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Lattice study of the leptonic decay constant of the pion and its excitations

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We present a calculation of the decay constant of the pion, and its lowest-lying three excitations, at three values of the pion mass between around 400 and 700 MeV, using anisotropic clover lattices. We use the variational method to determine an optimal interpolating operator for each of the states. We find that the decay constant of the first excitation, and more notably of the second, is suppressed with respect to that of the ground-state pion, but that the suppression shows little dependence on the quark mass.

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While obtaining the properties of the ground states of hadrons has long been a primary goal of lattice QCD, extracting information about excited hadrons poses numerous challenges. The main difficulty arises from the faster decay of their Euclidean correlation functions in comparison with those of the ground states, which leads to the worsening of the signal-to-noise ratio at increasing temporal separations. An additional complication arises from the cost of constructing the necessarily large correlation matrix needed to apply the variational method, and the implementation of an operator basis that respects the symmetries of the lattice, yet enables the continuum quantum numbers to be identified. Finally, in general we are dealing with resonances that are unstable under the strong interactions.

Despite all these obstacles, the latest developments in advanced computational lattice QCD techniques are enabling increasingly precise quantitative calculations that can both confront existing, and offer the prospect to predict the outcomes of forthcoming experiments. Such experiments are the aim of the 12 GeV upgrade of the Continuous Electron Beam Accelerator Facility (CEBAF) at Jefferson Lab [1] with, in particular, a meson spectroscopy program based on the photoproduction of meson excitations. The expectation is that new data produced in this upcoming experiment, combined with recent lattice QCD results aimed to extracting the spectrum of meson excited states [2, 3], will represent a unique opportunity for the study of the nature of confinement mechanism and for determining the role of the gluonic field in the hadron spectrum.

The work presented here is also devoted to the study of excited states, but its emphasis is on the computation of some of their properties. In general, our goal is to investigate quark distribution amplitudes which, in case of mesons, can be extracted from the vacuum-to-hadron matrix elements of quark bilinear operators. These amplitudes would provide predictions for form factors and transition form factors at high momentum transfers. In this talk, we describe calculations devoted to the study of the leptonic decay properties of the pion - the lightest system with simple quark-antiquark structure - and of its excitations.

1. Pseudoscalar leptonic decay constants

Charged mesons are allowed to decay, through quark-antiquark annihilations via a virtual W boson, to a pair of leptons. The decay width for any pseudo-scalar meson P of a quark content q_1 and $\bar{q}_2$ with mass m_P is given as [4]

$$\Gamma(P \to l\nu) = \frac{G_f^2}{8\pi} f_P^2 m_l^2 m_P \left(1 - \frac{m_l^2}{m_P^2}\right) |V_{q_1\bar{q}_2}|^2. \tag{1.1}$$

Here m_l is the mass of the lepton l, G_F is the Fermi coupling constant, $V_{q_1\bar{q}_2}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element between the constituent quarks in P, and f_P is the decay constant related to the wave-function overlap of the quark and antiquark at the origin. Thus, a charged pion can decay as $\pi \to l\nu$ (we assume here $\pi^+ \to l^+\nu_l$ or $\pi^- \to l^-\bar{\nu}_l$), and its decay constant f_π, which dictates the strength of these leptonic pion decays, has a significant influence on many areas of modern physics. Among others pseudo-scalar meson decay constants, f_π plays an important role for extracting CKM matrix elements. Following Eq. 1.1, the leptonic decay width Γ is proportional to $f_P|V_{q_1\bar{q}_2}|$, so theoretically predicted value of f_P can allow determination of the corresponding CKM element, which is crucial for testing the flavor sector of the Standard

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Model. As $|V_{ud}|$ has been quite accurately measured in super-allowed β-decays, measurements of $\Gamma(\pi^+ \to \mu^+\nu)$ yield a value of f_π. The most precise determination of f_π corresponds to [4]

$$f_{\pi^-} = (130.41 \pm 0.03 \pm 0.20)\ MeV. \tag{1.2}$$

Lattice QCD provides an *ab initio* means of computing the mass spectrum and decay constants of pseudo-scalar mesons nonperturbatively. Thus, for example, recent lattice predictions [5, 6, 7] for the ratio f_K/f_π of K^- and π^- decay constants were used in order to find a value for $|V_{us}|/|V_{ud}|$ which, together with precisely measured $|V_{ud}|$, provides an independent measure of $|V_{us}|$. The pion decay constant, determining the strength of $\pi\pi$ interactions, also serves as an expansion parameter in Chiral Perturbation Theory (ChiPT) [8, 9]. Therefore, reliable lattice QCD determinations of decay constants from first principles are of fundamental importance.

