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Deconfinement transition at weak coupling in Yang-Mills theory on a torus

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We describe a weak coupling realization of the deconfinement transition in gauge theory compactified on $\mathbb{R}^3 \times \mathbb{S}^1$. We consider Yang-Mills theory with a single Weyl fermion of mass m in the adjoint representation of the gauge group. The fermion is subject to periodic boundary conditions, $\lambda(0) = \lambda(L)$, where L is the size of the circle S_1. This theory reduces to thermal Yang-Mills theory in the limit $m \to \infty$. In the limit $m \to 0$ the deconfinement transition can be studied using weak coupling methods. The analysis is based on semi-classical objects characterized by topological and magnetic charges. At leading order the relevant configurations are monopole-instantons and monopole-anti-monopole pairs ("bions"). We argue that in the $m - L$ plane the weak coupling transition is continuously connected to the deconfinement transition in pure gauge theory.

31st International Symposium on Lattice Field Theory - LATTICE 2013
July 29 - August 3, 2013
Mainz, Germany

*Speaker.

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1. Introduction

Finding controlled approximations to study the deconfinement transition in QCD, or in gauge theories related to QCD, is desirable for many reasons. Recently there has been some progress in this direction by investigating novel compactifications [1, 2]. In this contribution we summarize recent work on gauge theory on $\mathbb{R}^3 \times \mathbb{S}^1$ [3, 4] . We will argue that we can construct a theory that is continuously connected (as a function of a mass parameter) to pure gauge theory at finite temperature, and that this theory posses a deconfinement transition that can be studied in weak coupling.

We consider gauge theory with a single Weyl fermion in the adjoint representation of the gauge group. The gauge group can be any semi-simple compact Lie group. The lagrangian is

$$\mathscr{L} = -\frac{1}{4g^2} F^a_{\mu\nu} F^{a\mu\nu} + \frac{i}{g^2} \lambda^a \sigma \cdot D^{ab} \lambda^b + \frac{m}{g^2} \lambda^a \lambda^a . \tag{1.1}$$

Both fermions and bosons satisfy periodic boundary conditions on the circle, $\lambda(0) = \lambda(L)$ and $A_\mu(0) = A_\mu(L)$. A proposed phase diagram for this theory as a function of the compactification scale L and the mass m is shown in Fig. 1. At $m = 0$ the theory reduces to $\mathscr{N} = 1$ SUSY Yang-Mills theory. The twisted partition function is equal to the Witten index, and there is no deconfinement transition as a function of L. We will show below that for small m there is a deconfinement transition at small L. In $SU(N)$ gauge theories this transition is characterized by the breaking of Z_N symmetry. As $m \to \infty$ the theory reduces to thermal pure gauge theory which is known to have a deconfinement phase transition at $L = \beta_c = 1/T_c$. This transition is second order for $SU(2)$ gauge theory, and first order for $SU(N \geq 3)$ or other higher rank gauge groups. Fig. 1 shows the minimal phase diagram consistent with these facts. It is possible that there are additional transitions at intermediate m that are not associated with a change of symmetry. It is also possible that the slope of the transition line is not positive everywhere. This would not invalidate the picture presented here, but it would make extrapolation from small m to large m more difficult. Both of these possibilities can be investigated using lattice simulations.

2. Weak coupling calculation

2.1 Effective theory for small $\mathbb{S}^1$

In this section we will focus on $SU(2)$ gauge theory. Classical vacua of the theory are labeled by the Polyakov line

$$\Omega = \exp\left(i \int A_4 dx_4 \right) . \tag{2.1}$$

The Polyakov line can be diagonalized, $\Omega = \mathrm{diag}(e^{i\Delta\theta/2}, e^{-i\Delta\theta/2})$. At a generic point on the moduli space $\Delta\theta \neq 0$ and the Polyakov line acts as a Higgs field that breaks the gauge symmetry to its abelian subgroup, $SU(2) \to U(1)$. We can construct an effective theory that describes the light fields in the limit that the size L of circle $\mathbb{S}^1$ is small.

