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Lattice QCD study of $B_s \to D_s \ell \bar{\nu}_\ell$ decay near zero recoil

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We study the hadronic matrix elements describing the $B_s \to D_s \ell \bar{\nu}_\ell$ decay in and beyond the Standard Model. By using the twisted mass QCD on the lattice with $N_f = 2$ dynamical flavors we compute the normalization $\mathscr{G}_s(1)$ of the form factor dominating $B_s \to D_s \ell \bar{\nu}_\ell$ in the SM. We also make the first lattice determination of $F_0(q^2)/F_+(q^2)$ and $F_T(q^2)/F_+(q^2)$ near zero recoil (near $q^2_{\rm max}$). We briefly discuss the non-strange case $B \to D \ell \bar{\nu}_\ell$ as well.

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1. Introduction

A precise knowledge of semileptonic decays of B-mesons brings several advantages to flavor physics. For example, the decay channel $B \to D\ell\bar{\nu}_\ell$ allows an independent estimate of the CKM matrix element (V_{cb}) which is extracted by comparing the theoretical determination of form factors with experimental measurements of the partial or total decay widths. In the limit of vanishing lepton mass, the differential decay rate of $B \to D\ell\bar{\nu}_\ell$ reads [1]

$$\frac{d\Gamma}{dw}(B \to D\,\ell\bar{\nu}_\ell) \;=\; \frac{G_F^2}{48\pi^3}\,(m_B+m_D)^2\, m_D^3 (w^2-1)^{3/2}\,|V_{cb}|^2\,|\mathscr{G}(w)|^2\,, \tag{1.1}$$

where $\mathscr{G}(w)$ is the relevant form factor and w is the product of the velocities of the hadrons ($w = v_B \cdot v_D$) in the HQET framework.
The main theoretical problem is the knowledge of $\mathscr{G}(w)$. At the zero recoil point, $w = 1$, heavy quark symmetries play a useful role in setting $\mathscr{G}(1) = 1$ in the limit of $m_{b,c} \to \infty$ [2]. Together with the short distance QCD corrections, the effect of the finiteness of m_b and m_c masses leads to the fact that $\mathscr{G}(1) \neq 1$, which is a non-perturbative effect that needs to be computed by means of lattice QCD.

Here we propose to study the $B_s \to D_s$ transitions in and beyond the Standard Model (SM). The decay mode $\bar{B}_s^0 \to D_s^+\ell\bar{\nu}_\ell$ could be studied at LHCb and especially at Super Belle. One more advantage in studying B_s decay is that the soft photon problem is less important than in the non-strange case in which the charged and neutral B-semileptonic decay modes are averaged [3]. From the lattice point of view, the non-strange heavy-light mesons are more difficult because a chiral extrapolation in the valence light quark is required, which is a source of systematic uncertainties. Working with the strange case is simpler because the light spectator is fixed to its known mass (m_s) and no extrapolation in the light quark mass is needed when computing the form factors on the lattice.

In the following, we first introduce the relevant form factors contributing to the semileptonic decays of B_s into D_s mesons in the SM. We then proceed to describe the simulation details as well as the strategy for our computation. We present our results at the zero recoil point for $\mathscr{G}_s(1)$ and then we look for New Physics (NP) beyond the SM in $B_s \to D_s\ell\bar{\nu}_\ell$ decays by introducing the scalar and the tensor form factors. More details can be found in Ref. [4].

2. $B_s \to D_s$ form factors

The hadronic matrix element governing the $B_s \to D_s$ decay is parametrized in the SM as

$$\langle D_s(p_{D_s})|V_\mu|\bar{B}_s(p_{B_s})\rangle = \; F_+(q^2)\,(p_{B_s}+p_{D_s})_\mu + q_\mu\;\left[F_0(q^2) - F_+(q^2)\right]\left(\frac{m_{B_s}^2 - m_{D_s}^2}{q^2}\right), \tag{2.1}$$

