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Studies of multi-strangeness baryon-bayon interactions from lattice QCD

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Derivation of baryon-baryon interactions from lattice QCD is highly awaited to investigate hypernuclear and neutron star structure and mechanism of supernova explosions since their experimental data are scarce. Owing to developments of computer performances and simulation techniques, lattice QCD calculations allow us to understand nuclear physics in terms of fundamental theory of the strong interaction (QCD). Our approach to baryon-baryon interactions is deriving a potential from inverting Schröedinger equation using Nambu-Bethe-Salpeter (NBS) wave function simulated by lattice QCD. This approach have been extended to the cases of multi-strange baryon-baryon interactions. Our numerical results are obtained from 2+1 flavor QCD gauge configuration provided by the PACS-CS Collaboration. The scattering parameters by these potentials are also discussed.

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1. Introduction

Investigations of the baryon-baryon (*BB*) interactions are crucial for the deeper understandings of atomic nuclei, structure of neutron stars and supernova explosions. Owing to developments of computer performances and simulation techniques, lattice QCD calculations allow us to understand nuclear physics in terms of fundamental theory of the strong interaction (QCD). A series of investigations by HAL QCD Collaboration [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] enables us to extract *BB* potentials from Nambu-Bethe-Salpeter (NBS) wave function by lattice QCD simulation. The method would potentially be a complement to baryon-baryon scattering experiments since potentials derived through our procedure are assured to reproduce the scattering phase shifts by the QCD lagrangian. Comparison between the HAL QCD method and the Lüscher's finite volume method [11], which relates the energy of the system to the scattering phase shift, has been performed in Ref. [12].

Using the potentials derived from our method, it is interesting to seek a possibility for an existence of a bound state in two-baryon systems other than the deuteron, especially in strange sector because it is difficult to access there experimentally. In the flavor SU(3) symmetric world simulated by lattice QCD, the investigation by our method indicates an existence of the bound state in the flavor singlet channel with strangeness $S = -2$ [13, 14], which correponds to the H-dibaryon state, predicted by R. L. Jaffe [15]. An extension of the HAL QCD method to the coupled channel formalism is achieved in Refs. [16, 17] to investigate the case of strangeness $S = -2$ system with isospin $I = 0$ with the (large) SU(3) breaking effects. By applying the conventional Lüscher formula, it is shown that the H-dibaryon state exists with the binding energy of about 13 MeV at $m_\pi \sim 390$MeV [18].

Bound state search on strangeness $S = -4$ $\Xi\Xi$ system is also interesting and heatedly discussed in papers [19, 20, 21]. Especially for 27-plet irreducible representation in flavor SU(3) symmetry, they expected a bound state of $\Xi\Xi$ $^1S_0$ state. Actually, NN $^1S_0$ state is nearly bound into di-neutron at physical point. It means that there is strongly attractive potential in 27-plet. According to the quark cluster model calculation [22], the short range repulsion in nuclear (or BB) force are originated by the Pauli effect and color-magnetic interaction between quarks. Therefore we think that this system is interesting to see how SU(3) breaking effects appears in BB potential and to reveal the roles of quarks in baryons based on QCD.

The paper is organized as follows. First, We briefly show the procedure of HAL QCD method to extract an energy independent potential in Sect. 2. Setups of our numerical simulations are given in Sect. 3. The results of BB interactions and scattering phase shifts in $S = -4$ sector are presented in Sect. 4 Conclusion and outlook are given in Sect. 5.

2. Formalism

We start with the (equal time) NBS wave function defined with local composite operators for a baryon, $B(\vec{x})$, as

$$\psi^{B_1B_2}(\vec{r},E) = \sum_{\vec{x}} \langle 0 \mid B_1(\vec{x}+\vec{r})B_2(\vec{x}) \mid E \rangle, \tag{2.1}$$

which is embeded in the normalized four point correlator given for moderate t by

$$R^{B_1 B_2}_{\mathscr{I}}(t,\vec{r}) = \sum_{\vec{x}} \frac{\langle 0 \mid B_1(t,\vec{x}+\vec{r}) B_2(t,\vec{x})\, \overline{\mathscr{I}}(0) \mid 0 \rangle}{e^{-(m_1+m_2)t}} = \sum_n A_{E_n} \psi^{B_1 B_2}(\vec{r},E_n) e^{-\tilde{E}_n t} + \cdots \tag{2.2}$$