In this limit we can focus on the lowest Kaluza-Klein modes, and the effective lagrangian involves three dimensional fields. There are two light bosonic fields. One is the massless photon associated with the unbroken $U(1)$ symmetry. We describe this field using the dual photon

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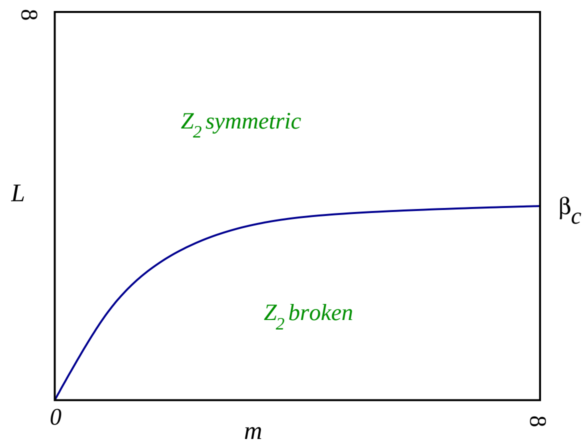

Figure 1: Schematic phase diagram of $SU(2)$ gauge theory with one adjoint Weyl fermion of mass m compactified on $\mathbb{R}^3 \times \mathbb{S}^1$. The length of the circle is denoted by L. As $m \to \infty$ the theory has a deconfining phase transition at $L = \beta_c$, where $\beta_c = 1/T_c$ is the inverse critical temperature of the pure gauge theory.

$\varepsilon_{ijk}\partial_k\sigma = F_{ij}$. We will see that near $m = 0$ the potential for $\Delta\theta$ is almost flat and the second light field is associated with fluctuations of the holonomy. We define $b = 4\pi\Delta\theta/g^2$. Finally, there is one light fermionic field λ^α which is associated with the abelian subgroup. For $m = 0$, these three fields can be written in terms of a chiral superfield $B = b + i\sigma + \sqrt{2}\theta^\alpha\lambda_\alpha$. The effective lagrangian for the bosonic fields is

$$\mathscr{L} = \frac{g^2}{32\pi^2 L}\left[(\partial_i b)^2 + (\partial_i\sigma)^2\right] + V(\sigma,b)\,, \tag{2.2}$$

where we have determined the kinetic terms at leading order in perturbation theory.

2.2 Perturbative effects

The scalar potential $V(\sigma,b)$ has an expansion of the form

$$V = \sum_n g^n V_0^n + \sum_n g^n e^{-\frac{c_0}{g^2}} V_1^n + \sum_n g^n e^{-\frac{2c_0}{g^2}} V_2^n + \ldots\,, \tag{2.3}$$

where V_0^n is related to perturbative effects and V_k^n is determined by semi-classical configuration with action $S = kc_0/g^2$. At one-loop order the perturbative part of the potential was computed by Gross, Pisarski and Yaffe [5]. In $\mathscr{N} = 1$ SUSY YM theory the potential vanishes because bosonic and fermionic contributions cancel. If the mass of the fermion is not zero then the cancellation is not exact. We find

$$V = -\frac{m^2}{2\pi^2 L^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\,|\mathrm{tr}\,\Omega^n|^2 = -\frac{m^2}{L^2}B_2\left(\frac{\Delta\theta}{2\pi}\right), \tag{2.4}$$

where B_2 is the second Bernoulli polynomial. There is no potential for the dual photon. The potential for the holonomy has a minimum at $\Delta\theta = 0, 2\pi$, which corresponds to the Z_2 broken phase. The center symmetric point $\Delta\theta = \pi$ is a local maximum of the potential.