where the vector ($F_+(q^2)$) and the scalar ($F_0(q^2)$) form factors are functions of $q^2 = (p_{B_s} - p_{D_s})^2$, that can vary within the range $q^2 \in [m_\ell^2, q^2_{\rm max}]$, where $q^2_{\rm max} = (m_{B_s} - m_{D_s})^2$.
We choose to work in the B_s rest frame and give a momentum to the D_s meson. We will take this momentum to be symmetric in its spatial components $p_{B_s} = \left(m_{B_s}, \vec{0}\right)$ and $p_{D_s} = (E_{D_s}, p, p, p)$. We also use the twisted boundary conditions (BCs) [5] for the quark field. This allows to shift the quantized values of p by a continuous amount $p = \dfrac{\theta\pi}{L}$ so that $|\vec{q}| = \sqrt{3}\,\dfrac{\theta\pi}{L}$. The θ's we choose correspond to small momenta, thus we are discussing the decay matrix element near zero-recoil (near $q^2_{\rm max}$). More specifically, our θ's correspond to the following recoils w for $B_s \to D_s\ell\bar{\nu}_\ell$:

$$w \in \{1,\, 1.004,\, 1.016,\, 1.036,\, 1.062\}\,. \tag{2.2}$$

Note that $w_{\max}$ for this decay mode, in the case of massless leptons, is 1.546.
One can separate the contribution proportional to the scalar form factor $F_0(q^2)$

$$P^0_\mu \langle D(p_{D_s})|V_\mu|B(p_{B_s})\rangle = \frac{m_{B_s}+m_{D_s}}{m_{B_s}-m_{D_s}} F_0(q^2) \text{ with } P^0_\mu = \frac{q_\mu}{q^2_{\max}}, \text{ and } q_\mu = (m_{B_s}-E_{D_s}, -\vec{p}_{D_s}), \quad (2.3)$$

and that of the vector form factor $F_+(q^2)$ as

$$P^+_\mu \langle D(p_{D_s})|V_\mu|B(p_{B_s})\rangle = \vec{q}^{\,2} \frac{2m_{B_s}}{m_{B_s}-E_{D_s}} F_+(q^2) \text{ with } P^+_\mu = \left(\frac{\vec{q}^{\,2}}{m_{B_s}-E_{D_s}}, \vec{q}\right). \quad (2.4)$$

3. Lattice strategy and setup

In this analysis, we use the gauge field ensembles produced by the European Twisted Mass Collaboration [6, 7] at four values of the lattice spacing corresponding to four values of the inverse bare gauge coupling β. Dynamical quark simulations have been performed using the tree-level improved Symanzik gauge action [8] and the Wilson twisted mass quark action [9] tuned to the maximal twist [10]. Bare quark mass parameters, corresponding to a degenerate bare mass value of the u/d quark, are chosen to have the light pseudoscalar mesons (PS) in the range $280 \le m_{\rm PS} \le 500$ MeV. The details of the simulation parameters are collected in Ref. [4].
We computed all the quark propagators by using stochastic sources, and then applied the so-called one-end trick to compute the needed correlation functions [7]. We work with ten heavy quark masses m_h starting from the charm quark mass, $m_h^{(0)} = m_c$, and then successively increase the heavy quark mass by a factor of $\lambda = 1.176$, $m_h^{(i)} = \lambda^i m_c$, so that after 9 steps one arrives at $m_h^{(9)} = m_b = \lambda^9 m_c$.
In order to extract the form factors $F_0(q^2)$ and $F_+(q^2)$ from lattice data, we compute the two- and three-point correlation functions, $\mathscr{C}_H^{(2)}(t)$ and $\mathscr{C}_\mu^{(3)}(\vec{q},t)$ and, from which we then extract the desired hadronic matrix elements as

$$\mathscr{C}_\mu^{(3)}(\vec{q};t) \xrightarrow{0\ll t\ll t_S} \frac{\mathscr{Z}_{B_s}\mathscr{Z}_{D_s}}{4m_{B_s}E_{D_s}} \exp(-m_{B_s}t) \times \exp[-E_{D_s}(t_S-t)] \langle D_s(\vec{k})|V_\mu|B_s(\vec{0})\rangle,$$