where $A_E = \langle E \mid \overline{\mathscr{I}}(0) \mid 0 \rangle$ and $\tilde{E} \equiv E - m_1 - m_2$. Source operator $\mathscr{I}(t_{src})$ located at $t = t_{src}$ creats states with baryon number $B = 2$. Total energy of the system E is related with the asymptotic momentum p in center-of-mass (CM) frame by

$$E_n = \sqrt{m_1^2 + p_n^2} + \sqrt{m_2^2 + p_n^2}. \tag{2.3}$$

For symplicity, we neglect the ellipse part in eq. (2.2) which represents inelastic contributions.

In this study, we focus on the $\Xi\Xi$ system with strangeness $S = -4$. We employ local composite interpolating baryon operators for Ξ multiplet as

$$\Xi^0_\alpha(\vec{x}) = \varepsilon^{abc}(s^T_a(\vec{x}) C\gamma_5 u_b(\vec{x})) s_{c\alpha}(\vec{x}), \quad \Xi^-_\beta(\vec{x}) = \varepsilon^{abc}(s^T_a(\vec{x}) C\gamma_5 d_b(\vec{x})) s_{c\beta}(\vec{x}) \tag{2.4}$$

with the totally anti-symmetric tensor ε^{abc} and the charge conjugation matrix C. The roman and greek indices in eq (2.4) represent the colors and spinors of quarks.

The most general form of the Schrödinger equation for wave function $\psi(\vec{r},E)$ is given by using a reduced mass $\mu = m_\Xi/2$ and an asymptotic momentum p in CM frame as,

$$\left[\frac{p_n^2}{m_\Xi} - H_0\right] \psi(\vec{r},E_n) = \int d^3\vec{r}'\, U(\vec{r},\vec{r}')\, \psi(\vec{r}',E_n) \tag{2.5}$$

where $H_0 = -\nabla^2/m_\Xi$ is the free Hamiltonian, $U(\vec{r},\vec{r}')$ is an energy independent non-local potential [2]. The derivative expansion is performed to handle the non-locality of potential as

$$U(\vec{r},\vec{r}') = (V_{LO} + V_{NLO} + \cdots)\delta(\vec{r} - \vec{r}'), \tag{2.6}$$

where N^nLO term is of $O(\vec{\nabla}^n)$. At low energies, efficiency of derivative expansion was confirmed in Ref. [6].

Alternative derivation of potential without assuming a single state saturation has been proposed in [8]. Within the non-relativistic expansion that $E - 2m_\Xi \simeq p^2/m_\Xi$, we can easily obtain the kinetic energy term by the time derivative of the normalized four-point correlator as

$$-\frac{\partial}{\partial t} R^{B_1 B_2}_{\mathscr{I}}(t,\vec{r}) = \sum_n \frac{p_n^2}{m_\Xi} A_{E_n} \psi^{B_1 B_2}(\vec{r},E_n) e^{-(E_n - m_1 - m_2)t}. \tag{2.7}$$

Thus we finally have

$$\left(-\frac{\partial}{\partial t} - H_0\right) R^{B_1 B_2}_{\mathscr{I}}(t,\vec{r}) = V_{LO}(\vec{r}) R^{B_1 B_2}_{\mathscr{I}}(t,\vec{r}). \tag{2.8}$$

This method requires a large t which is large enough that the inelastic contributions can be neglected from $R^{B_1 B_2}_{\mathscr{I}}(t,\vec{r})$. The potential through this equation is determined without ground state saturation for R and is guaranteed to be proper for the scattering phase-shift of two-Ξs by QCD lagrangian.

Table 1: Summary table of gauge ensembles and hadron masses in unit of MeV.