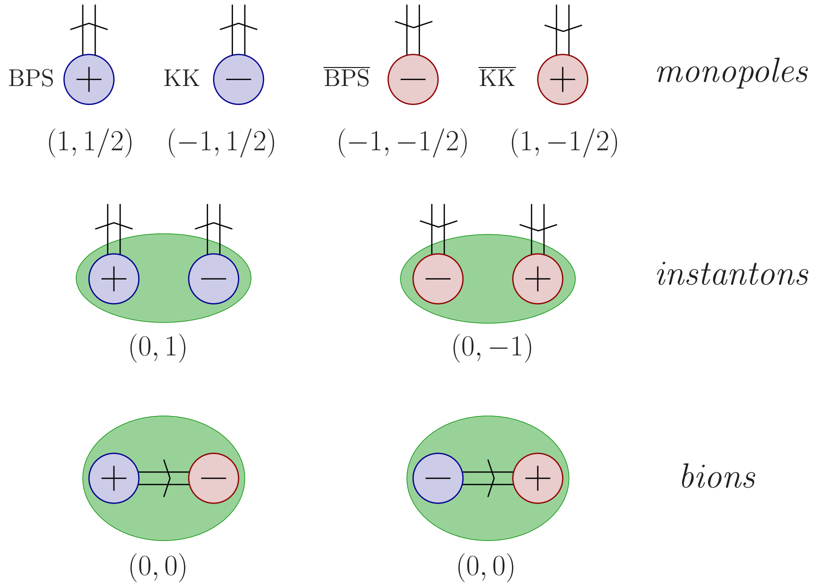

Figure 2: Topological objects in $SU(2)$ gauge theory on $\mathbb{R}^3\times\mathbb{S}^1$. The objects are labeled by (Q_M,Q_{top}) for the center-symmetric holonomy. Lines denoted fermionic zero modes, and the arrows point from $\lambda\lambda$ to $\bar\lambda\bar\lambda$ vertices.

2.3 Non-perturbative effects: SUSY Yang-Mills theory

For $m=0$ the potential for $\Delta\theta$ vanishes to all orders in perturbation theory. This implies that exponentially small corrections that arise from topological objects are important even at small coupling. Semiclassical objects on $\mathbb{R}^3\times\mathbb{S}^1$ can be classified by the asymptotic value of the holonomy Ω and by their topological and magnetic charges [5]

$$(Q_M,Q_{top}) = \left(\frac{1}{4\pi}\int_{S_2} B\cdot d\Sigma,\, \frac{1}{32\pi^2}\int_{\mathbb{R}^3\times\mathbb{S}^1} F^a_{\mu\nu}\tilde F^{\mu\nu a}\right). \tag{2.5}$$

Periodic instantons (calorons) are topological objects with $Q_{top}=k$ ($k\in Z$) and magnetic charge zero. Monopole-instantons, also known as dyons, are magnetically charged objects with fractional topological charge. Monopole-instantons come in two types, which we will refer to as BPS and KK monopole-instantons [6, 7]. Instantons can be viewed as bound states of BPS and KK monopoles. In particular, the magnetic charges of the two types of monopoles are opposite, and their topological charges add to an integer, see Fig. 2.

The coupling of the elementary BPS and KK monopoles to the low energy fields is given by

$$\mathcal{M}_1 = e^{-b+i\sigma}\lambda\lambda,\quad \mathcal{M}_2=\eta e^{+b-i\sigma}\lambda\lambda, \tag{2.6}$$
$$\overline{\mathcal{M}}_1 = e^{-b-i\sigma}\bar\lambda\bar\lambda,\quad \overline{\mathcal{M}}_2=\eta e^{+b+i\sigma}\bar\lambda\bar\lambda, \tag{2.7}$$

where $\eta=\exp(-2S_0)$ with $S_0=4\pi^2/g^2$ and we have suppressed overall numerical factors. Monopole-instantons carry fermionic zero modes and do not contribute to the bosonic potential. In the SUSY Yang-Mills case fermion zero modes are lifted by the integral over Grasssmann parameters, and monopole-instantons give a non-zero superpotential. We find [8]

$$\mathscr{W} = \frac{M_{PV}^3 L}{g^2}\left(e^{-B} + e^{-2S_0}e^{B}\right), \tag{2.8}$$

where M_{PV} is a Pauli-Villars mass parameter. This is the Affleck-Dine-Seiberg superpotential, which was originally determined by different methods [9]. The scalar potential is

$$V(b,\sigma) \sim \left|\frac{\partial \mathscr{W}}{\partial B}\right|^2 \sim \frac{M_{PV}^6 L^3 e^{-2S_0}}{g^6}\left[\cosh\left(\frac{8\pi}{g^2}(\Delta\theta - \pi)\right) - \cos(2\sigma)\right]. \tag{2.9}$$

We observe that the potential has a minimum at the center-symmetric point $\Delta\theta = \pi$, and that there is a mass gap for the dual photon. This means that for $m = 0$ the theory is in the confined phase for all L.

