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Omega-Omega interaction on the Lattice

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We investigate interactions between Omega baryons in lattice QCD. Employing the HAL QCD method, we extract the Omega-Omega potential from the Nambu-Bethe-Salpeter (NBS) wave function, which are calculated on 2+1 flavor full QCD gauge configurations generated by the CP-PACS/JLQCD Collaboration at $m_\pi = 875$ MeV and $m_\Omega = 2108$ MeV. Both a shape of the potential and the phase shift of the Omega-Omega scattering calculated with it indicate that the interaction is strongly attractive. We finally discuss a possibility for an existence of a weak bound state in this system.

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1. Introduction

While hyperon interactions become important in hight density matters such as the core of the neutron star [1], investigations so far have been mainly focused on the octet sector. A main reason is, of course, that all decuplet baryons except Omega are unstable due to strong decays. Furthermore, even for the Omega-Omega case, it is still difficult to investigate their interaction experimentally due to their short-life time via weak decays. Theoretically, there exists two model calculations for the omega-omega inyteraction in the $J = 0$ channel [2, 3] leading to controversial results: while one reported a weakly repulsive interaction, the other indicated a strong attraction. The recent lattice QCD calculation using the standard finite volume method[Lueshcer] concluded that the Omega-Omega interaction is weakly repulsive in this channel: it's scattering length a =-0.16±0.22fm [4].

Recently, a new but first-principle method was proposed to investigate nucleon-nucleon interactions in QCD on the lattice [5–7], where the potential can be extracted from the Nambu-Bethe-Salpeter (NBS) wave function. This method has been generalized to derive potentials including hyperons (YN and YY) [8–13] and the three-nucleon force [14–16]. In this report, we therefore employ the HAL QCD method to study the Omega-Omega interaction, by calculating the corresponding potential in lattice QCD. Our results suggest an attractive Omega-Omega interaction in the $J = 0$ channel, which seems rather strong.

2. Extraction of potentials

The potential method was originally introduced by the HAL QCD collaboration [5], where a non-local potential which is defined from the equal time Nambu-Bethe-Salpeter (NBS) wave functions. For the two Omega system, the NBS wave function is defined as

$$\psi_n(\vec{r}) \equiv \langle 0|\, \Omega(\vec{r},0)\Omega(\vec{0},0)\, |\Omega(k_n)\Omega(-k_n); in\rangle \tag{2.1}$$

where $|\Omega(k_n)\Omega(-k_n); in\rangle$is an eigenstate of two-omega in QCD with the energy $2\sqrt{m_\Omega^2 + k_n^2}$, $\Omega(x)$ and $\overline{\Omega}(x)$ are local operators for Omega, whose explicit definition will be given in section 3.1. One of the most import ant properties here is that the NBS wave function at large $r = |\vec{r}|$ in QCD has the same asymptotic form to that of the scattering wave in quantum mechanics. From this fact, one can define a non-local but energy independent potential from the NBS wave function as

$$(E_n - H_0)\,\psi_n(\vec{r}) = \int d^3r' U(\vec{r}, \vec{r'})\,\psi_n(\vec{r'}) \tag{2.2}$$

where $H_0 \equiv -\frac{1}{2\mu_\Omega}\nabla^2$ is the non-interaction part of the Hamiltonian, $\mu_\Omega \equiv \frac{m_\Omega}{2}$ is reduced mass, $E_n = k_n^2/(2\mu_\Omega$ is the kinetic energy in the center-of-mass frame, in two Omega system. The non-local potential $U(\vec{r}, \vec{r'})$ can be made to be energy-independent. Although we use the non-relativistic Schrödinger equation to define the potential, no non-relativistic approximation is made here [8]. Of course, potentials are not physical observables, but the potential defined above can reproduce physical observables such as phases of the S-matirx correctly by construction.