In this work, we focus on the leptonic decay constants of the excited states of the pion. Our motivation is the calculation of the moments of the quark distribution amplitudes in the pion and its excitations, for which the decay constants provide the overall normalization. However, the decay constants of the excitations can themselves provide insights into QCD-inspired descriptions of hadrons. The study based on Schwinger-Dyson equations [10] predicted significant suppression of the excited-state pion decay constant with comparison to that one of the ground state. Similar predictions, based on the QCD-inspired models and sum rules, also propose remarkably small values for f^1_π; e.g., [11] proposed the ratio $\frac{f^1_\pi}{f^0_\pi}$ to be of the order of one percent. There has also been an extraction of the decay constant of the first excitation of the pion using lattice QCD [12], in a calculation with two dynamical quark flavors; this calculation found an increasingly strong suppression of the decay constant of the first excited state relative to that of the ground state with decreasing quark mass. Here we use developments in excited state spectroscopy to compute not only the decay constant of the ground and first excited state, but of higher states also.

2. Computational Method

Our calculation of the excited-state spectrum of QCD is dependent on three novel elements: the use of an anisotropic lattice enabling the time-sliced correlation functions to be examined at small Euclidean times, the use of the variational method with a large basis of operators derived from a continuum construction yet which satisfy the symmetries of the lattice, and an efficient means of computing the necessary correlation functions through the use of "distillation".

The variational approach involves the solution of the generalized eigenvalue problem

$$C(t)v(t,t_0) = \lambda(t)C(t_0)v(t,t_0), \tag{2.1}$$

where $C(t)$ is an $N \times N$ matrix of correlators constructed from operators $\mathcal{O}_i, i = 0\dots N-1$ such that $C_{ij}(t) = \sum_{\vec{x}}\langle 0 \mid \mathcal{O}_i(\vec{x},t)\mathcal{O}^\dagger_j(0) \mid 0\rangle$. At sufficiently large $t > t_0$, the ordered eigenvalues satisfy

$$\lambda_n(t,t_0) \longrightarrow e^{-E_n(t-t_0)}$$

where E_n is the energy of the n^{th} state. The eigenvalues are normalized to unity at $t = t_0$, whilst the eigenvectors satisfy the orthogonality condition $v^{(n')\dagger}C(t_0)v^{(n)} = \delta_{n,n'}$, and thereby enable us to construct an *optimal* operator Ω_n that couples predominantly to the n^{th} state [13]:

$$\Omega_n = \sqrt{2E_n}v^{(n)}_i e^{-E_n t_0/2}\mathcal{O}_i.$$

The decay constant of the n^{th} excitation of the pion, Π_n, is given by the hadron-to-vacuum matrix matrix element of the axial vector current:

$$\langle 0 \mid A_\mu | \Pi_n >= p_\mu f_\pi^n.$$

For a state at rest, we employ the temporal component of the axial-vector current. Armed with the optimal interpolating operator for the n^{th} excited state, we now extract its decay constant f_π^n through the two-point correlation function

$$C^n(t) = \sum_{\vec{x}} \langle 0 \mid A_4(\vec{x},t) \Omega_n^\dagger(0) \mid 0 \rangle \longrightarrow e^{-m_n t} m_n f_\pi^n, \tag{2.2}$$

where now we have identified the energy with the mass of the n^{th} excited state m_n.

2.1 Distillation

Our efficient implementation of the variational method with a large number of interpolating operators relies on a novel smearing method denoted by "distillation" [14]. Our starting point is the solution of the Dirac equation

$$\tilde{\tau}^{(k)}_{\alpha\beta}(\vec{x},t;0) = M^{-1}_{\alpha\beta}(\vec{x},t;0)\xi^{(k)}, \tag{2.3}$$

where the $\xi^{(k)}, k = 1 \dots N_{\text{eigen}}$ are the lowest N_{eigen} eigenvectors of a three-dimensional Laplacian at timeslice $t' = 0$. The construction of the correlation functions from operators smeared both at the sink and the source has been described in detail [14], but the extension to the calculation of the smeared-local two-point functions needed here is straightforward:

$$\begin{aligned} C_{\mu,i}(t,0) &= \frac{1}{V_3} \sum_{\vec{x},\vec{y}} \langle 0 \mid A_\mu(\vec{x},t) \mathscr{O}_i^\dagger(\vec{y},0) \mid 0 \rangle \\ &= \frac{1}{V_3} \sum_{\vec{x}} \mathrm{Tr}[\gamma_\mu \tilde{\tau}(\vec{x},t;0) \Phi_i(0) \gamma_5 \tilde{\tau}(\vec{x},t;0)^\dagger], \end{aligned} \tag{2.4}$$

where the trace is over spin, colour and eigenvector indices, Φ is the representation of the operator $\mathscr{O}_i$ in terms of the eigenvectors, and the spatial-volume V_3 reflects the time-sliced sum at the source that is a strength of the distillation method. The correlator onto the optimal operator for the n^{th} excited state immediately follows from Eqn. 2.2.

