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The chromomagnetic operator on the lattice

M. Constantinou[a], **M. Costa**[a], **R. Frezzotti**[b], **V. Lubicz**[c], **G. Martinelli**[d], **D. Meloni**[c], **H. Panagopoulos**[*a], **S. Simula**[c]

[a] *Department of Physics, University of Cyprus, CY-1678 Nicosia, Cyprus*
[b] *Dipartimento di Fisica, Università di Roma "Tor Vergata" and INFN Sezione "Tor Vergata", I-00133 Rome, Italy*
[c] *Dipartimento di Fisica, Università Roma Tre, and INFN, Sezione di Roma Tre, I-00146 Rome, Italy*
[d] *SISSA, I-34136 Trieste, Italy*

E-mail: constantinou.martha@ucy.ac.cy, kosta.marios@ucy.ac.cy, roberto.frezzotti@roma2.infn.it, lubicz@fis.uniroma3.it, guido.martinelli@sissa.it, meloni@fis.uniroma3.it, haris@ucy.ac.cy, simula@roma3.infn.it

We study matrix elements of the "chromomagnetic" operator on the lattice. This operator is contained in the strangeness-changing effective Hamiltonian which describes electroweak effects in the Standard Model and beyond.

Having dimension 5, the chromomagnetic operator is characterized by a rich pattern of mixing with other operators of equal and lower dimensionality, including also non gauge invariant quantities; it is thus quite a challenge to extract from lattice simulations a clear signal for the hadronic matrix elements of this operator.

We compute all relevant mixing coefficients to one loop in lattice perturbation theory; this necessitates calculating both 2-point (quark-antiquark) and 3-point (gluon-quark-antiquark) Green's functions at nonzero quark masses. We use the twisted mass lattice formulation, with Symanzik improved gluon action.

For a comprehensive presentation of our results, along with detailed explanations and a more complete list of references, we refer to our forthcoming publication [1].

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1. Introduction

The electroweak effective Hamiltonian describing stangeness changing ($\Delta S = 1$) processes, in the Standard Model (SM) and beyond, contains four "magnetic" operators of dimension 5:

$$H_{\rm eff}^{\Delta S=1,\ d=5} = \sum_{i=\pm} (C_\gamma^i Q_\gamma^i + C_g^i Q_g^i) + \text{h.c.} \tag{1.1}$$

$$Q_\gamma^\pm = \frac{Q_d\, e}{16\pi^2} \left(\bar{s}_L\, \sigma^{\mu\nu}\, F_{\mu\nu}\, d_R \pm \bar{s}_R\, \sigma^{\mu\nu}\, F_{\mu\nu}\, d_L\right), \quad Q_g^\pm = \frac{g}{16\pi^2} \left(\bar{s}_L\, \sigma^{\mu\nu}\, G_{\mu\nu}\, d_R \pm \bar{s}_R\, \sigma^{\mu\nu}\, G_{\mu\nu}\, d_L\right)$$

The coefficients C_γ^i and C_g^i, multiplying the electromagnetic (EMO) and chromomagnetic (CMO) operators, respectively, may be calculated perturbatively via the OPE; they are suppressed within the SM, but become more pronounced beyond the SM, e.g. through penguin diagrams in SUSY.

The matrix elements of the CMO are parameterized as [2]:

$$\langle \pi^0 | Q_g^+ | K^0 \rangle = \frac{-11}{32\sqrt{2}\pi^2} \frac{M_K^2 (p_\pi \cdot p_K)}{m_s + m_d} B_{g1} \tag{1.2}$$

$$\langle \pi^+\pi^- | Q_g^- | K^0 \rangle = \frac{11\,\mathrm{i}}{32\pi^2} \frac{M_K^2 M_\pi^2}{f_\pi\,(m_s + m_d)} B_{g2} \tag{1.3}$$

$$\langle \pi^+\pi^+\pi^- | Q_g^+ | K^+ \rangle = \frac{-11}{16\pi^2} \frac{M_K^2 M_\pi^2}{f_\pi^2\,(m_s + m_d)} B_{g3} \tag{1.4}$$

These matrix elements are relevant for the study of $K^0 - \bar{K}^0$ mixing, ε'/ε, the $\Delta I = 1/2$ rule, and $K \to 3\pi$ decays. To leading order in χPT, the B-parameters are all related [3]:

$$Q_g^\pm = \frac{11}{256\pi^2} \frac{f_\pi^2 M_K^2}{m_s + m_d} B_g \left[U(D_\mu U^\dagger)(D^\mu U) \pm (D_\mu U^\dagger)(D^\mu U) U^\dagger\right]_{23} \tag{1.5}$$

Thus, a lattice study of, say, Eq. (1.2), provides information for Eqs. (1.3), (1.4) as well.

