PROCEEDINGS OF SCIENCE

$D\,K$ scattering and the D_s spectrum from lattice QCD

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We present results from Lattice QCD calculations of the low-lying charmed-strange meson spectrum using two types of Clover-Wilson lattices. In addition to quark-antiquark interpolating fields we also consider meson-meson interpolators corresponding to D-meson kaon scattering states. To calculate the all-to-all propagation necessary for the backtracking loops we use the (stochastic) distillation technique. For the charm quark we use the Fermilab method. Results for the $J^P = 0^+$ $D^*_{s0}(2317)$ charmed-strange meson are presented.

31st International Symposium on Lattice Field Theory - LATTICE 2013
July 29 - August 3, 2013
Mainz, Germany

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1. Introduction

Prior to the observation of charmed-strange mesons in experiment, quark models predicted the existence of resonances of appreciable width with $J^P = 0^+$ and $J^P = 1^+$ above the DK and D^*K thresholds. Results from various experiments instead show narrow states below threshold [1] dubbed $D^*_{s0}(2317)$ and $D_{s1}(2460)$ which form a doublet of $j^P = \frac{1}{2}$ states[1] in the heavy quark limit. The coupling of these states to the DK system near threshold was suggested as a mechanism for lowering their mass [2]. Beyond D_s mesons, the observed meson spectrum contains many states close to s-wave thresholds, leading to interpretations as shallow bound molecules, strongly bound tetraquarks or mesons with gluonic excitations. As an example of such a system we study the $D^*_{s0}(2317)$ in detail, using a combined basis of quark-antiquark ($\bar{q}q$) and meson-meson interpolators. Our results have been published in [3]. Section 2 takes a look at some previous results, Section 3 describes certain details of our simulation and Section 4 summarizes our results.

2. Previous results

All previous lattice results considered only quark-antiquark interpolators for studies of the $D^*_{s0}(2317)$. Early (quenched) lattice studies [4 – 7] found energy levels above the physical DK threshold. More recent dynamical LQCD calculations [8 – 13] found states close the the DK threshold which could not be convincingly related to the $D^*_{s0}(2317)$ due to the closeness of the DK threshold.

Figure 1 shows results from our previous simulation [10], where D_s meson states of $J^P = 0^+$ and 1^+ significantly above the physical states were observed (see left panel). Close to physical pion masses, these states were almost degenerate with the non-interacting $D^{(*)}K$ threshold as measured on the lattice. In particular it turned out that the thresholds in our simulation were quite unphysical, which was caused by an unphysical kaon mass.

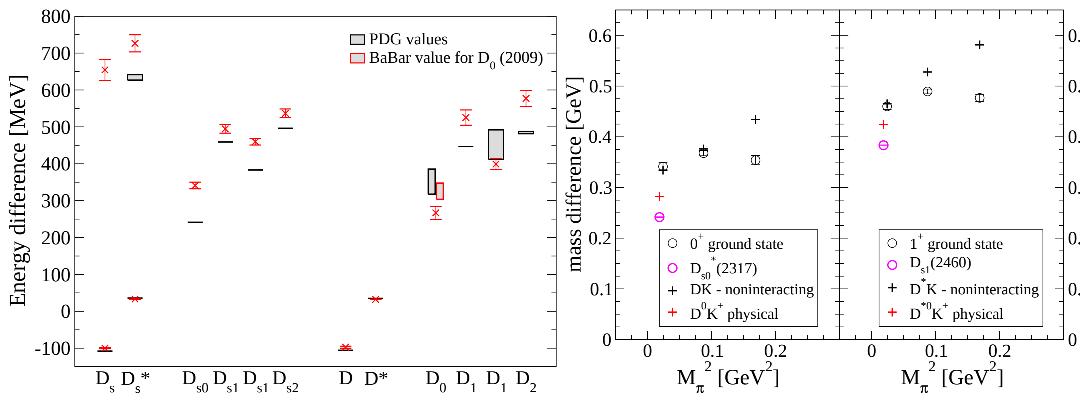

Figure 1: Previous results on the spectrum of D and D_s mesons from [10] using a basis of quark-antiquark interpolators. The left panel shows mass splittings with respect to the spin-averaged ground state $M_{\overline{1S}} = (M_{D_q} + 3M_{D^*_q})/4$ where $D^{(*)}_q$ stands for D and D_s mesons respectively. The right panel shows the splitting of the low-lying states with $J^p = 0^+$ and $J^p = 1^+$ as a function of the pion mass.

[1] here j denotes the light-quark spin

3. Calculational setup

For our simulation we use two types of gauge configurations with parameters listed in Table 1. Ensemble (1) has two mass-degenerate flavors of nHYP smeared Clover-Wilson quarks [14, 15]. The pion mass for this ensemble is $m_\pi = 266(3)(3)$MeV [16] while the strange and charm quark have been tuned to their physical values [17]. Ensemble (2) provided by the PACS-CS collaboration [18] has almost physical pion masses. For this ensemble the strange sea quark mass differs significantly from its physical value and we therefore use partially quenched strange quark masses $m_s^{val} \neq m_s^{sea}$ which we tune by either demanding that the ϕ meson made from just connected strange quarks assumes its physical value or by demanding that the mass of the unphysical η_s agrees with a high precision lattice determination [19]. Both determinations agree well and we obtain $\kappa_s = 0.13666$ which almost coincides with the determination in [20].

