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On one-loop corrections to matching conditions of Lattice HQET including $1/m_b$ terms

Piotr Korcyl*
NIC, DESY, Platanenallee 6, 15738 Zeuthen, Germany
E-mail: piotr.korcyl@desy.de

(for the ALPHA collaboration)

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HQET is an effective theory for QCD with N_f light quarks and a massive valence quark if the mass of the latter is much bigger than Λ_{QCD}. As any effective theory, HQET is predictive only when a set of parameters has been determined through a process called matching. The non-perturbative matching procedure including $1/m_b$ terms, developped by the ALPHA collaboration, consists of 19 carefully chosen observables which are precisely computable in lattice QCD as well as in lattice HQET. The matching conditions are then a set of 19 equations which relate the QCD and HQET values of these observables. We present a study of one-loop corrections to two generic matching observables involving correlation function with an insertion of the A_0 operator. Our results enable us to quantify the quality of the relevant observables in view of the envisaged non-perturbative implementation of this matching procedure.

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*Speaker.

In a problem involving a hierarchy of scales such as a lattice QCD simulation of heavy-light mesons one needs to employ an effective description of dynamics on one of the scales if lattices of affordable size are to be used. In a particular case of extraction of decay constants and form-factors of B-mesons, the ALPHA collaboration decided to use Heavy Quark Effective Theory [1] in order to account for the dynamics of the **b** quark. A fully non-perturbative strategy was set up [2, 3, 4] which consists of a non-perturbative matching step between HQET and QCD in a finite volume using the Schrödinger functional framework and of a non-perturbative evolution of HQET parameters using step scaling techniques up to volumes sufficiently large to perform full QCD calculations. The success of the matching step relies on a set of suitable QCD observables and their effective HQET counterparts which can be precisely evaluated in a Monte Carlo simulation. Apart of being precise, one also requires that the matching observables do not introduce artificially large $1/m_b^2$ corrections. The entire set of matching observables was investigated at tree-level of perturbation theory in Ref.[5] and the aim of this work is to report on the extention of that study to include one-loop corrections.

After introducing basic notation in section 1 we describe two examples of matching observables in section 2 and discuss how to estimate the size of such unwanted $1/m_b^2$ corrections using lattice perturbation theory in section 3. We conclude with some discussion in section 4.

1. HQET including the $1/m_b$ terms

We use the Eichten-Hill formulation of HQET [1] in which the Lagrangian at order $1/m_b$ is a sum of the leading, static, part and two $1/m_b$ corrections

$$\mathscr{L}_{\mathrm{HQET}} = \mathscr{L}_{\mathrm{stat}} - \omega_{\mathrm{kin}}\mathscr{L}_{\mathrm{kin}} - \omega_{\mathrm{spin}}\mathscr{L}_{\mathrm{spin}} \tag{1.1}$$

with $\mathscr{L}_{\mathrm{stat}} = \bar{\psi}_h D_0 \psi_h$. The power divergent mass-counter term was absorbed in m_{bare}, the only parameter of the static HQET action, which after an appropriate change of variables appears in a prefactor $e^{-m_{\mathrm{bare}}|x_0-y_0|}$ of some correlation functions.

The kinetic and chromomagnetic operators enter only as insertions in the static vacuum expectation values, namely for some operator $\mathscr{O}$ we have

$$\langle \mathscr{O} \rangle_{\mathrm{HQET}} = \langle \mathscr{O} \rangle_{\mathrm{stat}} + \omega_{\mathrm{kin}} \sum_x \langle \mathscr{O} \mathscr{L}_{\mathrm{kin}}(x) \rangle_{\mathrm{stat}} + \omega_{\mathrm{spin}} \sum_x \langle \mathscr{O} \mathscr{L}_{\mathrm{spin}}(x) \rangle_{\mathrm{stat}}. \tag{1.2}$$

