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Semileptonic D-decays with twisted mass QCD on the lattice

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We compute the hadronic matrix elements relevant to the semileptonic $D \to \pi/K$ decays in and beyond the Standard Model. Besides the usual vector and scalar form factors $[f_{+,0}(q^2)]$, we also present the first lattice QCD estimates for the penguin induced form factor $[f_T(q^2)]$. Our results are obtained from simulations of twisted mass QCD with $N_f = 2$ dynamical quarks at several lattice spacings allowing us to take the continuum limit.

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1. Introduction

The semileptonic decays of the D meson have been actively studied since many years both in theory and in experiment. A comparison of the experimental results for the full or differential decay width with the theoretical expressions, derived within the Standard Model (SM), allows us to extract the Cabibbo–Kobayashi-Maskawa (CKM) matrix elements $|V_{cd}|$ (from $D \to \pi\ell\nu$) and $|V_{cs}|$ (from $D \to K\ell\nu$). Alternatively, one can fix the CKM couplings from the leptonic decays, or by imposing the unitarity of the CKM matrix, and search for the effects of physics beyond the Standard Model (BSM). For that research to be possible a considerable experimental progress has been achieved in the past decade, mostly thanks to the experiments at CLEO [1]), BaBar [2] and Belle [3] where the partial decay width has been accurately measured as a function of q^2, the square of the four-momentum transfered to the leptons. On the theory side the most significant progress has been made in the field of lattice QCD where in the computation of the form factors $f_{+,0}^{D\to\pi}(q^2)$ and $f_{+,0}^{D\to K}(q^2)$ the effects of dynamical quarks have been included [4].

The theoretical SM estimate of the differential decay width in the $D-$ meson rest frame reads,

$$\frac{d\Gamma(D^0 \to \pi^- \ell \overline{\nu}_\ell)}{dq^2} = \frac{G_F^2 |V_{cd}|^2}{24\pi^3} |\vec{p}_\pi|^3 \, |f_+(q^2)|^2 \,, \tag{1.1}$$

where we neglected the term proportional to the mass of the lepton squared, which is a very good approximation since no τ-lepton can emerge from this decay. The vector form factor $f_+(q^2)$ encodes the non-perturbative QCD dynamics of the decay and should be estimated by means of numerical simulations of QCD on the lattice. In extensions of the SM predicting the existence of a charged scalar boson (such as any model in the scenario with two Higgs doublets, 2HDM), a term proportional to the scalar form factor $f_0(q^2)$ is not helicity suppressed any more, and its impact on the decay width might be significant. Finally, if a tensor coupling of quarks and leptons to gauge bosons BSM is allowed then a term proportional to the tensor form factor becomes numerically significant as well. This form factor is also required for the theoretical description of the rare $D \to \pi\ell^+\ell^-$ decays.

In summary, for a full phenomenological analysis of the semileptonic decays one needs not only a reliable QCD based estimate of the form factor $f_+(q^2)$ but also that of $f_0(q^2)$ and $f_T(q^2)$. [1] The above discussion and eq. (1.1) refer to $D \to \pi\ell\nu$, and it is equally applicable to $D \to K\ell\nu$ after replacing $\pi \to K$ and $V_{cd} \to V_{cs}$.

In this note we present our preliminary results for the form factors $f_{+,0,T}^{D\to K/\pi}(q^2)$, obtained by using the QCD gauge field configurations generated by employing the (maximally) twisted mass QCD on the lattice [5] with $N_f = 2$ dynamical light flavors, produced by the European Twisted Mass collaboration (ETMC) [6]. The results presented here represent a significant improvement of the previous (partial) study made in ref. [7], namely: i) simulations performed at smaller lattice spacing have been included in the analysis, ii) smearing of the source operators has been implemented in order to increase their overlap with the ground states of the *in* and *out* hadrons; c) the more refined value of the charm quark mass of ref. [8] has been used.