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In lattice QCD simulations, NBS wave unctions can be extracted from the two-Omega correlation function as

$$C_{\Omega\Omega}(\vec{r},t,t_0) = \frac{1}{V}\sum_{\vec{x}}\left\langle \Omega(t,\vec{x}+\vec{r})\Omega(t,\vec{x})\overline{\Omega}(t_0)\overline{\Omega}(t_0)\right\rangle = \sum_n \psi_n(\vec{r})a_n e^{-E_n(t-t_0)}+\cdots \tag{2.3}$$

$$\simeq a_0\psi_0(\vec{r})a_n e^{-E_0(t-t_0)}, \quad t-t_0\to\infty \tag{2.4}$$

with $a_n = \left\langle \Omega(k_n)\Omega(-k_n); \mathrm{in}\left|\overline{\Omega}(t_0)\overline{\Omega}(t_0)\right|0\right\rangle$, where E_0 is the smallest energy of the system, and ellipses represent inelastic contributions.

In this report, we have employed the improved extraction, called the time dependence method [7], which is given by

$$(\frac{1}{4m}\frac{\partial^2}{\partial t^2}+\frac{1}{m}\nabla^2-\frac{\partial}{\partial t})R(\vec{r},t,t_0) = \int d^3r' U(\vec{r},\vec{r}')R(\vec{r}',t,t_0) \tag{2.5}$$

where

$$R(\vec{r},t,t_0) \equiv \frac{C_{\Omega\Omega}(\vec{r},t,t_0)}{e^{-2m(t-t_0)}} = \sum_n a_n\psi_n(r)e^{-\Delta W_n(t-t_0)}+\cdots \tag{2.6}$$

with $\Delta W_n \equiv E_n - 2m_\Omega$. Since all two-Omega elastic scattering states with different n satisfy the same Schrödinger equation with the same energy-independent non-local potential, the large $t-t_0$ limit necessary for the ground state saturation is no longer required. A condition necessary for this method to work is that $t-t_0$ should be large enough to suppress both inelastic contributions in the two-Omega system and excited states in the single-Omega correlation function.

To extract potentials in practice, we employ the derivative expansion of the non-local potential [5]. We consider the leading-order term at low energies as

$$U(\vec{r},\vec{r}') = V_{\alpha'\beta'\alpha\beta}(\vec{r})\delta(\vec{r}-\vec{r}')+\mathcal{O}(\vec{\nabla}) \tag{2.7}$$

where $\alpha',\beta',\alpha,\beta = 0\sim 3$ are spin indices. In this report, we consider a case that the total spin of two Omega baryons is zero, whose potential depend on $r=|\vec{r}|$ only.

3. Symmetry of the Omega-Omega system

In this section we discuss a symmetry of the Omega-Omega system.

3.1 Symmetry

A single Omega baryon can not decay to a pair of an octet baryon and a pseudo-scalar meson in QCD, since the lowest energy pair, Ξ and K, has larger energy than the Omega baryon mass. On the lattice, we simply define local operators Omega and anti-Omega baryons as

$$\Omega_{\alpha,k}(x) \equiv \varepsilon^{abc}s_a^T(x)C\gamma_k s_b(x)s_{c\alpha}(x), \quad \overline{\Omega}_{\alpha,k}(x) \equiv \Omega^\dagger_{\alpha,k}\gamma^0 = \varepsilon^{abc}\overline{s}_{a\alpha}(x)\overline{s}_b^T(x)\gamma_{k_1}C\overline{s}_c(x) \tag{3.1}$$

where a,b,c are color indices, ε^{abc} is the totally anti-symmetric tensor, γ_k is the gamma matrix, α is the spinor index, and $C\equiv\gamma_4\gamma_2$ is the charge conjugation matrix.