Local operators have an effective description as well. We write it explicitely for the lattice discretized A_0 operator since this will be the operator we will need in the following. We have

$$Z_{A_0}^{\mathrm{HQET}} (A^{\mathrm{HQET}})_0 = Z_{A_0}^{\mathrm{HQET}} \Big[\bar{\psi}_\ell \gamma_0 \gamma_5 \psi_h + a c_{A_{0,1}} \bar{\psi}_\ell \frac{1}{2} \gamma_5 \gamma^k (\nabla_k^S - \overleftarrow{\nabla}_k^S) \psi_h + \\ + a c_{A_{0,2}} \bar{\psi}_\ell \frac{1}{2} \gamma_5 \gamma^k (\nabla_k^S + \overleftarrow{\nabla}_k^S) \psi_h \Big] \tag{1.3}$$

where ψ_ℓ denote relativistic, massless fermions, whereas ψ_h is a nonrelativistic heavy fermion with $P_+\psi_h = \psi_h$. The renormalization schemes for $Z_{A_0}^{\mathrm{QCD}}$ and $Z_{A_0}^{\mathrm{HQET}}$ will be specified in section 3.1. Notation for the finite differences ∇_k^S is taken from [6].

In order to define HQET and the currents at the next-to-leading order in $1/m_b$ one has to fix 3 parameters in $\mathscr{L}_{\mathrm{HQET}}$ and 2×3 parameters in $A_0(x)$ and $V_0(x)$ and 2×5 in $A_k(x)$ and $V_k(x)$ giving in total 19 parameters. They are usualy denoted collectively by ω_i, with $i = 1,\dots,19$. In this work we concentrate on two parameters appearing in Eq.(1.3), namely $c_{A_{0,2}} \equiv \omega_5$ and $Z^{\mathrm{HQET}}_{A_0} \equiv \omega_6$ and on the corresponding matching observables.

2. Two examples of matching observables

HQET parameters are determined by considering an appropriately choosen set of observables $\{\Phi_i\}_{i=1,\dots,19}$. The approach implemented by the ALPHA collaboration [5] consists in using the Schrödinger functional (SF) framework [7] to define correlation functions out of which the observables Φ_i are constructed. In this work we will need one boundary-to-boundary and one boundary-to-bulk correlation function, e.g.

$$F_1(\theta_\ell,\theta_h) = -\frac{a^{12}}{2L^6}\sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z}} \langle \bar{\zeta}'_\ell(\mathbf{u})\gamma_5\zeta'_h(\mathbf{v})\bar{\zeta}_h(\mathbf{y})\gamma_5\zeta_\ell(\mathbf{z})\rangle, \tag{2.1}$$

$$f_{A_0}(\theta_\ell,\theta_h,x_0) = -\frac{a^6}{2}\sum_{\mathbf{u},\mathbf{v}} \langle \bar{\zeta}_h(\mathbf{u})\gamma_5\zeta_\ell(\mathbf{v})\left(A_0\right)_I(x_0)\rangle \tag{2.2}$$

where ζ and $\bar{\zeta}$ denote fermionic fields living on the boundary. The θ angles are additional kinematic parameters which in the free theory correspond to the momenta of quark fields,

$$\psi_h(x+L\hat{k}) = e^{i\theta_h^k}\psi_h(x), \qquad\qquad \psi_\ell(x+L\hat{k}) = e^{i\theta_\ell^k}\psi_\ell(x). \tag{2.3}$$

The θ angles can be tuned such as to minimize $1/m_b^2$ effects [5]. In order to determine $c_{A_{0,2}}$ and $Z^{\mathrm{HQET}}_{A_0}$ the following observables were proposed

$$\Phi_5(\theta_\ell,\theta_{h_1},\theta_{h_2}) = \log\frac{f_{A_0}(\theta_\ell,\theta_{h_1},x_0=T/2)}{f_{A_0}(\theta_\ell,\theta_{h_2},x_0=T/2)}, \tag{2.4}$$

$$\Phi_6(\theta_\ell,\theta_h) = \log\frac{-Z_{A_0}f_{A_0}(\theta_\ell,\theta_h,x_0=T/2)}{\sqrt{F_1(\theta_\ell,\theta_h)}} \equiv \log Z_{A_0} + \phi_6(\theta_\ell,\theta_h). \tag{2.5}$$