[1] Semileptonic decay width derived by using a generic BSM effective lagrangian can be found in e.g. ref. [9].

This writeup is based on the statistics available at the time of the presentation at the conference. Since then, we increased the statistics and improved our analysis in several aspects. The final results will soon be presented in a separate publication.

2. Lattice and kinematical setup

To make this computation we benefited from the publicly available gauge field configurations that include $N_f = 2$ dynamical light quarks, generated by employing the twisted mass QCD action at maximal twist by the ETMC. (see refs. [6, 8]). In tab. 1 we provide the main information concerning the statistics and the parameters used in our calculations, together with the renormalization constants $Z_V(g_0^2)$ and $Z_T(g_0^2,\mu)$.

β	3.80	3.90	4.05	4.20
$L^3\times T$	$24^3\times 48$	$24^3\times 48$	$32^3\times 64$	$32^3\times 64$
# meas.	240	240	150	150
$\mu_{\rm sea}$	0.0080, 0.0110	0.0040, 0.0064, 0.0085, 0.0100	0.0030, 0.0060, 0.0080	0.0065
a [fm]	0.098(3)	0.085(3)	0.067(2)	0.054(1)
$Z_V(g_0^2)$ [10]	0.5816(2)	0.6103(3)	0.6451(3)	0.686(1)
$Z_T(g_0^2)$ [10]	0.73(2)	0.750(9)	0.798(7)	0.822(4)
μ_c [8]	0.2331	0.2150	0.1849	0.1566
μ_s [8]	0.0194	0.0177	0.0154	0.0129

Table 1: Summary of the lattices used in this work (for more information see ref. [8]). Notice that the non-perturbatively evaluated Z_T is converted to the $\overline{\rm MS}$ renormalization scheme at the scale $\mu = 2$ GeV.

As in our recent study [11], we employ a mixed-action setup. All quark propagators are computed by using stochastic sources, and in the computation of the correlation functions we used the so-called one-end trick [6]. In our kinematical setup the D-meson interpolating operator is placed at $t = T/2$ and is kept at rest ($|\vec{p}_D| = 0$), whereas the K/π-meson interpolating operator is placed at $t = 0$ and is given different values of the three-momentum in order to cover the physically relevant kinematical range, namely $q^2 \in [0,(m_D - m_{\pi/K})^2]$. The weak interaction hadronic matrix element entering the $D\to\pi\ell\nu_\ell$ decay amplitude is conveniently parameterized as

$$\langle\pi(p_\pi)|V_\mu|D(p_D)\rangle = (p_D+p_\pi)_\mu\, f_+(q^2) + q_\mu\,\frac{m_D^2-m_\pi^2}{q^2}\left[f_0(q^2)-f_+(q^2)\right], \tag{2.1}$$

where $V_\mu = \bar{c}\gamma_\mu d$, $q = p_D - p_\pi$, and the form factors $f_{+,0}^{D\to\pi}(q^2)$ are functions of $q^2\in[0,q^2_{\rm max}]$. The case $q^2 = q^2_{\rm max} \equiv (m_D - m_\pi)^2$ is peculiar, as only $f_0(q^2_{\rm max})$ can be determined. As for the matrix element of $T_{0i} = \bar{c}\sigma_{0i}d$, it is parameterized via

$$\sum_{i=1}^{3}\langle\pi(\vec{p}_\pi)|T_{0i}|D(\vec{0})\rangle = -i\frac{2m_D f_T(q^2)}{m_D+m_\pi}\,\vec{p}_\pi\,. \tag{2.2}$$

Similar relations are valid for the case of $D\to K\ell\nu$ too.