	κ_{ud}	κ_s	π	K	N	Λ	Σ	Ξ
Esb1	0.13700	0.13640	701(1)	789(1)	1585(5)	1644(5)	1660(4)	1710(5)
Esb2	0.13727	0.13640	570(1)	713(1)	1411(12)	1504(10)	1531(11)	1610(9)
Esb3	0.13754	0.13640	411(2)	635(2)	1215(12)	1351(8)	1400(10)	1503(7)

3. Numerical simulations

In the calculation we employ $2+1$-flavor full QCD gauge configurations from Japan Lattice Data Grid(JLDG)/International Lattice Data Grid(ILDG) [25]. The PACS-CS Collaboration generated $L^3 \times T = 32^3 \times 64$ lattice with a renormalization-group improved gauge action at $\beta = 6/g^2 = 1.90$ and a non-perturbatively $O(a)$ improved Wilson-clover quark action with $C_{SW} = 1.715$, corresponding to lattice spacings of $a = 0.091$ fm ($a^{-1} = 2.176$ GeV) [23]. The spacial volume of them is about $(2.9\text{ fm})^3$ in physical unit. In order to investigate quark mass dependences of BB interaction, three sets of hopping parameters for the u and d quark masses with fixed $\kappa_s = 0.13640$ are considered named as Esb1, Esb2 and Esb3 respectively corresponding to $m_\pi \simeq 700, 570$ and 410MeV, given in Table 1.

Quark propagators are calculated with the spatial wall source at $t = t_{src}$ with the Dirichlet boundary condition in temporal direction at $t = 32 + t_{src}$ which rules out an opposite propagation of two baryons in temporal direction. The wall source is placed at 16 different time slices on each of different gauge configuration ensembles, in order to enhance the signals, together with the average over forward and backward propagations in time direction. An average over the cubic group is taken for the sink operator, in order to obtain the S-wave in the BB wave function.

4. Results

We discuss the results of BB potentials in strangeness $S = -4$ sector. Fig. 1 shows the results of the S-wave $\Xi\Xi$ system for three different gauge ensembles. Left panel of fig. 1 shows the potential in the $^1S_0$ and $I = 1$ channel and right shows the phase-shifts by solving Lippmann-Schwinger equation in the infinite volume using the extracted potentials. From the figure of potentials (left panel), we observe a repulsion at short range region while an attraction at long distance which is similar to the case of the NN potential belonging to the same multiplet in flavor $SU(3)$ symmetry if it is in the symmetric limit. Comparing the results from different gauge ensembles, we can check the quark mass dependence of the potential. While short range repulsion is increasing as the light quark masses are decreasing, we can not see a clear enhancement of a range of attractive pocket which is expected of the decreasing exchanged meson masses.

The $\Xi\Xi$ scattering phase shift is plotted in the right panel of fig. 1. The phase shift rises rapidly at small energy and gradually diminishes its value. The tendency of shape of the phase shift is similar to the NN scattering case but it looks far from making a bound state. Paying attention to the quark mass dependence of the $\Xi\Xi$ scattering phase shift, it becomes smaller as a growth of repulsive core of potential. Thus we can not see any indication of bound $\Xi\Xi$ state in the $^1S_0$ and $I = 1$ channel at least within the mass range of $m_\pi = 410$ - 700 MeV.

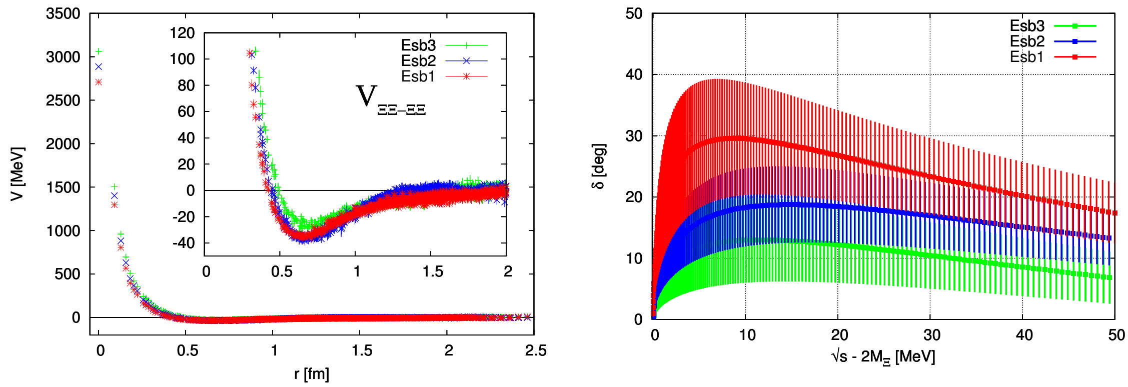

Figure 1: Simulation results for $^1S_0$ $\Xi\Xi(I=1)$ case. (Left) The $\Xi\Xi$ central potentials are plotted as a function of r. (Right) The scattering phase shifts calculated with the extracted $\Xi\Xi$ potential are plotted as a function of CM energy.