where we fix $t_S = T/2$ (T is the size of the temporal extension of the lattice). m_H, E_H and $\mathscr{Z}_H$ are extracted from the large time behavior of the two-point correlation functions,

$$\mathscr{C}_H^{(2)}(t) \xrightarrow{t\gg 0} |\mathscr{Z}_H|^2 \frac{\cosh[m_H(T/2-t)]}{m_H} \exp[-m_H T/2],$$

where H stands for either D_s or B_s meson. In the computation of correlation functions we applied the smearing procedure on the source operators (see [4] for details).

4. Extraction of $\mathscr{G}_s(1)$

Often used parametrization of the matrix element (2.1) is the one motivated by the heavy quark effective theory (HQET) and reads

$$\frac{1}{\sqrt{m_{B_s}m_{D_s}}} \langle D_s(v_{D_s})|V_\mu|B_s(v_{B_s})\rangle = (v_{B_s}+v_{D_s})_\mu h_+(w) + (v_{B_s}-v_{D_s})_\mu h_-(w). \quad (4.1)$$

The desired $\mathscr{G}_s(w)$ is then written as

$$\mathscr{G}_s(w) = h_+(w)\left[1-\left(\frac{m_{B_s}-m_{D_s}}{m_{B_s}+m_{D_s}}\right)^2 H(w)\right], \quad \text{where} \quad H(w) = \left(\frac{m_{B_s}+m_{D_s}}{m_{B_s}-m_{D_s}}\right)\frac{h_-(w)}{h_+(w)}.$$

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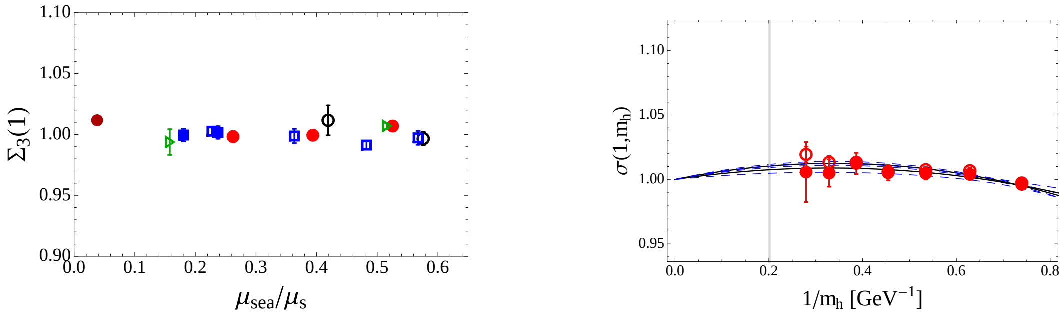
To determine $\mathscr{G}_s$ at the zero recoil point, $w = 1$, one needs to combine $h_+(1)$ with $H(1)$. The form factor $h_+(1)$ can be obtained from $F_0(q^2_{\max})$ as,

$$h_+(1) \; = \; \frac{m_{B_s} + m_{D_s}}{\sqrt{4 m_{B_s} m_{D_s}}} F_0(q^2_{\max}) \, ,$$

whereas $H(1)$ is not directly accessible from the lattice data. $H(w)$ is computed at different values of w (very close to $w = 1$) and then extrapolated to $H(1)$.
In that way we computed $\mathscr{G}_s(1)$ for each value of the heavy quark mass m_h. Note that the charm and strange quark masses are kept fixed. We then form the ratios

$$\Sigma_k(1, m_h^{(k)}, a^2) \; = \; \frac{\mathscr{G}_s(1, m_h^{(k+1)}, a^2)}{\mathscr{G}_s(1, m_h^{(k)}, a^2)} \, ,$$

where we indicate the dependence on the lattice spacing.
In Fig. 1(a), we see that our lattice data exhibit very little or no dependence on the light sea quark mass, or on the lattice spacing. Each of the ratios is then extrapolated to the continuum limit and to the physical sea



(a) (b)