Φ_5 is defined in such a way as to cancel all renormalization factors, whereas in Φ_6 only the renormalization factor of A_0, remains uncancelled. A generic matching condition for the 'Φ_5-type' observables can be written as

$$\Phi_{i,\mathrm{QCD}}\left(\bar{m}(m),a=0,L\right) \stackrel{!}{=} \Phi_{i,\mathrm{HQET}}\left(a,L,\omega(\bar{m}(m),a)\right) = \Phi_{i,\mathrm{stat}}\left(a,L\right) + \sum_j \Phi_{ij,1/m}(a,L)\;\omega_j(\bar{m}(m),a), \tag{2.6}$$

where L is the size of the finite SF volume in which the observables Φ_i are defined, a is the lattice spacing and $\bar{m}(m)$ is the **b** quark mass defined in the lattice minimal subtraction scheme [7] at the scale m of the **b** quark mass. The scale m can be given by m_{pole} or $\bar{m}$ or in any other scheme since at one-loop precision the scheme is not relevant. In the following we will work with dimensionless quantities so we introduce z as a parameter to fix the heavy quark mass

$$z = \bar{m}(m)L. \tag{2.7}$$

The perturbative analysis of the observables Eq.(2.5) was made using `pastor`, an automatic tool for generation and calculation of lattice Feynman diagrams [8] with SF boundary conditions.

For a given discretized action, correlation function and parameters such as L/a and the dimensionless heavy quark mass z, `pastor` generates the Feynman rules, all Feynman diagrams and a C++ program to evaluate each diagram. The calculations were performed using the Wilson plaquette gauge action and $\mathcal{O}(a)$-improved Wilson fermions.

In Ref.[5] a tree-level analysis of the entire set of matching observables was presented. The purpose of this work is, using an example of two matching observables, to confirm that $1/m_b^2$ corrections are small also at one-loop level. Similar results for other observables were reported in [9, 10].

3. One-loop contributions to matching observables

3.1 $c_{A_{0,2}}$

$f_{A_0}^{\mathrm{stat}}(\theta_l, \theta_h, x_0)$ does not depend on θ_h, therefore $\Phi_{5,\mathrm{stat}}$ vanishes. We expand the matching condition Eq.(2.6) in g^2 and get (abbreviating $(\theta_\ell, \theta_{h_1}, \theta_{h_2})$ by θ)

$$\Phi^{(0)}_{5,\mathrm{QCD}}(\theta,z) + g^2\Phi^{(1)}_{5,\mathrm{QCD}}(\theta,z) = z^{-1}\sum_t \left(\hat{\omega}_t^{(0)}\hat{\Phi}^{(0)}_{5,t}(\theta) + g^2\hat{\omega}_t^{(1)}(z)\hat{\Phi}^{(0)}_{5,t}(\theta) + g^2\hat{\omega}_t^{(0)}\hat{\Phi}^{(1)}_{5,t}(\theta)\right), \tag{3.1}$$

with $\hat{\omega}_j = \bar{m}\omega_j$ and $\hat{\Phi}_j = L\Phi_j$. The sum over t refers to different subleading contributions, namely $t = \{\mathrm{kin}, \mathrm{spin}, c_{A_{0,1}}, c_{A_{0,2}}\}$. Separating different orders in g^2 we get

$$\begin{aligned}
\Phi^{(0)}_{5,\mathrm{QCD}}(\theta,z) &= z^{-1}\sum_j \hat{\omega}_j^{(0)}\hat{\Phi}^{(0)}_{5,j}(\theta),\\
\Phi^{(1)}_{5,\mathrm{QCD}}(\theta,z) &= z^{-1}\sum_j \left(\hat{\omega}_j^{(1)}(z)\hat{\Phi}^{(0)}_{5,j}(\theta) + \hat{\omega}_j^{(0)}\hat{\Phi}^{(1)}_{5,j}(\theta)\right).
\end{aligned} \tag{3.2}$$