2.4 Non-perturbative effects: Non-zero mass case

In this section we show how to rederive this result without using supersymmetry, and then extend the calculation to $m \neq 0$. The relation $V \sim |\partial\mathscr{W}/\partial B|^2$ implies that the monopole contribution to the superpotential corresponds to a monopole-anti-monopole contribution to the scalar potential. In particular, the potential for the dual photons is generated by magnetic "bions" $[\mathscr{M}_1\overline{\mathscr{M}}_2]$ and $[\mathscr{M}_2\overline{\mathscr{M}}_1]$, and the potential for the holonomy is generated by neutral "bions" $[\mathscr{M}_1\overline{\mathscr{M}}_1]$ and $[\mathscr{M}_2\overline{\mathscr{M}}_2]$ [10].

Calculating the contribution of neutral bions is subtle because the topological charge is zero and there is no barrier between the semi-classical contribution and the perturbative vacuum. The amplitude is of the form

$$\mathscr{A}_{[\mathscr{M}_1\overline{\mathscr{M}}_1]} \sim e^{-2b}\int d^3r\, e^{-S_{12}(r)}, \qquad S_{12}(r) = -2\frac{4\pi L}{g^2 r} + 4\log(r) \tag{2.10}$$

where d^3r is the integral over the monopole separation. The first term in S_{12} is the scalar attraction between the monopoles, and the second term is due to approximate fermion zero modes. The integral over r diverges at small r. In [3] we show how to compute the amplitude by analytic continuation in g^2 [3]. This method was introduced by Bogomolny and Zinn-Justin (BZJ) in the context of instanton-anti-instanton calculations in quantum mechanics. We show that the total contribution from neutral bions is given by

$$V(b,\sigma) \sim \frac{M_{PV}^6 L^3 e^{-2S_0}}{g^6}\cosh\left(\frac{8\pi}{g^2}(\Delta\theta - \pi)\right), \tag{2.11}$$

in agreement with the calculation based on the superpotential.

Once we know how to compute the potential without supersymmetry it is straightforward to extent the result to $m \neq 0$. There are three contributions: 1) The perturbative potential given in equ. (2.4), 2) the potential from neutral and charged bions, 3) a contribution from monopoles in which the fermion zero mode is lifted by the mass term. We find [3]

$$\tilde{V} = \cosh 2b' - \cos 2\sigma + \frac{\tilde{m}}{2\tilde{L}^2}\cos\sigma\left(\cosh b' - \frac{b'\sinh b'}{3\log\tilde{L}^{-1}}\right) - \frac{1}{1728\log^3\tilde{L}^{-1}}\left(\frac{\tilde{m}}{\tilde{L}^2}\right)^2 (b')^2, \tag{2.12}$$