If a distance between two Omega baryons becomes large, we can neglect interactions between them. Such asymptotic Omega-Omega states can be classified by the orbital angular momentum

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	$P=+$	$P=-$
$J=0$	$S=0,L=0$; $S=2,L=2$	$S=1,L=1$; $S=3,L=3$
$J=1$	$S=2,L=2$	$S=1,L=1$; $S=3,L=3$
$J=2$	$S=2,L=0$; $S=0,L=2$; $S=2,L=2$; $S=2,L=4$	$S=1,L=1$; $S=3,L=1$; $S=1,L=3$; $S=3,L=3$; $S=3,L=5$
$J=3$	$S=2,L=2$; $S=2,L=4$	$S=3,L=1$; $S=1,L=3$; $S=3,L=3$; $S=3,L=5$
$J=4$	$S=2,L=2$; $S=0,L=4$; $S=2,L=4$; $S=2,L=6$	$S=3,L=1$; $S=1,L=3$; $S=3,L=3$; $S=1,L=5$; $S=3,L=5$; $S=3,L=7$

Table 1: Decomposition of the Omega-Omega system with conserved quantum numbers J and P in terms of states with a given (L,S).

(L), the total spin (S), the total angular momentum (J) and parity(P), even though conserved quantum numbers in QCD are J and P only. The two fermion state must change a sign under an exchange of them, while the asymptotic Omega-Omega state with given L and S has a factor $(-1)^{S+L+1}$ by the exchange, so that we should have $S+L=$ even. In table1 we decompose states with conserved quantum numbers J and P in terms of asymptotic Omega-Omega states with given L and S.

3.2 Spin Projection

Since we employ wall sources, which has $L=0$ and $P=+$, in our lattice calculations, the J is determined by the total spin S generated by the sources.

To construct the Omega-Omega operator which creates the state with the definite total spin S, let us first define a single spin 3/2 Omega baryon operator with a given S_z as

$$\Omega_{\frac{3}{2},\frac{3}{2}} \equiv -(\psi\Gamma_+\psi)\psi_{\frac{1}{2}} \tag{3.2}$$

$$\Omega_{\frac{3}{2},\frac{1}{2}} \equiv \frac{1}{\sqrt{3}}[\sqrt{2}(\psi\Gamma_Z\psi)\psi_{\frac{1}{2}} + (\psi\Gamma_+\psi)\psi_{-\frac{1}{2}}] \tag{3.3}$$

$$\Omega_{\frac{3}{2},-\frac{1}{2}} \equiv \frac{1}{\sqrt{3}}[(\psi\sqrt{2}\Gamma_Z\psi)\psi_{-\frac{1}{2}} + (\psi\Gamma_-\psi)\psi_{\frac{1}{2}}] \tag{3.4}$$

$$\Omega_{\frac{3}{2},-\frac{3}{2}} \equiv (\psi\Gamma_-\psi)\psi_{-\frac{1}{2}} \tag{3.5}$$

where $\Gamma_\pm \equiv \frac{1}{2}(C\gamma^2 \pm iC\gamma^1)$, $\Gamma_Z \equiv \frac{-i}{\sqrt{2}}C\gamma^3$, so that spin 1 di-quark operators, $\psi\Gamma_+\psi$, $\psi C\Gamma_Z\psi$ and $\psi\Gamma_-\psi$, have $S_z = 1,0,-1$, respectively, in non-relativistic limit.

Combining these operators, we can construct spin 3 ,spin 2 , spin 1,spin 0 states of Omega-Omega. The spin 0 state, used in this research, is given by

$$(\Omega\Omega)_{0,0} \equiv \frac{1}{2}\left(\Omega_{\frac{3}{2},\frac{3}{2}}\Omega_{\frac{3}{2},-\frac{3}{2}} - \Omega_{\frac{3}{2},\frac{1}{2}}\Omega_{\frac{3}{2},-\frac{1}{2}} + \Omega_{\frac{3}{2},-\frac{1}{2}}\Omega_{\frac{3}{2},\frac{1}{2}} - \Omega_{\frac{3}{2},-\frac{3}{2}}\Omega_{\frac{3}{2},\frac{3}{2}}\right). \tag{3.6}$$