In order to isolate the leading $1/z$ dependence we define a ratio R of the one-loop correction to the tree-level contribution

$$R_5 = \frac{\Phi^{(1)}_{5,\mathrm{QCD}}(\theta,z)}{\Phi^{(0)}_{5,\mathrm{QCD}}(\theta,z)} = \frac{\sum_j \hat{\omega}_j^{(0)}\hat{\Phi}^{(1)}_{5,j}(\theta)}{\sum_j \hat{\omega}_j^{(0)}\hat{\Phi}^{(0)}_{5,j}(\theta)} + \frac{\sum_j \hat{\omega}_j^{(1)}(z)\hat{\Phi}^{(0)}_{5,j}(\theta)}{\sum_j \hat{\omega}_j^{(0)}\hat{\Phi}^{(0)}_{5,j}(\theta)} = \alpha(\theta) + \gamma(\theta)\log(z) + \mathcal{O}(1/z), \tag{3.3}$$

where we used the fact that the only way a z-dependence can appear on the right-hand side of the above equation is through $\hat{\omega}_j^{(1)}(z)$ which must be of the functional form $\hat{\omega}_j^{(1)}(z) = a_j + b_j \log z$ (a_j, b_j constants). When R is plotted on a linear-log plot, it measures:

- $1/z^2$ *corrections*: deviations from a linear behaviour,
- *coefficient of the subleading logarithm* : slope of the data.

Plots shown on figure 1 present the results for the Φ_5 observable. The left plot 1(a) shows the one-loop contributions to $\Phi_{5,\mathrm{QCD}}$ extrapolated to the continuum as a function of z which extrapolates to a vanishing static limit. The $1/z^2$ corrections seem to be surprisingly small. The right plot 1(b) contains data for the corresponding R ratio which confirms this observation; the logarithmic dependence as well as higher corrections in $1/z$ are very small. Thus, the one-loop results do not favour any of the analyzed combination of θ angles.

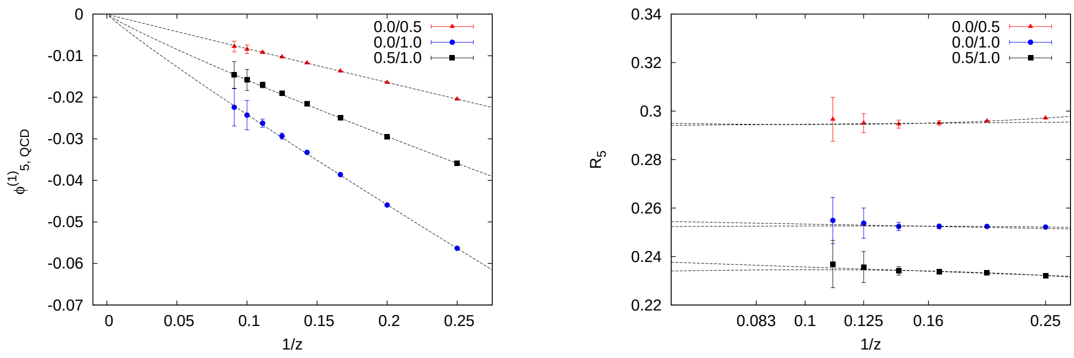

Figure 1: Results for Φ_5. Figure on the left presents the z dependence of the one-loop contributions to QCD observables together with a fit of the form $f(z) = \beta_0/z + \beta_1 \log z/z$. Figure on the right shows the corresponding R ratio. To each data set two fits were performed with anstätze $f(z) = \alpha + \gamma \log z$ and $f'(z) = \alpha' + \gamma' \log z + \delta'/z$. One can estimate higher-order corretions by calculating $\frac{f(4)-f'(4)}{f(4)} \sim 0.0003$, which turns out to be very small.