Figure 1: *(a) Values of Σ_3 as a function of $\mu_{\rm sea}/\mu_s$. ○ for $\beta = 3.80$, □ for $\beta = 3.90$ (24^3), ■ for $\beta = 3.90$ (32^3), ● for $\beta = 4.05$, and ▷ for $\beta = 4.20$. The result of continuum extrapolation is indicated at $\mu_{\rm sea}/\mu_s = m_{ud}/m_s = 0.037(1)$.(b) Heavy quark mass dependence of the ratio σ extrapolated to the physical value of the heavy quark mass. The vertical line represents the value of the inverse physical b quark mass. Filled symbols correspond to $\sigma(1, m_h)$ extrapolated to the continuum limit with all parameters free, whereas the empty symbols refer to the results obtained by imposing $\beta_k^s = 0$.*

quark mass by fitting the data to a linear function of $m_l^{\rm sea}$ and a^2,

$$\Sigma_k(1, m_h^{(k)}, a^2) = \alpha_k^s + \beta_k^s \frac{m_l^{\rm sea}}{m_s} + \gamma_k^s \frac{a^2}{a^2_{3.9}} \, . \tag{4.2}$$

Since our data do not exhibit a dependence on the sea quark mass we also extrapolated $\Sigma_k(1)$ by imposing $\beta_k^s = 0$ in Eq. (4.2). For higher masses, the results of the continuum extrapolation have larger error bars in the case of a free β_k^s, as it can be seen in Fig. 1(b).
We identify $\sigma_k(1, m_h)$ with the continuum limit of $\Sigma_k(1, m_h^{(k)}, a^2)$. Since $\lim\limits_{m_h \to \infty} \mathscr{G}_s(1) \to$ constant, the successive ratios $\sigma_k(1, m_h)$ satisfy $\lim\limits_{m_h \to \infty} \sigma_k(1, m_h) = 1$. In the continuum limit, we then fit our lattice data to

$$\sigma(1, m_h) \; = 1 + \frac{s_1}{m_h} + \frac{s_2}{m_h^2} \, . \tag{4.3}$$

Clearly, the problem of larger errors for large quark masses is circumvented by the above interpolation formula because $\sigma(1,\infty) = 1$ ensures that the data with larger error bars become practically irrelevant in the fit.
Finally, after the interpolation to $\sigma(1,m_h)$, we obtain $\mathscr{G}_s(1)$ at the physical b quark mass by using a product of $\sigma_k(1)$ factors: $\mathscr{G}_s(1) \equiv \mathscr{G}_s(1,m_h = m_b) \equiv \underbrace{\mathscr{G}_s(1,m_h = m_c)}_{=1} \sigma_0 \sigma_1 \cdots \sigma_7 \cdots \sigma_8$, leading to

$$\mathscr{G}_s(1) = 1.073(17) \;\; (\beta_k^s = 0)\,, \qquad\qquad \mathscr{G}_s(1) = 1.052(46) \;\; (\beta_k^s \neq 0)\,. \tag{4.4}$$

The result on the left, which is more accurate, agrees with the only existing unquenched Lattice QCD estimate, obtained for the light non-strange spectator quark [15].
Instead of starting from $\mathscr{G}_s(1,m_h = m_c)$, we could have started from a $k < 9$, computed $\mathscr{G}_s(1,m_h = \lambda^{(k+1)} m_c)$ in the continuum limit, and then applied $\sigma_{k+1} \cdots \sigma_8$ to reach the physical b-quark mass. For example, by taking $k = 3$, we obtain

$$\mathscr{G}_s(1) = \mathscr{G}_s(1,\lambda^4 m_c)\, \sigma_4 \sigma_5 \cdots \sigma_8 = 1.059(47) \qquad (\beta_k^s \neq 0)\,, \tag{4.5}$$

fully consistent with Eq. (4.4).