4. Results

4.1 Lattice set up

In our calculation we employ 700 gauge configurations generated by CP-PACS and JLQCD Collaborations [17] with the renormalization group improved gauge action and the non-perturbatively

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$\mathscr{O}(a)$ improved Wilson quark action [17] at $\beta = 1.83$ ($a \simeq 0.12$fm) on the$16^3 \times 32$ lattice, whose physical extension becomes $L = 1.92$ fm. The hopping parameter is (κ_{ud} , κ_s) = (0.13760, 0.13710), which gives m_π = 875(1) MeV and m_K = 916(1) MeV. The periodic boundary condition is imposed on the quark fields along the spatial direction, while the Dirichlet boundary condition is employed along the temporal direction on the time-slice t = $\frac{T}{2}$. To improve statistics, we calculate nine sources on different time slices per config, where the Dirichlet boundary is always separated from the source by $\frac{T}{2}$. Statistical errors are estimated by the Jackknife method where the bin-size is taken to be 1 configuration.

4.2 Omega-Omega Potential

We show the central potential between Omega-Omega in the $^1S_0$ channel at $t - t_0 = 7,8,9$ in Fig1, where we use the notation $^{2S+1}L_J$ to specify quantum numbers of the channel. Overall structures of potentials are similar to those of NN potentials previously obtained in the lattice QCD [5]. This potential have repulsive core and deep attractive pocket, whose depth is 70~80 MeV. We observe that time dependence is small except for a long distance part at $t = 9$, which seems to be affected by finite volume effect. In future studies, we should investigate a volume dependence of the potential.

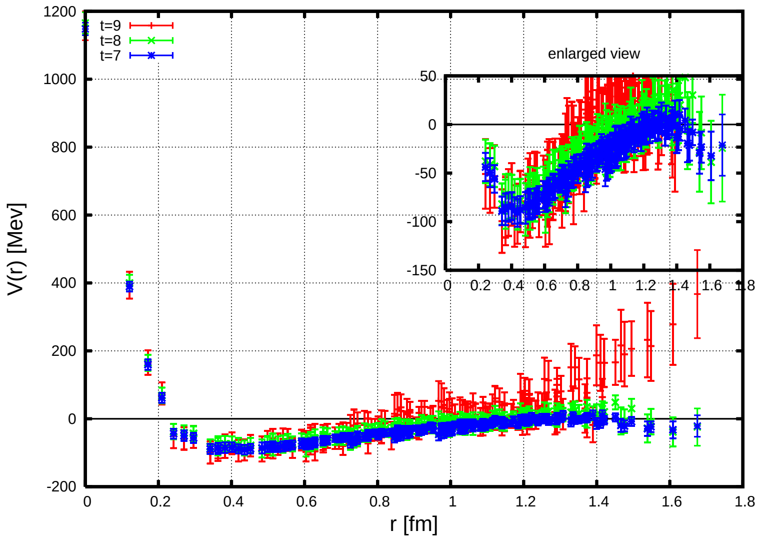

Figure 1: The central potential $V_c(r)$between Omega-Omega in the $^1S_0$ channel at $t - t_0 = 7$(blue) ,8(green), 9(red).

4.3 Phase shift and Scattering length

To calculate phase shift and scattering length, we first fit the potential with the form that

$$V(r) = a_1 e^{-a_2 r^2} + a_3 e^{-a_4 r^2} + a_5 e^{-a_6 r^2}, \tag{4.1}$$

which gives, for example, $a_1 = -1.5(0.5) \times 10^2$MeV, $a_2 = 2.0(0.6)$fm^{-2}, $a_3 = 2.0(0.9) \times 10^2$MeV, $a_4 = 1.2(1.1) \times 10$fm^{-2}, $a_5 = 1.0(0.1) \times 10^3$MeV, $a_6 = 6.9(1.1) \times 10$fm^{-2} at $t - t_0 = 7$.