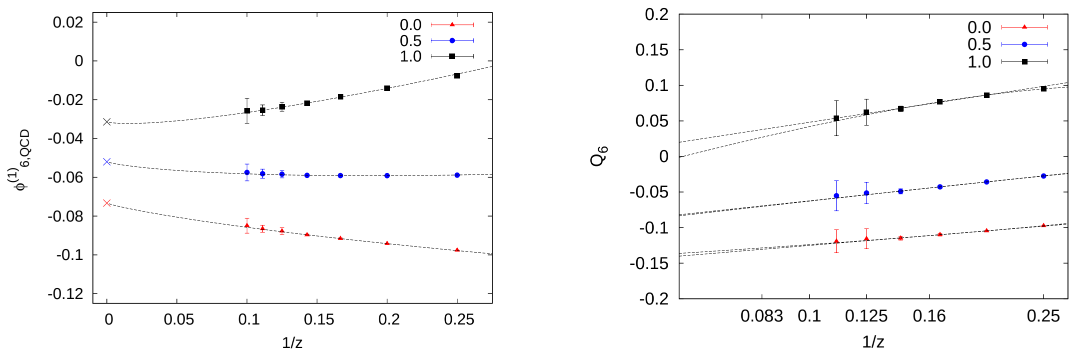

Figure 2: Results for Φ_6. Figure on the left presents the z dependence of the one-loop contributions to QCD observables together with a fit of the form $f(z) = \beta_0 + \beta_1/z + \beta_2 \log z/z$. Figure on the right shows the corresponding R ratio. To each data set two fits were performed with anstätze $f(z) = \alpha + \gamma \log z$ and $f'(z) = \alpha' + \gamma' \log z + \delta'/z$. One can estimate higher-order corretions by calculating $\frac{f(4)-f'(4)}{f(4)} \sim 0.003$, which turns out to be small.

3.2 $Z_{A_0}^{\mathrm{HQET}}$

In order to fix the renormalization constant $Z_{A_0}^{\mathrm{HQET}}$ we have to match the renormalized observables. Writing explicitly the renormalization factors, Eq.(2.6) becomes

$$\lim_{a/L\to 0}\left[\log Z_{A_0}^{\mathrm{QCD}} + \phi_{6,\mathrm{QCD}}(z,a/L)\right] \stackrel{!}{=} \log Z_{A_0}^{\mathrm{HQET}}(\mu,a) + \phi_{6,\mathrm{HQET}}(a/L,\omega(z,a/L)) =$$
$$= \log Z_{A_0}^{\mathrm{HQET}}(\mu,a) + \Big(\phi_{6,\mathrm{stat}}(a/L) + \sum_j \phi_{6,j}(a/L)\ \omega_j(z,a/L)\Big), \quad (3.4)$$

where the continuum limit is taken keeping the renormalized mass $\bar{m}$ and coupling g^2 fixed. In order to estimate the $1/m_b^2$ corrections to the observable ϕ_6 it is enough to work at the static order at which the renormalization factor $Z_{A_0}^{\rm stat}$ is known. Hence, the matching condition Eq.(3.4) takes the form

$$\lim_{a/L\to 0}\left[\log Z_{A_0}^{\rm QCD}+\phi_{6,{\rm QCD}}(z,a/L)\right]\stackrel{!}{=}\log C_{A_0}^{\rm match}+\log Z_{A_0}^{\rm stat}(\mu=\bar{m}(m),a)+\phi_{6,{\rm stat}}(a/L)+\mathscr{O}(1/z). \tag{3.5}$$

The QCD side is renormalized in a scheme enforcing the current algebra relations at $z=0$ [11]. On the HQET side we use an intermediate renormalization scheme, the lattice minimal subtraction scheme, which only cancels the logarithmic divergence present in $\phi_{6,{\rm stat}}(a/L)$ [12, 13], i.e.

