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Surface operator study in SU(2) gauge field theory

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The surface operator in an SU(2) non-Abelian gauge field theory is studied. We analyze abelian projection of the SU(2) symmetry to the U(1) group calculating the surface parameter using multi-level and multi-hit algorithms. The surface parameter dependence on the surface area and volume is studied in confinement and deconfinement phases. It is shown that at the deconfinement phase the spatial surface operator exhibits nontrivial area dependence. In the confinement phase the operator is trivial with no area and volume dependence. It is shown also that the temporal surface operator exhibits the same phase behavior.

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1. Introduction

The most important probes for the phase states of a four-dimensional gauge field theory are the Wilson and t'Hooft line operatros that are defined on one-dimensional curves in the space-time. For example, these line-operators define order parameters for the confinement-deconfiment phase transition of the QCD vacuum. However, for more detail understanding of four-dimensional gauge field theory dynamics and vacuum topology we need additional probes expressed by operators defined on the subspaces with higher dimensions.

Possible candidates are operators that are defined on the two-dimensional surface in the four-dimensional space-time. In the present work the surface operator in an SU(2) non-Abelian gauge field theory is studied. We analyze abelian projection of the SU(2) symmetry to the U(1) group calculating the surface operator using multilevel and multi-hit algorithms for the sake of statistical confidence. The surface operator dependence on the surface area and volume is studied in confinement and deconfinement phases. It is shown that at the deconfinement phase the spatial surface operator exhibits nontrivial area dependence. In the confinement phase the operator is trivial with no area and volume dependence. It is shown also that the temporal surface operator exhibits the same phase behavior.

2. Surface operator on the lattice

In general case the surface operator can be defined by the following expression:

$$W = e^{i\kappa \int\limits_S F_{ij}\, d\sigma^{ij}} \tag{2.1}$$

where F_{ik} - the gauge field tensor, $d\sigma_{ik}$ - surface element (here we do not distinguish between upper and lower indexes, because all calculations are performed in Euclidean space-time after Wick rotation), i, $k = 1, 2, 3$ - indexes of the space-time directions. The field flow through a lattice plaquette can be related with the plaquette angle θ_p as follows:

$$\kappa \int\limits_S F_{ij}\, d\sigma^{ij} = \theta_p \tag{2.2}$$

Thus, we define the surface operator on the lattice as follows:

$$W_p(S) = Re \prod_S e^{\iota\theta_p}, \tag{2.3}$$

The phase of a wave function is changing by the plaquette angle value by moving along the contour of the plaquette. This phase related with chromomagnetic field flow through plaquette surface as follows:

$$\kappa \int\limits_S \mathbf{H} \cdot d\mathbf{S} = \kappa \oint \mathbf{A} \cdot d\mathbf{l} = \theta_p, \tag{2.4}$$

where integration over $d\mathbf{l}$ carried out on a path covering the area S. This equation provides a simple connection of the surface and line operators in the trivial vacuum.

In this work we consider pure gauge field theory with $SU(2)$ group symmetry broken up to $U(1)$. Thus the θ_p related with $F_{\mu\nu}$ as follows:

$$F_p = \widehat{1}\, cos\,\theta_p + \imath n_i \sigma_i \, sin\,\theta_p, \tag{2.5}$$

where n_i - vector on the unit sphere, σ_i - Pauli matrices, F_p is a value of the gauge field tensor $F_{\mu\nu}$ on the plaquette. Thus, for the θ_p we can write the following expression:

$$\theta_p = \arccos\left(\frac{1}{2} Tr\, F_p\right). \tag{2.6}$$

All phases are calculated on the surface of the three dimension cube in the space-time. The function $\arccos(x)$ is defined within the range $[0,\pi]$. In the gauge group $U(1)$ the range of variation of the angle is $[0,2\pi]$. Thereby, on the one side of cube the phase is selected as $\arccos\left(\frac{1}{2} Tr\, F_p\right)$, on the opposite side as $\arccos\left(\frac{1}{2} Tr\, F_p\right) + \pi$.