The EMO has been studied in simulations with $N_f = 0$ [4] and $N_f = 2$ [5] dynamical flavors, focusing on:

$$\langle \pi^0 | Q_\gamma^+ | K^0 \rangle = \mathrm{i}\, \frac{Q_d\, e\sqrt{2}}{16\pi^2 M_K}\, p_\pi^\mu\, p_K^\nu\, F_{\mu\nu}\, B_T\, R_T(q^2) \qquad [R_T(0) = 1] \tag{1.6}$$

The parameter B_T appears, e.g., in the branching ratio of $K_L \to \pi^0 e^+ e^-$ in SUSY models.

2. Operator Mixing – Lattice Action – Symmetries

A formidable issue in the study of the CMO is the fact that it mixes with a large number of other operators under renormalization. Even in dimensional regularization (DR), which has the simplest mixing pattern, the CMO ($\mathcal{O}_{CM} \equiv \mathcal{O}_1$) mixes with a total of 9 other operators ($\mathcal{O}_2 - \mathcal{O}_{10}$), forming a basis of dimension-five, Lorentz scalar operators with the same flavor content as the CMO. Among them, there are also gauge noninvariant operators ($\mathcal{O}_9\,, \mathcal{O}_{10}$); these are BRST invariant and vanish by the equations of motion, as required by renormalization theory.

$$\begin{aligned}
\mathcal{O}_1 &= g\,\overline{\psi}_s \sigma_{\mu\nu} G_{\mu\nu} \psi_d & \mathcal{O}_6 &= \overline{\psi}_s (\overrightarrow{\not{D}} + m_d)^2 \psi_d + \overline{\psi}_s (-\overleftarrow{\not{D}} + m_s)^2 \psi_d \\
\mathcal{O}_2 &= (m_d^2 + m_s^2)\overline{\psi}_s \psi_d & \mathcal{O}_7 &= m_s \overline{\psi}_s (\overrightarrow{\not{D}} + m_d)\psi_d + m_d \overline{\psi}_s (-\overleftarrow{\not{D}} + m_s)\psi_d \\
\mathcal{O}_3 &= m_d m_s \overline{\psi}_s \psi_d & \mathcal{O}_8 &= m_d \overline{\psi}_s (\overrightarrow{\not{D}} + m_d)\psi_d + m_s \overline{\psi}_s (-\overleftarrow{\not{D}} + m_s)\psi_d \\
\mathcal{O}_4 &= \overline{\psi}_s \overleftarrow{D}_\mu \overrightarrow{D}_\mu \psi_d & \mathcal{O}_9 &= \overline{\psi}_s \overleftarrow{\partial} (\overrightarrow{\not{D}} + m_d)\psi_d - \overline{\psi}_s (-\overleftarrow{\not{D}} + m_s) \overrightarrow{\partial} \psi_d \\
\mathcal{O}_5 &= \overline{\psi}_s (-\overleftarrow{\not{D}} + m_s)(\overrightarrow{\not{D}} + m_d)\psi_d & \mathcal{O}_{10} &= \overline{\psi}_s \overrightarrow{\partial} (\overrightarrow{\not{D}} + m_d)\psi_d - \overline{\psi}_s (-\overleftarrow{\not{D}} + m_s) \overleftarrow{\partial} \psi_d
\end{aligned} \tag{2.1}$$

On the lattice, the mixing pattern can become considerably more complicated, given that certain symmetries are violated; there can be mixing with additional operators of dimension five (with finite coefficients) or less (with power-divergent coefficients). A generic hypercubic- and gauge-invariant lattice discretization will result in mixing with 2+8+32 candidate operators of dimension 3, 4, 5, respectively. It is thus imperative to make a judicious choice of lattice action, with a large set of discrete symmetries, so as to exclude as many as possible of these candidates.