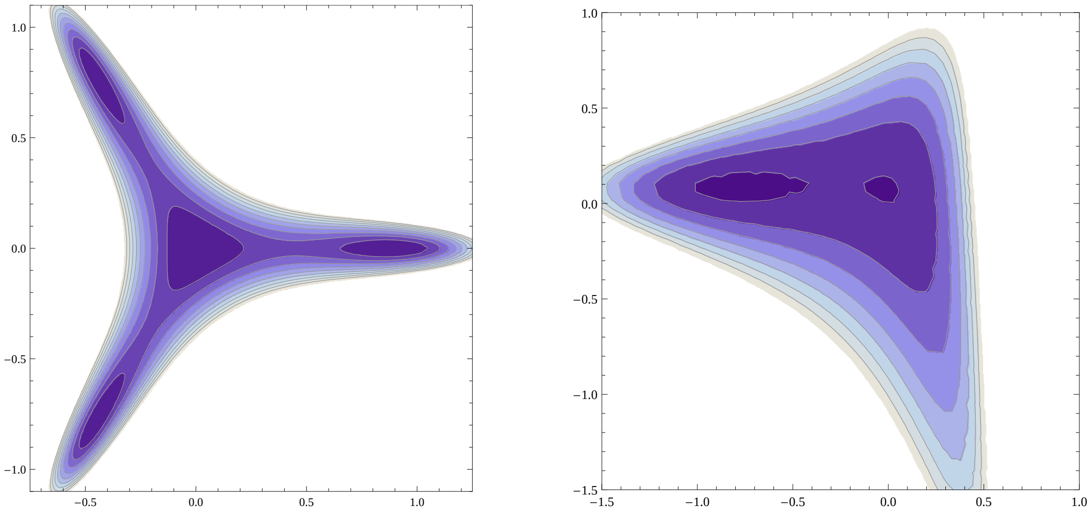

Figure 3: Contour plots of the effective potential for the holonomy in $SU(3)$ (left panel) and G_2 gauge theory (right panel). The potentials are shown at the critical length L_c corresponding to the first order deconfinement transition. The holonomy is written as $\Omega = \exp(i\vec{H}\cdot(\vec{b}_0+\vec{b}))$ and plotted as a function of $b_{1,2}$. See [4] for the definition of the Cartan vector $\vec{H}$ and the center symmetric holonomy $\vec{b}_0$.

where we have introduced dimensionless variables $b' = b - 4\pi^2/g^2$, $\tilde{L} = \Lambda L$, and $\tilde{m} = m/\Lambda$. Λ is the scale parameter, and $\tilde{V}$ is a dimensionless potential, see equ. (2.35) in [3]. The competition between the center stabilizing bions and the center de-stabilizing monopoles and perturbative terms leads to a phase transition at $L_c = \Lambda^{-1}(\tilde{m}/8)^{1/2}$, consistent with the phase diagram shown in Fig. 1.

3. Extension to other gauge groups and outlook

In [4] we show how to extend this analysis to higher rank gauge groups. We consider both gauge groups with and without a non-trivial center. If the case of gauge groups with a trivial center, like G_2, the deconfinement transition is not associated with a change of symmetry. For a general gauge group of rank r there are r-1 fundamental BPS and one fundamental KK monopole-instanton [8]. The monopole and bion induced potentials can be expressed in terms of the roots of the Lie algebra.

The phase diagram for higher rank gauge groups has the same structure as the $SU(2)$ phase diagram shown in Fig. 1, except that the phase transition is first order. In Fig. 3 we show contour plots of the potential for the holonomy at the deconfinement transition in the case of $SU(3)$ and G_2 gauge theory. Both transitions are clearly first order, but in the case of G_2 the holonomy is non-vanishing in both phases. There are a variety of issues that can be studied:

- The large N_c limit is smooth provided the mass of the lightest higgsed gluon, $m_w \sim 2\pi/(N_cL)$, is kept fixed as $N_c \to \infty$. The effective potential has multiple branches labeled by $k = 0,\ldots,N_c-1$, in agreement with Witten's arguments [11, 12].
- We can compute the shift in L_c due to a non-zero theta term. We observe that the critical $T_c \sim L_c^{-1}$ is reduced [13], in agreement with lattice calculations reported in [14].

- We have studied the distribution of the eigenvalues of the Polyakov line in the confined and deconfined phases. We observe the expected eigenvalue repulsion in the confined phase, and clustering in the deconfined phase. In the case of G_2 we observe that the Polyakov line jumps from a slightly negative value below T_c to a positive value above T_c. This behavior was also seen in lattice calculations [15].

Recent work has also begun to address the role of fermions in the fundamental representation. For large quark masses one finds the expected effects due to explicit breaking of the center symmetry [16]. For small quark masses the theory flows to strong coupling, and a dual description is required.

Acknowledgments: I would like to hank my collaborators Erich Poppitz and Mithat Ünsal. This work was supported by the US Department of Energy grant DE-FG02-03ER41260.

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