5. The scalar and the tensor form factors

Latest experimental results by the BaBar Collaboration for the ratio $R(D)$ of the branching fractions $\mathscr{B}(B \to D\mu\bar{\nu}_\mu)$ and $\mathscr{B}(B \to D\tau\bar{\nu}_\tau)$ suggest a disagreement with respect to the SM prediction [12]. This discrepancy, which is around 2σ, might provide us with a first evidence for New Physics (NP) effects in semitauonic B decay [13, 14].
In the models with two Higgs doublets (2HDM), the charged Higgs boson can mediate the tree level processes, including $B \to D\ell\bar{\nu}_\ell$, and considerably enhance the coefficient multiplying the scalar form factor in the decay amplitude. By estimating the scalar form factor, involved in the SM theoretical prediction of these branching fractions, we can interpret the discrepancy between the experimentally measured $R(D)$ and its theoretical prediction within the SM. Moreover, in the models of physics Beyond Standard Model (BSM) in which the tensor coupling to a vector boson is allowed, a third form factor might become important. In the present study, we make the first Lattice QCD estimate of the tensor form factor F_T which, in the B_s rest frame, is defined via

$$\langle D(\vec{k})|T_{0i}|B(\vec{0})\rangle = \frac{-2i\, m_{B_s} k_i}{m_{B_s} + m_{D_s}}\, F_T(q^2)\,, \quad \text{where} \quad T_{\mu\nu} = \bar{c}\sigma_{\mu\nu} b\,.$$

We focus on the determination of the ratios

$$R_0(q^2) = \frac{F_0(q^2)}{F_+(q^2)} \qquad \text{and} \qquad R_T(q^2) = \frac{F_T(q^2)}{F_+(q^2)}\,,$$

at different values of $q^2 \lesssim q^2_{\rm max}$, but near the zero recoil (cf. Eq. (2.2)).

$\underline{R_0(q^2)}$: we consider in this case ratios of R_0 computed at successive heavy quark masses and with a given value of w, namely

$$\Sigma^0_{(k)}(w, m_h^{(k)}, a^2) = \frac{R_0(w,\, m_h^{(k+1)},\, a^2)}{R_0(w,\, m_h^{(k)},\, a^2)}\,.$$

In order to obtain the continuum values $\sigma^0(w,m_h)$ of $\Sigma^0_{(k)}(w,m_h^{(k)},a^2)$, we apply the continuum extrapolation by using an expression analogous to the one given in Eq. (4.2). As in the previous section we observe that

our data do not show a significant dependence on the sea quark mass nor on the lattice spacing.
Using the HQET mass formula for m_{B_s,D_s} [16], and knowing that $h_+(w)$ scales as a constant with the inverse heavy quark mass, we deduce that the heavy quark interpolation can be performed by using [4]

$$\sigma^0(w,m_h) = \frac{1}{\lambda} + \frac{s'_1(w)}{m_h} + \frac{s'_2(w)}{m_h^2}. \tag{5.1}$$

The physical value of the ratio $R_0(w)$ is then obtained by $R_0(w,m_b) = R_0(w,\lambda^{k+1}m_c)\,\sigma_k^0(w)\cdots\sigma_8^0(w)$,where $\sigma_k^0(w) \equiv \sigma^0(w,m_h^{(k)})$. In Fig. 2(a), we illustrate the ratio σ^0 for one specific value of w showing the data