We have adopted the twisted mass action for valence quarks and the Osterwalder - Seiler action for sea quarks [6] (along with a compensating ghost action for valence quarks). For our one-loop perturbative calculation we only need the valence quark action, which reads (in the physical basis):

$$S_F[\psi_f,\bar{\psi}_f,U] = a^4 \sum_f \sum_x \bar{\psi}_f(x)\left[\gamma\cdot\widetilde{\nabla} - i\gamma_5 W_{\rm cr}(r_f) + m_f\right]\psi_f(x) \tag{2.2}$$

$$\gamma\cdot\widetilde{\nabla} \equiv \frac{1}{2}\sum_\mu \gamma_\mu(\nabla^\star_\mu + \nabla_\mu) \qquad W_{\rm cr}(r_f) \equiv -a\frac{r_f}{2}\sum_\mu \nabla^\star_\mu\nabla_\mu + M_{\rm cr}(r_f)$$

(r_f: Wilson parameter for flavour $f = u, d, s$; $M_{\rm cr}(r_f) = -M_{\rm cr}(-r_f)$: corresponding critical mass).

For gluons we have used the Symanzik improved action; for our perturbative results we employed several standard choices of values for the Symanzik coefficients appearing in that action [1].

A number of discrete symmetries [6] are present in our action; the CMO is invariant – up to a possible minus sign – under them, and the same must then hold for all other operators which mix with the CMO. As a result, alongside the 10 operators which mix in DR, only 3 additional ones appear on the lattice, and they all have dimension less than five:

$$\mathcal{O}_{11} = i r_d \overline{\psi}_s\gamma_5(\overrightarrow{\not{D}} + m_d)\psi_d + i r_s \overline{\psi}_s(-\overleftarrow{\not{D}} + m_s)\gamma_5\psi_d\,,\ \mathcal{O}_{12} = i(r_d m_d + r_s m_s)\overline{\psi}_s\gamma_5\psi_d\,,\ \mathcal{O}_{13} = \overline{\psi}_s\psi_d \tag{2.3}$$

3. Renormalization Matrix

Renormalized operators $\mathcal{O}_i^R$ are related to bare ones $\mathcal{O}_i$ via a 13×13 renormalization matrix Z:

$$\mathcal{O}_i = \sum_{j=1}^{13} Z_{ij}\mathcal{O}_j^R \qquad (\mathcal{O} = Z\mathcal{O}^R,\ \ \mathcal{O}^R = Z^{-1}\mathcal{O}) \tag{3.1}$$

The matrix elements Z_{ij} depend both on the regularization X ($X = L$ (lattice), DR (dimensional), etc.) and on the renormalization scheme Y ($Y = \overline{\rm MS}$, RI', etc.); where confusion might arise, one should denote them as $Z_{ij}^{X,Y}$. At tree level: $Z = \mathbb{1}$. For $\mathcal{O}_1^R$, we only need the first row of Z^{-1} (and thus, to one loop, only the first row or Z: Z_{1i}). Clearly: $Z_{1,1} = 1 + \mathcal{O}(g^2)$, $Z_{1i} = \mathcal{O}(g^2)$ $(i > 1)$.

To obtain Z_{1i}, we have calculated, to one loop and in an arbitrary covariant gauge, the 2-point (quark-antiquark) and 3-point (quark-antiquark-gluon) bare amputated Green's functions of $\mathcal{O}_1$; these are related to the corresponding renormalized Green's functions through:

$$\begin{aligned} &\langle\psi^R\mathcal{O}_1^R\overline{\psi}^R\rangle_{\rm amp} = Z_\psi\sum_{i=1}^{13}(Z^{-1})_{1i}\langle\psi\mathcal{O}_i\overline{\psi}\rangle_{\rm amp}\,, && \psi = Z_\psi^{1/2}\psi^R \\ &\langle\psi^R\mathcal{O}_1^R\overline{\psi}^R A_\nu^R\rangle_{\rm amp} = Z_\psi Z_A^{1/2}\sum_{i=1}^{13}(Z^{-1})_{1i}\langle\psi\mathcal{O}_i\overline{\psi}A_\nu\rangle_{\rm amp}\,, && A_\nu = Z_A^{1/2}A_\nu^R \end{aligned} \tag{3.2}$$

The renormalization functions Z_ψ, Z_A (as well as those for the coupling constant (Z_g), the fermion mass (Z_m), and the ghost field (Z_c)) were not all available for the actions considered in

this work, and had to be calculated as a prerequisite. We mention in passing that Z_ψ and Z_m do not depend on flavor in mass-independent schemes. We also note that both the 2-point and 3-point functions are necessary in order to fix all Z_{1i}, but they are also sufficient.