The extraction of the hadronic matrix element on the lattice proceeds through the computation of the three- and two-point correlation functions. For example, to access $\langle \pi(p_\pi)|V_\mu|D(p_D)\rangle$ we compute the three-point correlation functions:

$$C^V_\mu(\vec{q};t) = \sum_{\vec{x},\vec{y}} \langle P_{cu}(\vec{0},0) V_\mu(\vec{x},t) P^\dagger_{ud}(\vec{y},T/2) e^{-i\vec{q}(\vec{x}-\vec{y})}\rangle \,,$$

where $\vec{q} = -\vec{p}_\pi$. The interpolating source operators, $P_{cu} = \bar{\mathbf{u}}\gamma_5\mathbf{c}$ and $P_{ud} = \bar{\mathbf{u}}\gamma_5\mathbf{d}$, are defined in terms of the quark field obtained by applying the Gaussian smearing, $\mathbf{Q} = H \cdot Q$, where the operator H and the choice of smearing parameters are the same as those discussed in ref. [17]. In terms of the quark propagators the correlation function (2.3) reads

$$C^V_\mu(\vec{q};t) = \langle \sum_{\vec{x},\vec{y}} \mathrm{Tr}\left[\gamma_5 S_u(0,y;U)\gamma_5 S^{\vec{\theta}}_d(y,x;U)\gamma_\mu S_c(x,0;U)\right]\rangle \,, \tag{2.3}$$

where $y_t = T/2$, $x_t = 0$, and U denotes the background gauge field configuration. We give the light meson a three-momentum by imposing the twisted boundary condition on the d quark propagator [12, 13, 14], labelled by the superscript $\vec{\theta}$. In addition to the above three-point correlation functions, we also studied large time behavior of the two-point correlators to determine m_D, $E_\pi(\vec{p}_\pi)$, $\mathscr{Z}_D(\vec{0}) \equiv \mathscr{Z}_D$, and $\mathscr{Z}_\pi(\vec{p}_\pi)$, which are needed in order to remove the sources from the three-point correlation functions and to access the vector current matrix element. It appears that the energy of the state can be very well reconstructed from its mass by using the lattice dispertion relation as shown in [11]. We extract the desired hadronic matrix element from the ratio

$$R_\mu(t) = \frac{4m_D E_\pi(\vec{p}_\pi) C_\mu(t)}{\mathscr{Z}_D \mathscr{Z}_\pi(\vec{p}_\pi)\exp(-m_D t)\exp\left[-E_\pi(\vec{p}_\pi)(T/2-t)\right]} \xrightarrow{0 \ll t \ll T/2} \langle \pi(\vec{p}_\pi)|V_\mu|D(\vec{0})\rangle \,. \tag{2.4}$$

In a completely similar way one extracts the matrix element of the tensor density, $\langle \pi(\vec{p}_\pi)|T_{0i}|D(\vec{0})\rangle$. The above discussion can be trivially extended to the $D \to K\ell\nu$ case.

3. Getting to the physical result

We show in fig. 1 an example of the effective mass of the D-meson as obtained from the computation of the two-point correlation functions with both local or smeared sources. The matrix elements $\langle \pi(\vec{p}_\pi)|V_0|D(\vec{0})\rangle$ are extracted from the plateau of the ratio defined in eq. 2.4.Combining those with the matrix elements $\langle \pi(\vec{p}_\pi)|V_i|D(\vec{0})\rangle$, we were able to extract the form factors $f_{+,0}(q^2)$. Similarly we obtain our results for $f_T(q^2)$. Results for the tensor form factor are given by using the $\overline{\mathrm{MS}}$ scheme at $\mu = 2$ GeV. The above procedure is repeated for all of our 10 lattice ensembles. While for the $D \to K\ell\nu$ decay one needs to extrapolation only the spectator quark mass, the case of the $D \to \pi$ transition is more delicate as both the spectator and the quark entering the vertex need to be considered.

