqrt{\sigma(T)}$ and $m_s(T) = 2\sigma(T)/T$.

large R behavior of the correlator in the high temperature regime. The mass m_s looks similar to the mass of the "spurious states" observed in [3, 4], which in fact, were characterized by a large overlap with the torelon pair states. A similar mass was observed also in the finite temperature monopole spectrum in [18]. This new scale presumably measures the interaction between quarks and antiquarks belonging to different mesons and it becomes dominant when T_c is approached, this picture strongly resembles the one of deconfinement as "melting" of hadrons into individual quarks. It would be very interesting to extend our study to non-Abelian gauge theories. We expect these correlators to be very noisy; however using an exponential error reduction algorithm and a reasonable computational power, we believe it is now possible to test our observable also in these theories.

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