[An alternative definition of the CMO: $\widetilde{\mathcal{O}}_{CM} \equiv m\,\mathcal{O}_{CM}$ appears in the study of 4-fermi operators. In this case, the renormalization matrix reads: $\tilde{Z}_{ij} = Z_m Z_{ij}$ ($m^R \equiv Z_m^{-1}\, m$). Similarly, a factor of Z_g must be included in Z_{ij}, if Green's functions are computed using: $(1/g)\mathcal{O}_{CM}$, rather: $\mathcal{O}_{CM}$.]

The one-particle irreducible (1PI) Feynman diagrams contributing to the 2-point and 3-point Green's functions of $\mathcal{O}_1$ are shown in the left and right panels of Figure 1, respectively.

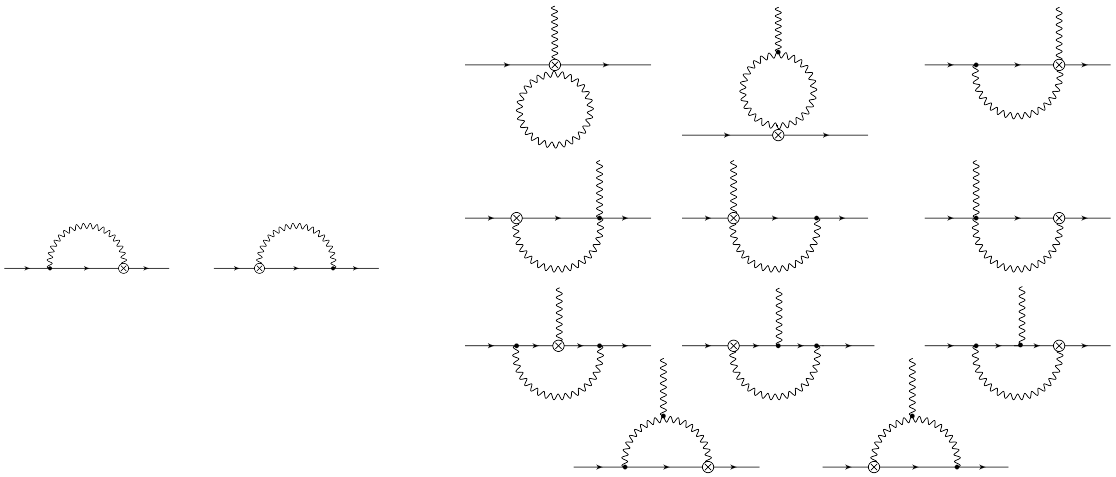

Figure 1: 1PI one-loop diagrams contributing to the 2-point and 3-point Green's functions. A wavy (solid) line represents gluons (quarks); an operator insertion is denoted by $\otimes$.

4. Results

We calculated the 2- and 3-point bare Green's functions of Eq. (3.2), first in DR and then in the far more complicated case of the lattice. The purpose of the calculation in DR is twofold: First, it provides the mixing coefficients $Z_{1i}^{DR,\overline{\rm MS}}$, which are interesting on their own right; second, and most important, it leads to the renormalized Green's functions in $\overline{\rm MS}$, which are then necessary for extracting the real quantities of interest: $Z_{1i}^{L,\overline{\rm MS}}$.