Using this fit result, we solve the Schrödinger equation in the infinite volume, and we obtain the phase shift $\delta(k)$ in the $^1S_0$ channel. The scattering phase shift $\delta(k)$ at a given momentum k can be obtained as ratio

$$\tan\delta(k) = \lim_{x_1,x_2\to\infty} \frac{\psi_k(x_2)\sin(kx_1) - \psi_k(x_1)\sin(kx_2)}{\psi_k(x_1)\cos(kx_2) - \psi_k(x_2)\cos(kx_1)}. \tag{4.2}$$

We show a center of mass energy dependence of the scattering phase shift in Fig. 2, where $E_{\rm CM} = k^2/(2\mu_\Omega)$. As shown in the figure, while phase shift calculated from the potential at t=7 suggests an existence for a bound state, phase shifts from data at t=8 and 9 indicate that the Omega-Omega interaction is strongly attractive but it may not be strong enough to form a bound state at this quark mass.

The scattering length a and effective range r_e are defined by

$$k\cot\delta(k) = \frac{1}{a} + \frac{1}{2}r_e k^2 + O(k^4). \tag{4.3}$$

The fit of $k\cot\delta(k)$ near $k = 0$ by the above formula gives $1/a = -2.4(2.5)\times10^{-1}, 9.0(6.9)\times10^{-2}, 2.9(1.2)\times10^{-1}$ fm^{-1}, and $r_e = 5.0(5.1)\times10^{-3}, 5.1(1.0)\times10^{-3}, 5.2(1.8)\times10^{-3}$ fm at $t - t_0 = 7, 8, 9$, respectively. Unfortunately, errors of both $1/a$ and r_e are quite large, as expected from Fig. 2.

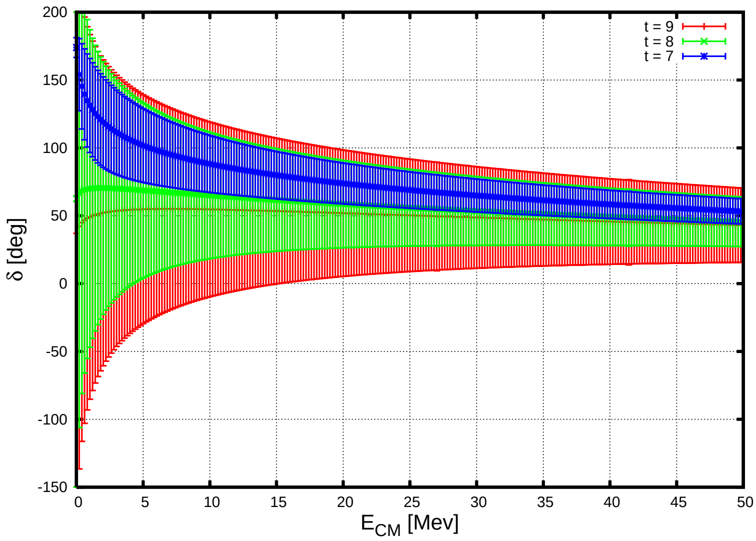

Figure 2: Scattering phase shift at $t - t_0$ =7(blue) ,8(green) , 9(red), as a function of the center of mass energy $E_{\rm CM}$.

5. Summary

In this paper, we have investigated the Omega-Omega interaction in the channel with the orbital angular momentum L=0 and the total spin S=0 in 2+1 flavor lattice QCD, using the method developed by the HAL QCD collaboration. We find that the central potential obtained from the

NBS wave function for the $^1S_0$ channel shows attractions at long distance and the repulsive core at short distance. The phase shift derived from this potential shows that the Omega-Omega interaction is strongly attractive, while an existence for an Omega-Omega bound sate is unfortunately inconclusive at the pion mass in this study. In future we will examine the Omega-Omega interaction at more lighter pion masses in larger volumes.

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