3.1 Method 1: Extrapolation at fixed q^2

We first interpolate our form factors in such a way that they correspond to several fixed values of q^2_{fix} in the continuum. Those form factors are then simultaneously extrapolated to the continuum and to the chiral limit by fitting our data to

$$f^{\mathrm{latt}}_{+,0,T}(q^2_{\mathrm{fix}}, a^2, m^2_\pi) = f_{+,0,T}(q^2_{\mathrm{fix}}, 0, 0)\left(1 + \alpha\, a^2 + \beta m^2_\pi\right) \,, \tag{3.1}$$

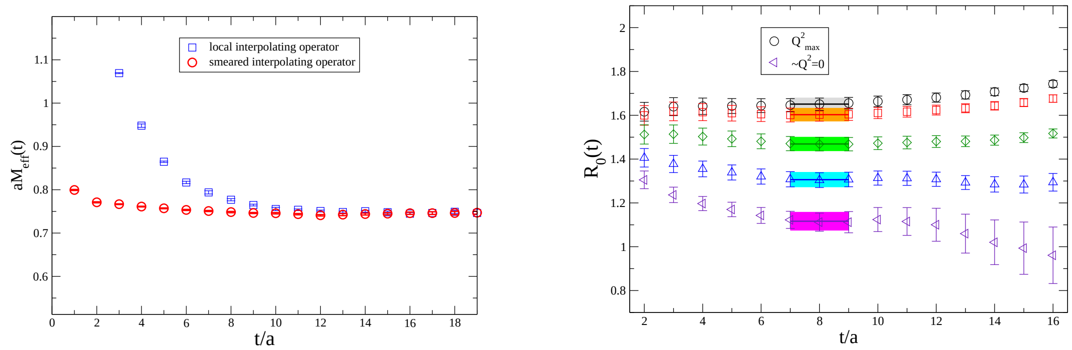

Figure 1: Left panel: effective mass of the two points correlation function for the D meson. Right panel: extraction of the $\langle \pi | V_0 | D \rangle$ matrix element according to eq 2.4. The examples are shown is for $L = 24$, $\beta = 3.90$, $\mu_{sea} = 0.0085$.

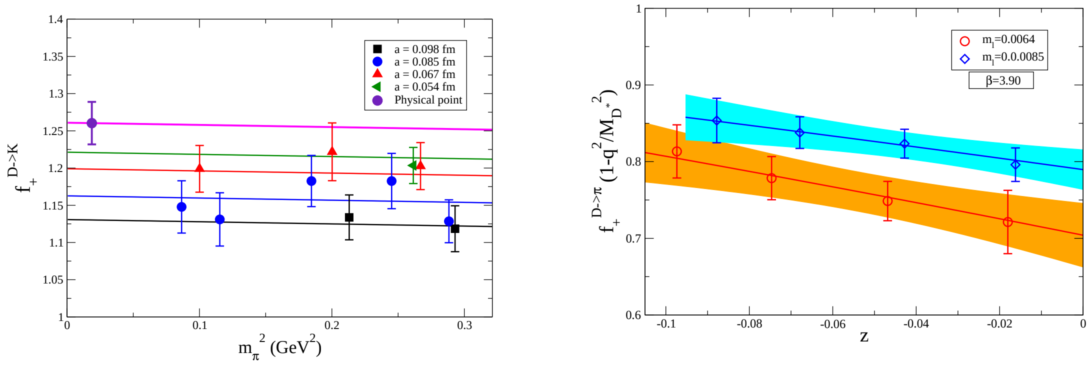

Figure 2: Left panel: chiral and continuum extrapolation of the $f_+^{D\to\pi}$ form factor at a fixed value of q^2, according to a linear polynomial in a^2 and m_π^2, cf. eq. (3.1). Right panel: linear fit of $F_+^{D\to\pi}(z)$, defined as in eq. (3.2) for a particular value of the lattice spacing and for two different values of the light sea quark mass.

where a is the lattice spacing and $f_{+,0,T}(q^2_{\text{fix}}) \equiv f_{+,0,T}(q^2_{\text{fix}}, 0, (m_\pi^{phys})^2)$ is our desired result. In this way we were able to cleanly disentangle the chiral and continuum extrapolation from the study of the shape of the form factor as a function of q^2. However, since the range of three-momenta needed to cover the entire physical region $q^2 \in [0, q^2_{\text{max}}]$ grows with the decrease of the pion mass and of the lattice spacing, with this method we cannot cover the whole range of physical q^2's needed for $D \to \pi \ell \nu$ decay in the continuum. An example of the simultaneous chiral and continuum extrapolation of the form factor $f_+^{D\to\pi}(q^2_{\text{fix}})$ for $q^2_{\text{fix}} = 1.35\ \text{GeV}^2$ is shown in the left panel of fig. 2.