4.1 Dimensional Regularization and $\overline{\rm MS}$ Renormalization

In $D = 4 - 2\varepsilon$ dimensions, renormalizability requires that the $\mathcal{O}(1/\varepsilon)$, 1PI part in the bare Green's functions of Eq. (3.2) has polynomial dependence on m_s, m_d, q_s, q_d, q_A ($q_s/q_d/q_A$: momenta of the external antiquark/quark/gluon). In fact, there appear in total 7+4 types of such dependence, as follows:

$$\begin{aligned}
\langle \psi\, \mathcal{O}_1\, \overline{\psi} \rangle^{1-loop}_{\rm amp}\Big|_{1/\varepsilon} &= \rho_1\,(q_s^2+q_d^2) + \rho_2\,(m_s^2+m_d^2) + \rho_3\, i\,(m_d \not{q}_d + m_s \not{q}_s) \\
&\quad + \rho_4\, i\,(m_s \not{q}_d + m_d \not{q}_s) + \rho_5\, q_s\cdot q_d + \rho_6\, \not{q}_s \not{q}_d + \rho_7\, m_s m_d \\
\langle \psi\, \mathcal{O}_1\, \overline{\psi} A_\nu \rangle^{1-loop}_{\rm amp}\Big|_{1/\varepsilon} &= R_1\, g\,(q_s+q_d)_\nu + R_2\, g\,(\gamma_\nu \not{q}_d + \not{q}_s \gamma_\nu) \\
&\quad + R_3\, i\, g\,(m_s+m_d)\gamma_\nu + R_4\,(-2i\, g\, \sigma_{\rho\nu} q_{A\rho})
\end{aligned} \tag{4.1}$$

There exist also 1-particle reducible diagrams contributing to the 3-point function, both at tree level and at one loop; these contain non-polynomial $\mathcal{O}(1/\varepsilon)$ terms, which however cancel by virtue of the 2-point relation. Computing the coefficients $\rho_1-\rho_7$, R_1-R_4 we find:

$$\rho_1=\frac{g^2C_F}{16\pi^2}\frac{1}{\varepsilon}(-3)\quad \rho_2=\frac{g^2C_F}{16\pi^2}\frac{1}{\varepsilon}(-6)\quad \rho_3=\frac{g^2C_F}{16\pi^2}\frac{1}{\varepsilon}(3)\quad \rho_4=\rho_5=\rho_6=\rho_7=0 \tag{4.2}$$

$$\{R_1\,,R_2\,,R_3\,,R_4\}=\frac{g^2}{16\pi^2}\frac{1}{\varepsilon}\left\{-6C_F,\frac{3N_c}{4},(\frac{-3}{2N_c}+\frac{3N_c}{4}),(\frac{1}{N_c}-\frac{\alpha}{2N_c}+\frac{7N_c}{4}+\frac{3\alpha N_c}{4})\right\} \tag{4.3}$$

(N_c : number of colors, $C_F=(N_c^2-1)/(2N_c)$, α : gauge parameter).

Demanding that all $\mathcal{O}(1/\varepsilon)$ dependence on the right-hand sides of Eqs. (3.2) disappears (as it ought to, since the corresponding renormalized Green's functions on the left-hand sides must be finite) provides 7+4 constraint equations on the 10 coefficients Z_{1i}. This set of equations is self-consistent and complete; solving them, we obtain:

$$\begin{aligned}
&Z_{1,1}^{DR,\overline{\rm MS}}=1+\frac{g^2}{16\pi^2}\frac{1}{\varepsilon}(-\frac{N_c}{2}+\frac{5}{2N_c}) && Z_{1,2}^{DR,\overline{\rm MS}}=-2Z_{1,10}^{DR,\overline{\rm MS}}=\frac{g^2}{16\pi^2}\frac{1}{\varepsilon}(-3N_c+\frac{3}{N_c})\\
&Z_{1,5}^{DR,\overline{\rm MS}}=\frac{g^2}{16\pi^2}\frac{1}{\varepsilon}(\frac{2N_c}{3}-\frac{3}{N_c}) && Z_{1,7}^{DR,\overline{\rm MS}}=-Z_{1,9}^{DR,\overline{\rm MS}}=\frac{g^2}{16\pi^2}\frac{1}{\varepsilon}(-\frac{3N_c}{4}+\frac{3}{2N_c})\\
&Z_{1,3}^{DR,\overline{\rm MS}}=Z_{1,4}^{DR,\overline{\rm MS}}=Z_{1,6}^{DR,\overline{\rm MS}}=Z_{1,8}^{DR,\overline{\rm MS}}=0
\end{aligned} \tag{4.4}$$

An immediate, well-known by-product of $Z_{1,1}^{DR,\overline{\rm MS}}$ is the anomalous dimension $\widetilde{\gamma}_{CM}$ for the operator $\widetilde{\mathcal{O}}_{CM}$: $\widetilde{\gamma}_{CM}=g^2/(16\pi^2)\cdot(4N_c-8/N_c)$. We note that the mixing coefficients for the gauge noninvariant operators $\mathcal{O}_9$, $\mathcal{O}_{10}$ do not vanish.