$$Z_{A_0}^{\rm stat}(\mu,a)=1-\gamma_0\log(a\mu)g^2+\mathscr{O}(g^4),\qquad \gamma_0=-\frac{1}{4\pi^2}, \tag{3.6}$$

whereas the finite factor $C_{A_0}^{\rm match}$ can be used to fix the finite translation factor between the two schemes. We explicitly indicated in Eq.(3.5) that the HQET side was renormalized at the scale $\mu=\bar{m}(m)$. In this situation the expansion of the factor $C_{A_0}^{\rm match}$ is known [14]

$$C_{A_0}^{\rm match}=1+B_{A_0}g^2+\mathscr{O}(g^4),\qquad B_{A_0}=-0.137(1). \tag{3.7}$$

Eq.(3.5) can be rewritten as

$$\Phi_{6,{\rm QCD}}(z)=\Phi_{6,{\rm stat}}(z,a/L)+\log C_{A_0}^{\rm match}(g^2)+\mathscr{O}(1/z), \tag{3.8}$$

We expand both sides of Eq.(3.8) in the coupling g^2 and get

$$\begin{aligned}\Phi_{6,{\rm QCD}}^{(0)}(z)&=\phi_{6,{\rm stat}}^{(0)}+\mathscr{O}(1/z),\\ \Phi_{6,{\rm QCD}}^{(1)}(z)&=\phi_{6,{\rm stat}}^{(1)}(a/L)-\gamma_0\log(a\bar{m})+B_{A_0}+\mathscr{O}(1/z).\end{aligned}$$

Subtracting $\gamma_0\log z$ from both sides of the last equation yields

$$\Phi_{6,{\rm QCD}}^{(1)}(z)-\gamma_0\log z=\phi_{6,{\rm stat}}^{(1)}(a/L)-\gamma_0\log(a/L)+B_{A_0}+\mathscr{O}(1/z)\equiv\Phi_{6,{\rm stat}}^{(1)}+\mathscr{O}(1/z), \tag{3.9}$$

where we consistently used the facts that $\mu=\bar{m}(m)$ and $z=L\bar{m}(m)$. The sum of subleading terms denoted by $\mathscr{O}(1/z)$ must vanish in the static limit, therefore we can assume that at one-loop level it can be parametrized by $\alpha_0/z+\alpha_1/z\log z$ (α_0, α_1 functions of θ angles only). Then, in order to make visible the $1/m_b^2$ corrections we define the quantity Q as

$$\begin{aligned}Q_6&=z\left[\Phi_{6,{\rm QCD}}^{(1)}(z)-\gamma_0\log z\right]-z\left[\Phi_{6,{\rm stat}}^{(1)}\right]\\ &=z\left[\mathscr{O}(1/z)+\mathscr{O}(1/z^2)\right]=\alpha_0+\alpha_1\log(z)+\mathscr{O}(1/z)\end{aligned}$$

In analogy to the ratio R of the previous subsection, when Q is plotted on a linear-log plot one can read off

- the $1/z^2$ *corrections*: as deviations from a linear behaviour,
- the *coefficient of the subleading logarithm* : as the slope of the data.

Results for the matching observable Φ_6 are presented on figure 2. The left plot 2(a) shows the z-dependence of the combination $\Phi_{6,{\rm QCD}}^{(1)}(z)-\gamma_0\log z$ together with the static observable $\Phi_{6,{\rm stat}}^{(1)}$. On the right plot 2(b) we show the data for the quantity Q_6. Again, the subleading logarithm as well as higher-order corrections in $1/z$ are small. The one-loop results favour small θ angles.

4. Conclusions

In this work we presented a pertubative study of matching observables proposed to match non-perturbatively lattice HQET to QCD. We extended the tree-level investigation of Ref.[5] to one-loop order and discussed in details results for two matching observables. By defining suitable quantities (R and Q) we were able to show that the matching observables do not receive large $1/m_b^2$ corrections at one-loop level, thus confirming the tree-level conclusions. Complete results for the remaining matching conditions will be presented elsewhere [15].

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