ID	$N_L^3 \times N_T$	N_f	a[fm]	L[fm]	#configs	m_π[MeV]	m_K[MeV]
(1)	$16^3 \times 32$	2	0.1239(13)	1.98	280/279	266(3)(3)	552(2)(6)
(2)	$32^3 \times 64$	2+1	0.0907(13)	2.90	196	156(7)(2)	504(1)(7)

Table 1: Gauge configurations used for the study of D_s mesons. N_L and N_T denote the number of lattice points in space and time directions, N_f refers to the number of dynamical sea quarks and a denotes the lattice spacing.

The Fermilab method [21, 22] is used for the charm quarks. The particular approach we use is outlined in detail in [17] for ensemble (1), and we repeat the same steps for ensemble (2). This leads to the improvement coefficient $c_{sw} = c_E = c_B = 1.64978$ and to the heavy quark hopping parameter $\kappa_c = 0.12686$. Within this approach the meson rest masses are affected strongly by discretization effects, while splittings between different mesons involving heavy quarks are expected to be close to physical. Therefore we always quote mass splittings and show mass splittings in all our plots. The errorbars include both the statistical uncertainty and the uncertainty due to scale setting.

Quark propagators are calculated using the distillation method [23, 24]. While full distillation is feasible for small physical volumes, larger volumes where many eigenvectors of the lattice Laplacian are needed are more efficiently handled within stochastic distillation [24]. We adopt this approach for ensemble (2), grouping 192 eigenvectors into 12 groups of 16 vectors in what Reference [24] refers to as an interlacing scheme. We calculate these propagators for 8 equidistant time slices and use them for quark lines connecting source and sink timeslices. For backtracking quark lines we also use time-interlacing [24], where we combine 8 timeslices in an interlacing scheme for a total of 8 additional sets of quark sources. We use four sets of random numbers for each quark type. This means that in total we calculate the matrix inverse on $2*(12*8*4*4) = 3072$ sources for each quark type. To achieve this the highly efficient SAP-GCR inverter from Lüscher's DDHMC package [25, 26] is used.

The distillation method allows for a large freedom in the choice of source and sink interpolators, which we exploit for our choice of basis within the variational method [27–29]. We calculate the correlation matrix

$$C_{ij}(t) = \sum_{t_i} \langle 0 | O_i(t_i + t) O_j^\dagger(t_i) | 0 \rangle = \sum_n \mathrm{e}^{-tE_n} \langle 0 | O_i | n \rangle \langle n | O_j^\dagger | 0 \rangle \ , \tag{3.1}$$

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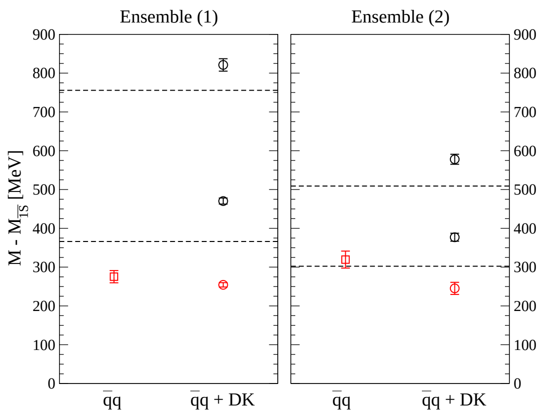

Figure 2: Energy levels obtained in our simulation. The left panel shows results from ensemble (1) with $m_\pi = 266$MeV and the right panel shows results from ensemble (2) which has almost physical pion and kaon masses. In both cases results from just $\bar{q}q$ interpolators and from a combined basis of $\bar{q}q$ and D-meson kaon interpolators are shown. The dashed lines indicate the two lowest D-meson-kaon noninteracting scattering thresholds as obtained in the finite volume.

for interpolating fields O_i with quantum numbers $J^P = 0^+$ and $I = 0$. We use four quark antiquark interpoltors and three meson-meson interpolators

$$\begin{aligned}
O_1^{\bar{s}c} &= \bar{s}c\,, & O_1^{DK} &= [\bar{s}\gamma_5 u](p=0)\,[\bar{u}\gamma_5 c](p=0) + \{u \to d\}\,, \\
O_2^{\bar{s}c} &= \bar{s}\gamma_i \overrightarrow{\nabla}_i c\,, & O_2^{DK} &= [\bar{s}\gamma_t\gamma_5 u](p=0)\,[\bar{u}\gamma_t\gamma_5 c](p=0) + \{u \to d\}\,, \\
O_3^{\bar{s}c} &= \bar{s}\gamma_t\gamma_i \overrightarrow{\nabla}_i c\,, & O_3^{DK} &= \sum_{p=\pm e_{x,y,z}\,2\pi/L} [\bar{s}\gamma_5 u](p)\,[\bar{u}\gamma_5 c](-p) + \{u \to d\}\,, \\
O_4^{\bar{s}c} &= \bar{s}\overleftarrow{\nabla}_i \overrightarrow{\nabla}_i c\,. & &
\end{aligned} \tag{3.2}$$

4. Results

Figure 2 shows the results for the low lying states on both ensembles. In both cases further higher lying energy levels are observed which are of poor statistical quality and which are away from the region of interest. When using the combined basis of $\bar{q}q$ and meson-meson interpolators we observe an additional low-lying level which could not be reliably extracted with the basis of just $\bar{q}q$ interpolators. Furthermore a second additional level appears which is dominated by the meson-meson interpolator with non-vanishing momentum $D(1)K(-1)$. This observation is confirmed by the observed energy in vicinity of the noninteracting level.