3. Calculational Details

We employ the dynamical $N_f = 2 \oplus 1$ anisotropic lattices generated by the Hadron Spectrum collaboration, using an anisotropic "clover" action with stout smearing in the spatial direction only to preserve the transfer matrix; the spatial lattice spacing $a_s \simeq 0.12$ fm, and the renormalized anisotropy $\xi \equiv a_s/a_t \simeq 3.5$. Details of the action, the generation of the lattices, and the setting of the physical scale and quark masses, are contained in ref. [15]. The parameters of the lattices used here are shown in Table 1. Our operator basis is constructed from the product of quark bilinears, and of discretisation of the covariant derivative to describe the spatial structure, namely

$$\bar{\psi} \Gamma D_{i_0} D_{i_1} \dots \psi,$$

which are then projected onto the A_1^{-+} *irrep* of the cubic group. We employ up to three derivatives, yielding a basis of 12 operators.

N_s	N_t	$a_t m_l$	$a_t m_s$	$a_t m_\pi$	r_0/a_s	N_{cfg}	N_{eigen}
16	128	-0.0743	-0.0743	0.1483(2)	3.21(1)	535	64
16	128	-0.0808	-0.0743	0.0996(6)	3.51(1)	470	64
16	128	-0.0840	-0.0743	0.0691(6)	3.65(1)	480	64

Table 1: Lattice extents (N_s and N_t), the masses of light quark $a_t m_l$ and strange quark $a_t m_s$, the pion mass $a_t m_\pi$, the Sommer scale r_0, and the number N_{cfg} of gauge-field configurations. The final column shows the number N_{eigen} of eigenvectors employed in the implementation of "distillation" [14].

m_π (MeV)	$a_t f_\pi^0$	$a_t f_\pi^1$	$a_t f_\pi^2$	$a_t f_\pi^3$
702	0.0551(19)	0.0319(67)	0.0023(306)	0.0320(415)
524	0.0440(49)	0.0254(57)	0.0080(338)	0.0311(172)
391	0.0368(34)	0.0207(63)	0.0062(115)	0.0240(234)

Table 2: Unrenormalized values of ground and first three excited-state decay constants $|f_\pi^N|$ (in lattice units) on each of our ensembles.

4. Results

Our results for the pseudoscalar decay constants using the unimproved local axial vector current on each of our ensembles are presented in Table 2. In order to remove dependence both on the lattice spacing, and on the renormalization constant, we show in Figure 1 the ratio of the decay constants of the first and ground states as a function of the pion mass. Whilst we see that the first excited state decay constant is indeed suppressed relative to that of the ground state, that suppression is quite small and largely independent of the quark mass over the mass range of this calculation. This is in contrast to ref. [12], which shows an increasing suppression with pion mass over the somewhat similar range of pion masses used in this calculation, and an even stronger suppression if an improved axial-vector current is employed; they obtain $|f_\pi^1/f_\pi^0| = 0.38(11)$ and $0.078(93)$ for the unimproved and improved currents respectively in the chiral limit.

Our ability to isolate the many energy levels in the spectrum enables us to determine the decay constants not only of the first two states, but of the second and third excited states. These are shown for each of our ensembles in Figure 2. If the decay constant of the first excited state is only mildly suppressed, we see that of the second excited state is strongly so, though once again that suppression does not exhibit a strong dependence on the quark mass for the unimproved operator over the range of pion masses studied.