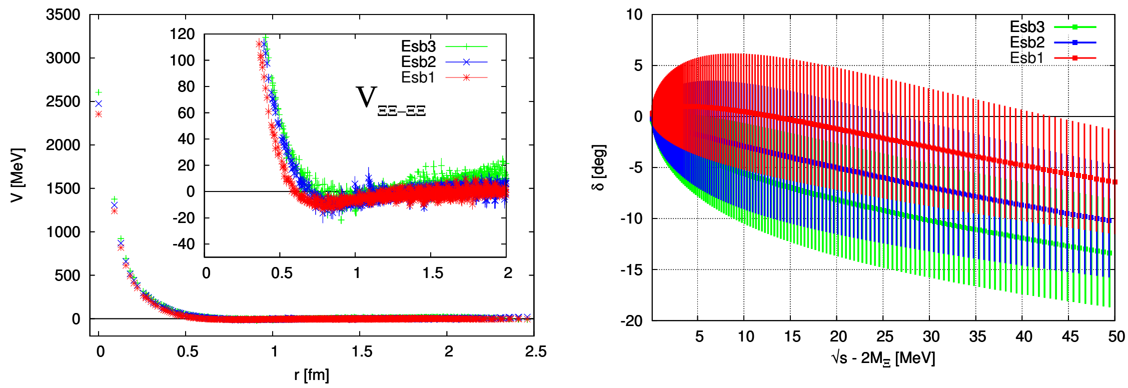

Figure 2: Simulation results for $^3S_1$ $\Xi\Xi(I=0)$ case. (Left) The $\Xi\Xi$ central potentials are plotted as a function of r. (Right) The scattering phase shifts calculated with the extracted $\Xi\Xi$ potential are plotted as a function of CM energy.

Fig. 2 shows the potential for $^3S_1(I=0)$ channel (left panel) and corresponding phase shift. For the left panel of Fig. 2, again we observe a repulsive core and an attractive pocket whose depth is much shallower than the $^1S_0$ and $I=1$ channel. We can not see any enhancement of attraction of potential in long range region as the light quark masses decreasing. A repulsion at short distance is expected to be much stronger than the $^1S_0(I=1)$ potential since the Pauli blocking effect is stronger than $^1S_0$ channel according to the results of constituent quark model calculation. Our result in Fig. 2 is consistent with this conjecture. Again we can find similar quark mass dependence to the $^1S_0(I=1)$ potential, such as increment of short range repulsion and long range attraction as decreasing light quark masses.

The scattering phase shift of $^3S_1(I=0)$ channel is plotted in the right panel of fig. 2. The phase shift slowly drops up to 50 MeV of CM energy, which means that potential in the $^1S_0$ and $I=1$ channel is essentially repulsive and there is no chance to have a bound state. We claerly find that repulsion of $\Xi\Xi$ force in $^3S_1(I=0)$ channel gets stronger as decreasing light quark masses.

5. Conclusion

We have investigated the $S = -4$ BB potentials from $2+1$ flavor lattice QCD. We employ full QCD gauge configurations generated by PACS-CS Collaboration with three different sets of hopping parameters which correspond to the pion mass of about 700, 570 and 410 MeV in order to check the quark mass dependence of $\Xi\Xi$ force. We find that diminution of u and d quark masses leads to a increment of short range repulsion of $\Xi\Xi$ force but do not have very much effects for an attraction at long distances. Although strong attraction can be seen in ${}^1S_0(I=1)$ channel, it is not strong enough to have a bound state within a mass region of $m_\pi = 410$ - 700 MeV. On the other hand, the $\Xi\Xi$ force in ${}^3S_1(I=0)$ channel is essentially repulsive with very shallow attractive pocket. Thus we conclude that there is no bound $\Xi\Xi$ state for both ${}^1S_0(I=1)$ and ${}^3S_1(I=0)$ channels.

Our conclusion is inconsistent with the result of multivolume exploration of the $\Xi^-\Xi^-$ bound state at a pion mass of about 390 MeV. Our potential is guaranteed to reproduce the proper scattering phase shift, so that the origins of such discrepancy should be solved in the future works.

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