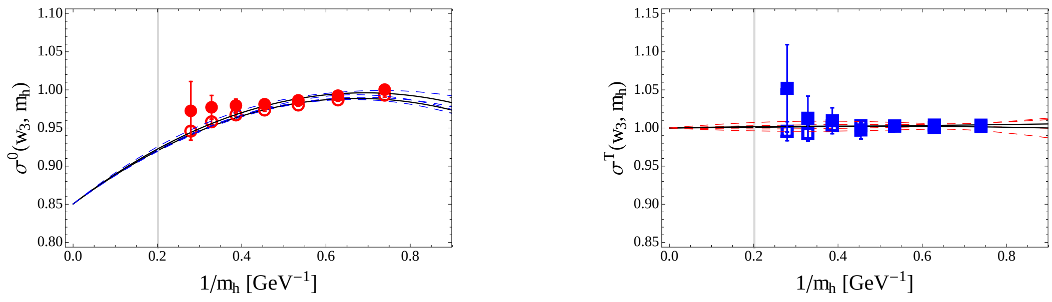


Figure 2: *(a) $\sigma^0(w)$ as a function of the inverse of the heavy quark mass, for $w = w_3 = 1.016$; (b) Same as (a) but for $\sigma^T(w)$. We see that the data obtained by assuming the independence of R_0 and R_T on the sea quark mass (empty symbols) scale better than the results obtained by allowing a linear dependence on the sea quark mass (filled symbols).*

obtained by assuming both the dependence and the independence of $R_0(w)$ on the sea quark mass.

$R_T(q^2)$: It can be shown that the heavy quark behavior of the form factor $F_T(q^2)$ is similar to that of $F_+(q^2)$ [4]. We again define the ratios computed at two successive quark masses that differ by a factor of λ,

$$\Sigma^T_{(k)}(w,m_h^{(k)},a^2) = \frac{R_T(w,m_h^{(k+1)},a^2)}{R_T(w,m_h^{(k)},a^2)}.$$

As in the previous cases, we extrapolate $\Sigma^T_{(k)}(w)$ to the continuum limit and observe that the result $\sigma_k^T(w,m_h)$ does not depend on the sea quark mass nor on the lattice spacing (within our error bars). The ratios $\sigma_k^T(w,m_h)$ are then fitted in the inverse heavy quark mass $\sigma^T(w,m_h) = 1 + \frac{s''_1(w)}{m_h} + \frac{s''_2(w)}{m_h^2}$, which is illustrated in Fig. 2(b) for $w = 1.016$. The ratio $R_T(w)$ at $m_h = m_b$ is then obtained by using the following chain $R_T(w,m_b) = R_T(w,\lambda^{k+1}m_c)\,\sigma_k^T(w)\cdots\sigma_8^T(w)$, where $\sigma_k^T(w) \equiv \sigma^T(w,m_h^{(k)})$. We checked that our results for $R_T(w)$ obtained by starting from either $k = 2$, 3, or 4, are completely consistent. The results are given in Ref. [4].
To our knowledge, the only existing result for $R_T(q^2)$ is the one of ref. [17] for the non-strange case ($B \to D\,\ell\,\bar{\nu}_\ell$) in which the constituent quark model was used. Their result for the ratio F_T/F_+ was 1.03(1) and, in their work, this ratio was predicted to be a constant with respect to the momentum transfer q^2 (or the recoil w), but no reference to the renormalization scheme or scale could have been made.

6. Conclusion

We computed the form factor $\mathscr{G}_s(1)$ by using the twisted mass QCD on the lattice. That form factor is necessary for the theoretical description of the $B_s \to D_s\ell\bar{\nu}_\ell$ decay in the Standard Model and with the massless lepton in the final state $\ell \in \{e, \mu\}$. In doing so, we implemented the method proposed in [18] that allows to reach the physical value through the interpolation of suitable ratios computed with successive heavy "*b*" quark masses for which a value for $m_h \to \infty$ is fixed by symmetry. Our final result is $\mathscr{G}_s(1) = 1.052(46)$. Moreover, following the same methodology and restricting our attention to the small recoil region, we have determined the ratio of the scalar form factor to the vector one and for the first time in LQCD the tensor form factor with respect to the vector one. Of several w's, we quote

$$\left.\frac{F_0(q^2)}{F_+(q^2)}\right|_{q^2=11.5\mathrm{GeV}^2} = 0.77(2) \quad \text{and} \quad \left.\frac{F_T(q^2)}{F_+(q^2)}\right|_{q^2=11.5\mathrm{GeV}^2} = 1.08(7)\,.$$

The above results are important for the discussion of this decay in various scenarios of physics BSM.
The same analysis has been done for the case of the non-strange decay mode and the results are fully consistent with those present for the $B_s \to D_s \ell \bar{\nu}_\ell$, but with larger statistical errors. More details can be found in [4].

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