5. Discussion and Conclusions

In this talk, we have undertaken the first steps in investigating the properties of the excited meson states in QCD by computing the decay constants both of the pion, and its lowest three excitations. Our results show that the optimal operators obtained through the variational method remain faithful interpolating operators for the lowest lying excitations in the spectrum when computing hadron-to-vacuum matrix elements of local operators. We find that both the first- and, more notably, the second-excited-state decay constants are suppressed relative to those of the ground state, but that this suppression is largely independent of the pion mass. This observation differs from that

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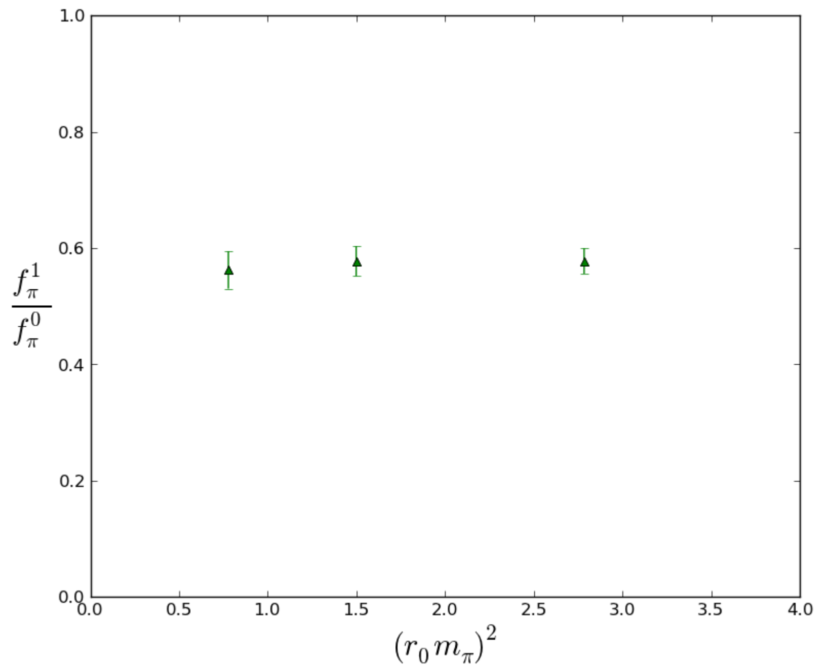

Figure 1: The dimensional ratio of the leptonic decay constants f_π^1 and f_π^0 as a function of m_π^2 in units of the Sommer scale r_0.

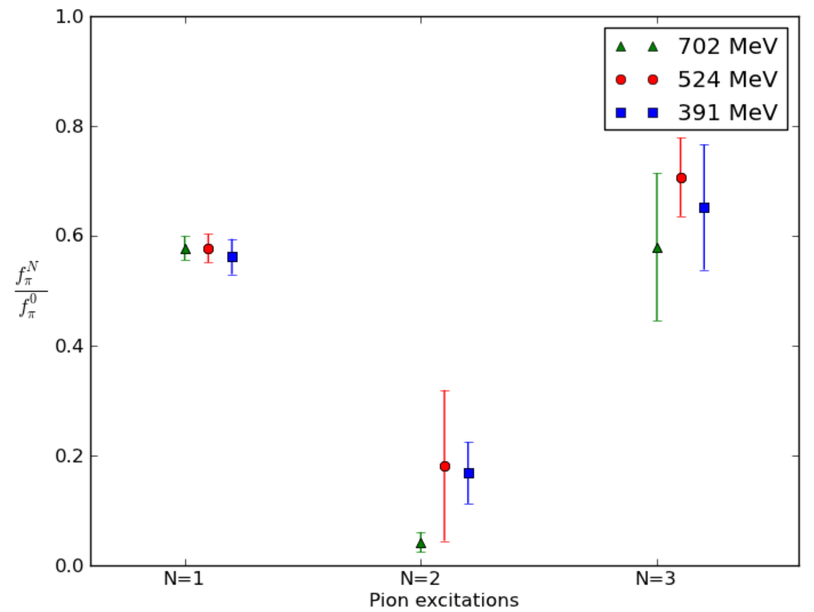

Figure 2: Ratios of the excited-state decay constants f_π^N to the ground-state decay constant f_π^0 for the first 3 pion excitations ($N = 1, 2, 3$).

of ref. [12]; our present calculation does not include operator mixing that they find largely responsible for the quark mass dependence of the excited-state decay constants, but note that the anisotropic lattice used in this calculation has fine spacing in the temporal direction. Furthermore, our basis of interpolating operators includes only "single-hadron" operators, whose coupling to multihadron decay states is expected to be suppressed by the volume; we note that the second excited state is at or above the energy level of the lowest-lying non-interacting two-meson state on each of our lattices [2]. Future work will therefore include the inclusion of the operator improvement term and calculations at larger volumes to expose any contributions from multihadron states. Nonetheless, this work clearly demonstrates that the properties of excited state hadrons are accessible to lattice calculation.

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