The situation allows for two possibilities: (1) A "bound state" of unknown nature below DK threshold or (2) a resonance with a strong decay into DK. The crucial insight is that we can test the plausibility of these interpretations as they predict different scattering lengths. Using Lüscher's formula [31] and an effective range approximation we determine the scattering length a_0 and the effective range parameter r_0. For both ensembles we observe a negative scattering length consistent with the bound state interpretation [32 – 34]. We plot our results for a_0 along with results from the indirect determination in Ref. [30] in Figure 3. Our results agree within the remaining uncertainties.

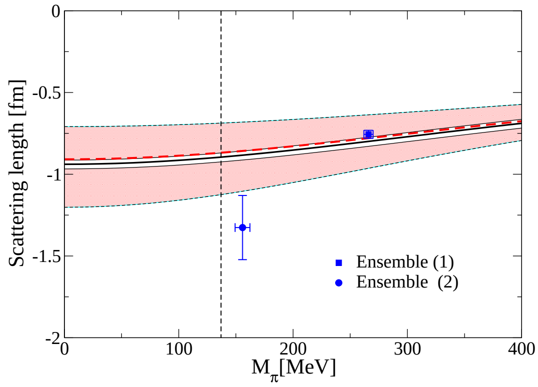

Figure 3: Scattering length a_0 for D-meson kaon scattering (I=0) in s-wave. We compare our results to the expectation from [30]. The vertical line corresponds to the physical pion mass. Two sets of curves are shown. The dashed curves with the red error band are the prediction using only lattice data as input to constrain the low energy constants in the effective field theory. For the second set of curves (thick solid) with small error band the experiment value for the mass of the $D^*_{s0}(2317)$ was used to constrain the low energy constants.

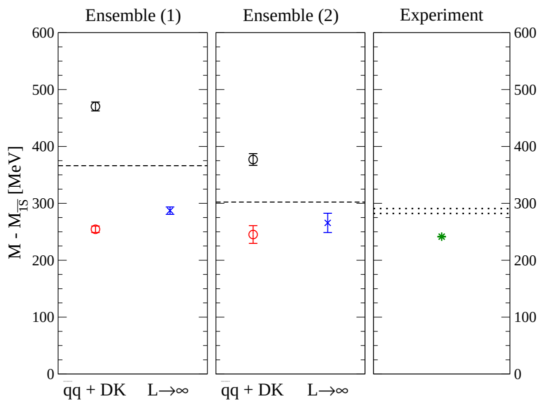

Figure 4: Energy levels on the lattice along with our estimate of the bound state position in infinite volume $M^{D^*_{s0}(2317)}_{L\to\infty}$ (left and middle panels). The dashed lines indicate the two lowest D-meson-kaon noninteracting scattering thresholds as obtained in the finite volume. In the right panel the situation in experiment is shown. The dotted lines represent the thresholds for D^0K^+ and K^0D^+.

In infinite volume a bound state corresponds to a pole of the S-matrix. This translates to the pole condition $\cot\delta(p_b) = i$ with the (imaginary) binding momentum p_b. Taking our extracted a_0 and r_0 we can determine this binding momentum and the associated energy level. The final result is given alongside our lattice energy levels and the physical mass of the $D^*_{s0}(2317)$ in Figure 4.

To summarize, we used both quark-antiquark and meson-meson interpolators to determine the low-lying energy levels in the channel of the $D^*_{s0}(2317)$. We observe energy levels associated with the D-meson-kaon scattering levels on the lattice as well as a bound state of unknown nature below threshold. Notably, this is the same situation than observed for the $D^*_{s0}(2317)$ meson in experiment and the observed energy of the infinite volume bound state is compatible with the $D^*_{s0}(2317)$.

Acknowledgments

We thank Anna Hasenfratz and the PACS-CS collaboration for providing gauge configurations and Martin Lüscher for making his DD-HMC software available. D. M. would like to thank E. Eichten, F.-K. Guo, M. Hansen, A. Kronfeld, Y. Liu and J. Simone for insightful discussions. The calculations were performed on computing clusters at TRIUMF, the University of Graz and IJS Ljubljana. This work is supported in part by the Austrian Science Fund FWF project I1313-N27, by the Slovenian Research Agency ARRS project number N1-0020 and by the Natural Sciences and Engineering Research Council of Canada. Fermilab is operated by Fermi Research Alliance, LLC under Contract No. De-AC02-07CH11359 with the United States Department of Energy.

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