The $\mathcal{O}(\varepsilon^0)$ parts of the right-hand side of Eqs. (3.2) are the $\overline{\rm MS}$-renormalized Green's functions; while they were not necessary for $Z_{1i}^{DR,\overline{\rm MS}}$, we do need them for $Z_{1i}^{L,\overline{\rm MS}}$ below.

4.2 Lattice Regularization and $\overline{\rm MS}$ Renormalization

The relations which we must turn to now are formally the same as the ones we studied in the previous subsection (Eqs. (3.2)); however all renormalization functions Z now stand for $Z^{L,\overline{\rm MS}}$, and the bare Green's functions on the right-hand sides must be calculated using the lattice regularization. The $\overline{\rm MS}$-renormalized Green's functions on the left-hand sides coincide with those which we calculated in the previous subsection, since they must be regularization-independent.

Renormalizability implies that, modulo terms which vanish as $a\to 0$, $\langle\psi^R\,\mathcal{O}_1^R\,\overline{\psi}^R\rangle_{\rm amp}-\langle\psi\,\mathcal{O}_1\,\overline{\psi}\rangle_{\rm amp}$ is polynomial in m's, q's (of $2^{\rm nd}$ degree, but also $a^{-1}\cdot(1^{\rm st})$, $a^{-2}\cdot(0^{\rm th})$):

$$\begin{aligned}
\langle\psi^R\,\mathcal{O}_1^R\,\overline{\psi}^R\rangle_{\rm amp}-\langle\psi\,\mathcal{O}_1\,\overline{\psi}\rangle_{\rm amp}=&\,\rho_1\,(q_s^2+q_d^2)+\rho_2\,(m_s^2+m_d^2)+\rho_3\,i\,(m_d q\!\!\!/_d+m_s q\!\!\!/_s)+\rho_4\,i\,(m_s q\!\!\!/_d+m_d q\!\!\!/_s)\\
&+\rho_5\,q_s\cdot q_d+\rho_6\,q\!\!\!/_s\,q\!\!\!/_d+\rho_7\,m_s m_d+\rho_8\,(r_d\,\gamma_5\,q\!\!\!/_d+r_s\,q\!\!\!/_s\,\gamma_5)+\rho_9\,i\,(r_d m_d+r_s m_s)\,\gamma_5+\rho_{10}\cdot 1
\end{aligned} \tag{4.5}$$

Similarly, $\langle\psi^R\,\mathcal{O}_1^R\,\overline{\psi}^R A_\nu^R\rangle_{\rm amp}-\langle\psi\,\mathcal{O}_1\,\overline{\psi}\,A_\nu\rangle_{\rm amp}$ must be polynomial ($1^{\rm st}$ degree, but also $a^{-1}\cdot(0^{\rm th})$):

$$\begin{aligned}
\langle\psi^R\,\mathcal{O}_1^R\,\overline{\psi}^R A_\nu^R\rangle_{\rm amp}-\langle\psi\,\mathcal{O}_1\,\overline{\psi}\,A_\nu\rangle_{\rm amp}=&\,R_1\,g\,(q_s+q_d)_\nu+R_2\,g\,(\gamma_\nu q\!\!\!/_d+q\!\!\!/_s\gamma_\nu)+R_3\,i\,g\,(m_s+m_d)\gamma_\nu\\
&+R_4\,(-2i\,g\,\sigma_{\rho\nu}q_{A\rho})+\mathbf{R_5}\,g\,(r_d-r_s)\,\gamma_5\,\gamma_\nu
\end{aligned} \tag{4.6}$$

The 10+5 coefficients ρ_i, R_i depend on a as: a^{-2}, a^{-1}, $\log(a\,\bar{\mu})$ (the scale $\bar{\mu}$ appears through the $\overline{\rm MS}$-renormalized Green's functions); they also depend on: N_c , α, and the Symanzik coefficients.