In lattice calculations we use the link variable $U_{ij} \in SU(2)$, where i,j is number of lattice sites, located at the ends of the link. Variable U_{ij} related with A_μ as follows:

$$U_{ij} = e^{\imath g_0 A_\mu a}, \tag{2.7}$$

where a is distance between sites and A_μ is taken at the middle of the link ij. According to Wilson [7] action for pure gauge theory can be written as follows:

$$S = \sum_{\Box} S_{\Box}, \tag{2.8}$$

$$S_{\Box} = \beta\left[1 - \frac{1}{2} Re\, Tr\left(U_{ij} U_{jk} U_{kl} U_{li}\right)\right], \tag{2.9}$$

where $\beta = 4/g_0^2$ and $\Box$ is a plaquette. The partition function is

$$Z = \int (dU)\, e^{-S(U)}. \tag{2.10}$$

Any observed value of a physical quantity A we can calculated using following expression:

$$\langle A \rangle = Z^{-1} \int (dU)\, A(U)\, e^{-S(U)}, \tag{2.11}$$

where $A(U)$ is physical quantity calculated on the lattice configurations U and the integration is over all configurations with weight equal $e^{-S(U)}$.

For study the surface operator we prepared two sets of configurations, in the confinement and deconfinement phase. In order to ensure that our lattice configuration exactly are in one phase we calculate Polyakov loop, which is defined as

$$L(T) = \frac{1}{2} Tr\left(e^{\imath g_0 \int_0^{1/T} A_0\, dt}\right), \tag{2.12}$$

where t is cyclic variable with period $1/T$, T is temperature on the lattice. Polyakov loop is the order parameter for the phase transition confinement-deconfinement. In the confinement phase it is

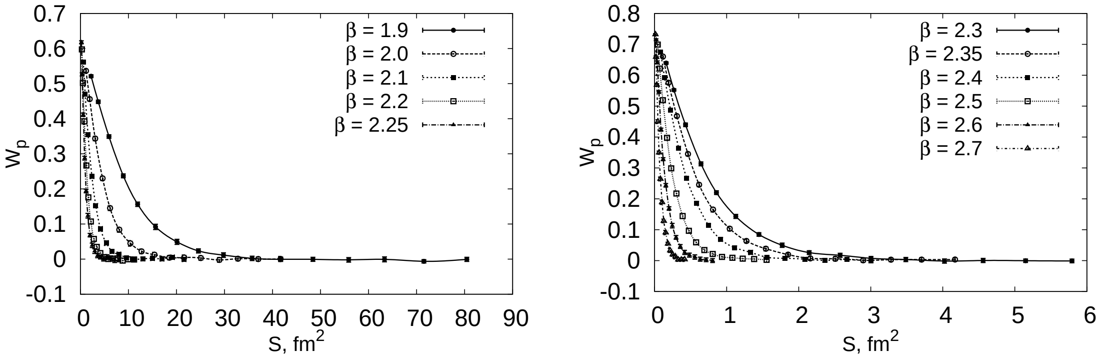

Figure 1: Dependence of the Witten parameter from area surface in the confinement (left panel) and deconfinement (right panel) phase and comparison of the fitting.

equal zero, and in the deconfinement phase it is different from zero. In the Table 1 we show lattice characteristic which used in calculation. On the lattice the Polyakov loop is defined by following expression:

$$L(\mathbf{x}) = \frac{1}{2} Tr \prod_{t=0}^{N_t-1} U_0(t,\mathbf{x}), \tag{2.13}$$

where $U_0(t,\mathbf{x})$ is the time direction link. To calculate the surface operator on the lattice, we select

Phase	Size lattice	β	$L(T)$
Deconfinement	$4*30^3$	> 2.25	0.349 ± 0.002
Confinement	41^4	≤ 2.25	0.0002 ± 0.0006

Table 1: Used lattice for search volume dependence.

a cube in the 3d space. Then, the phase is calculated on the each plaquette on the surface of the cube and result is obtained as a sum of these phases. After that we calculate our parameter at different points in the lattice configuration and average them. And the final result is obtained by averaging on the set of configurations.

We consider a cube with length of edge from $1a$ to $13a$ (a is the lattice scale), which corresponds area surface from 6 to 1014 plaquettes. For best accuracy we use multilevel [6], multi-hit [9] algorithms and MPI parallelism.

3. Results

All calculation performed on 50 configurations in 1000 points on the each lattice configuration. For both phases results are shown at the figure 1. To extract area and volume dependence of the surface operator we fit the obtained data by the following expression:

$$W_p(S,V) = e^{-\sigma S - \gamma V}, \tag{3.1}$$

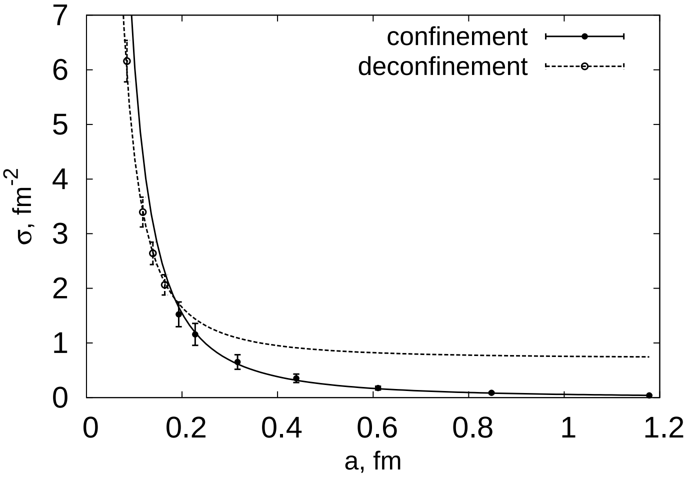

Figure 2: Dependence of the area coefficient σ on the lattice spacing. Solid line shows fit of the coefficient dependence in the confinement phase, dashed line is the fit in the deconfinement phase.

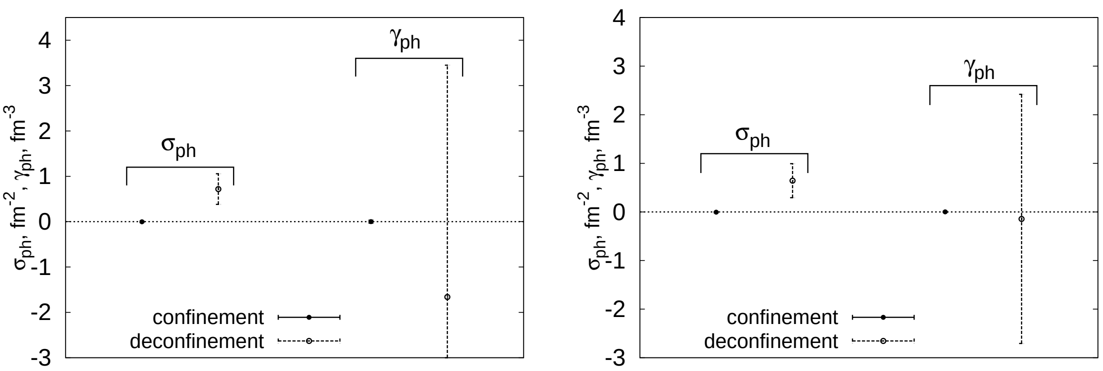

Figure 3: Values of the area (σ_{ph}) and volume (γ_{ph}) coefficients for the spatial (left panel) and temporal (right panel) surfaces in the confinement and deconfinement phases.

where σ is area coefficient, γ is volume coefficient, S is the surface area surface, V is volume covered by the surface. The parameters values are obtained with the help of minuit2 library from ROOT[1] package.

The figure 2 shows different asymptotic behavior of the σ parameter at large a in different phases, what points on possible order parameter implied in σ. At small a the parameter σ is divergent. This divergent behavior is related to the ultraviolet divergence of the self energy that is proportional to the area of the closed surface. This divergence can be qualitatively related to self energy of colored dipoles on the surface that define the surface operator. It is an analogue of the divergence in case of the Wilson lines:

$$\langle Tr P exp \left\{ -\int_C \hat{A}_\mu \, dx_\mu \right\} \rangle \sim exp\left\{ -const\, g^2 L/a \right\}, \tag{3.2}$$

[1]See http://root.cern.ch/drupal/

where L is the perimeter of the Wilson line C, a is the lattice spacing, g^2 is a coupling and we keep only the most divergent piece. For more information about magnetic degrees of freedom and surface operators one can refer to the article [11]. Thus, we parametrize the area and volume coefficients as follows:

$$\sigma(a) = \sigma_{ph} + \sigma_{div}/a^2, \\ \gamma(a) = \gamma_{ph} + \gamma_{div}/a^3 \tag{3.3}$$

where σ_{ph} and γ_{ph} are the physical coefficients and σ_{div} and γ_{div} are the coefficients of the divergent part of area and volume law. After fitting we obtain that the $\sigma_{ph} = 0$ in confinement phase and $\sigma_{ph} \neq 0$ in deconfinement case for both spatial and temporal surfaces (see figure 3). Thus, the σ_{ph} can be considered as an order parameter for the confinement-deconfinement phase transition. For the temporal surfaces (left panel of the figure 3) there is no volume law in the both phases. The volume law for the spatial surface needs more precise study to make statistical errors smaller.

In conclusion we can say following: 1) area law of the temporal surface operator corresponds to the perimeter law of the Wilson loop; 2) temporal surface operator has no volume dependence in both phases; 3) volume dependence of the spatial surface needs more precise study.

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