Thus, enforcing Eqs. (3.2) leads to 10+5 constraints for the 13 functions Z_{1i} , in such a way as to absorb the above polynomial differences. These constraints are self-consistent and complete.

It is a highly nontrivial task to show that the left-hand sides of Eqs. (4.5, 4.6) are polynomial. This is especially true for Eq. (4.6): The $\overline{\rm MS}$-renormalized 3-point function has already an extremely complicated dependence on momenta and masses (involving Spence functions even for $m = 0$), while the lattice bare 3-point function contains $\sim 10^5$ loop integrals depending on masses and external momenta. One relatively easy way to show the above property is to make a special nondegenerate choice for the external momenta, e.g. the "democratic" one: $q_s{-}q_d{+}q_A{=}0$, $q_s^2{=}q_d^2{=}q_A^2{=}\bar{\mu}^2$. In order to subject our results to as stringent a test as possible, we showed polynomiality without making any simplifying assumptions on the values of the external momentum 4-vectors, not even momentum conservation. An independent (far simpler) check on the lattice Green's functions is that the coefficients of $\log(a)$ must match those of $-1/(2\varepsilon)$ in DR.

Solving the constraint equations we find, in the case of the tree-level Symanzik gluon action:

$$Z_{1,1}^{L,\overline{\rm MS}} = 1+\frac{g^2}{16\pi^2}\left(N_c(-12.8455+\frac{1}{2}\log(a^2\bar{\mu}^2))+\frac{1}{N_c}(9.3779-\frac{5}{2}\log(a^2\bar{\mu}^2))\right), \tag{4.7}$$

$$Z_{1,2}^{L,\overline{\rm MS}} = \frac{g^2 C_F}{16\pi^2}(2.7677+6\log(a^2\bar{\mu}^2)),\;\; Z_{1,3}^{L,\overline{\rm MS}}=0,\;\; Z_{1,4}^{L,\overline{\rm MS}}=0,$$

$$Z_{1,5}^{L,\overline{\rm MS}} = \frac{g^2}{16\pi^2}\left(N_c(5.3894-\frac{3}{2}\log(a^2\bar{\mu}^2))+\frac{1}{N_c}(-5.5061+3\log(a^2\bar{\mu}^2))\right),\;\; Z_{1,6}^{L,\overline{\rm MS}}=0,$$

$$Z_{1,7}^{L,\overline{\rm MS}} = -Z_{1,9}^{L,\overline{\rm MS}} = -\frac{Z_5^{L,\overline{\rm MS}}}{2},\;\; Z_{1,8}^{L,\overline{\rm MS}} = \frac{g^2 C_F}{16\pi^2}(-3.9654),\;\; Z_{1,10}^{L,\overline{\rm MS}} = \frac{g^2 C_F}{16\pi^2}(5.5061-3\log(a^2\bar{\mu}^2)),$$

$$Z_{1,11}^{L,\overline{\rm MS}} = \frac{1}{a}\frac{g^2 C_F}{16\pi^2}(-4.0309) = -Z_{1,12}^{L,\overline{\rm MS}},\;\; Z_{1,13}^{L,\overline{\rm MS}} = \frac{1}{a^2}\frac{g^2 C_F}{16\pi^2}(47.7929)$$

Systematic errors coming from numerical loop integration are much smaller than the precision presented in the above results. Also, certain mixing coefficients vanish at one loop, but not beyond.

4.3 Non-perturbative results – Preliminary

In the calculation of on-shell matrix elements, by virtue of the equations of motion, some of the operators $\mathscr{O}_1$ – $\mathscr{O}_{13}$ will not appear. The remaining ones: $\mathscr{O}_1$, $\mathscr{O}_2$, $\mathscr{O}_3$, $\mathscr{O}_4$, $\mathscr{O}_{12}$, $\mathscr{O}_{13}$ will be present, and it is imperative to have a stringent estimate of the corresponding mixing coefficients. For operators of the same dimensionality as the chromomagnetic one, i.e. $\mathscr{O}_1$, $\mathscr{O}_2$, $\mathscr{O}_3$, $\mathscr{O}_4$, our one-loop results are expected to provide satisfactory accuracy; however, for operators of lower dimensionality ($\mathscr{O}_{12}$, $\mathscr{O}_{13}$), given that their coefficients are power divergent, perturbation theory is expected to provide only a ballpark estimate at best. Fortunately, it is precisely for the coefficients of these latter operators that we can have best access to non-perturbative estimates.

Imposing conditions such as:

$$\lim_{m_s,\,m_d\to 0}\langle\pi(0)|\mathscr{O}_1^{\rm sub}|K(0)\rangle = \lim_{m_s,\,m_d\to 0}\langle\pi(0)|\mathscr{O}_1+\frac{c_{13}}{a^2}\mathscr{O}_{13}|K(0)\rangle = 0 \tag{4.8}$$

$$\langle 0|\mathscr{O}_1^{\rm sub}|K(0)\rangle_{m_s,\,m_d} = \langle 0|\mathscr{O}_1+\frac{c_{13}}{a^2}\mathscr{O}_{13}+\frac{c_{12}}{a}\mathscr{O}_{12}|K(0)\rangle_{m_s,\,m_d} = 0 \tag{4.9}$$

we can fit the values of $c_{13}(g_0)$, $c_{12}(g_0)$ to data from simulations with varying quark masses.

In a preliminary series of simulations [1], we have extracted c_{13} at different values of the coupling ($\beta \equiv 6/g_0^2 = 1.90,\ 1.95,\ 2.10$). The results for c_{13} closely follow a quadratic dependence on g_0 , thus resembling a one-loop effect; nevertheless there is a notable difference, as was expected:

$$Z_{1,13}^{\text{non-pert}} \sim a^{-2}\,\frac{g^2 C_F}{16\pi^2}\,(33.7) \qquad\qquad Z_{1,13}^{\text{pert}} = a^{-2}\,\frac{g^2 C_F}{16\pi^2}\,(47.793) \tag{4.10}$$

[For a discussion on the possible choices for the coupling constant, see Ref. [1].]

5. Checks – Extensions

Besides a series of controls which we have applied to our results, some further ones may be applied: (i) A calculation of 4-point Green's functions will provide important consistency checks, but no new information, on Z_{1i} . On the other hand, 5-point functions and beyond are irrelevant: Being superficially convergent, they have a straightforward continuum limit. (ii) Non-perturbative estimates of all mixing coefficients would be very important cross checks.

Depending on the method one wishes to employ for computing matrix elements of the CMO non-perturbatively, a renormalization scheme other than $\overline{\text{MS}}$ may be more appropriate. In particular, one may employ an extension of the RI' scheme, in which RI'-like conditions need to be imposed on both 2-point and 3-point functions. The new mixing coefficients $Z_{ij}^{L,RI'}$ are related to $Z_{ij}^{L,\overline{\text{MS}}}$ via a (13×13) regularization-independent conversion matrix, whose elements are finite functions of the renormalized coupling. In fact, all relevant matrix elements are directly obtainable from our results on the renormalized Green's functions, with no further calculation required.

A further extension of the present work would be to apply methods of improved perturbation theory ("boosted" coupling, "cactus" diagrams, etc.) to our results. Another direction is to compute $\mathcal{O}(a^2 g^2)$ corrections to Green's functions; these, combined with non-perturbative evaluations, lead to an improvement in the non-perturbative estimates of the mixing coefficients.

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References

[1] M. Constantinou, M. Costa, R. Frezzotti, V. Lubicz, G. Martinelli, D. Meloni, H. Panagopoulos, S. Simula, *A lattice study of the chromomagnetic operator*, in preparation.

[2] G. D'Ambrosio, G. Isidori, G. Martinelli, *PLB* **480** (2000) 164 [arXiv:hep-ph/9911522].

[3] S. Bertolini, J.O. Eeg, M. Fabbrichesi, *NPB* **449** (1995) 197 [arXiv:hep-ph/9409437].

[4] D. Becirevic, V. Lubicz, G. Martinelli, F. Mescia, *PLB* **501** (2001) 98 [arXiv:hep-ph/0010349].

[5] I. Baum, V. Lubicz, G. Martinelli, L. Orifici, S. Simula, *PRD* **84** (2011) 074503 [arXiv:1108.1021].

[6] R. Frezzotti, G.C. Rossi, *JHEP* **0408** (2004) 007 [arXiv:hep-lat/0306014]; *JHEP* **0410** (2004) 070 [arXiv:hep-lat/0407002].