3.2 Method 2: Extrapolation of the parameters

An alternative approach consist in discussing the dependence of the form factors on the variable z, which is related to q^2 via, $z = (\sqrt{t_+ - q^2} - \sqrt{t_+})/(\sqrt{t_+ - q^2} + \sqrt{t_+})$, where $t_+ = (m_D + m_\pi)^2$. There are several ways to parameterize the dependence of the form factors on z. One of them was recently proposed in ref. [15] in which the authors factored out the contribution coming from the nearest pole, and then parameterize the rest as a polynomial in z, defining

$$F_{+,T}(q^2) \equiv f_{+,T}(q^2) \cdot \left(1 - \frac{q^2}{m^2_{1^-}}\right), \qquad F_0(q^2) \equiv f_0(q^2) \cdot \left(1 - \frac{q^2}{m^2_{0^+}}\right), \tag{3.2}$$

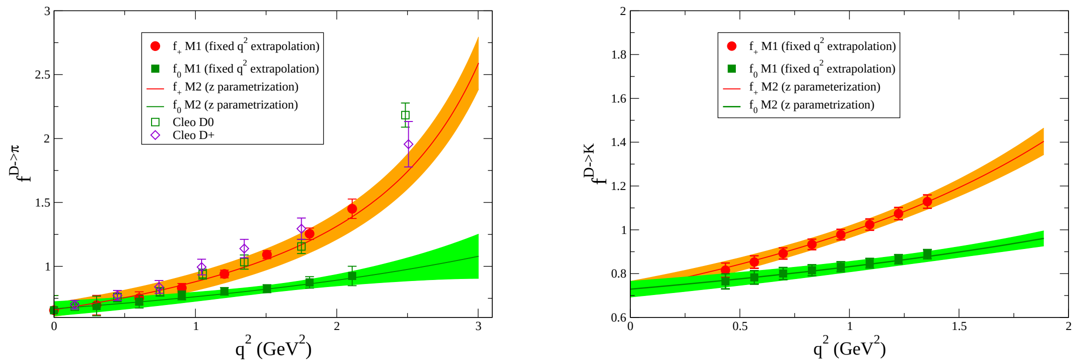

Figure 3: Form factors $f_{+,0}^{D\to\pi}$ (left panel) and $f_{+,0}^{D\to K}$ (right panel) extrapolated to the continuum limit and m_π^{phys}. For the case of $f_+^{D\to\pi}$ we compare also with the experimental results from CLEO [1].

where $m_{1^-,0^+}$ are the masses of the lowest vector/scalar meson exchanged in the t-channel ($m_{1^-} = m_{D^*}$ and $m_{0^+} = m_{D_0^*}$ in the case of $D\to\pi$). The lattice data $F_{+,0,T}(z)$, for each of our 10 ensembles, are then fit to a polynomial in z, namely $F_i(z) = c_0^{(i)}(a^2,m_\pi^2)+\cdots+c_n^{(i)}(a^2,m_\pi^2)z^n$. In the present study we limited our analysis to $n=1$ since that adequately describes our data. In the right panel of fig. 2 we show an example of $F_+^{D\to\pi}(z)$ as a linear function of z for two different values of the sea quark mass. The coefficients $c_i(a^2,m_\pi^2)$ can be extrapolated to the continuum limit in a way similar to eq. (3.1). The resulting form factor is converted back to its q^2-dependence and we arrive to the analytical parameterization of the form factors in the continuum and at the physical pion mass.

For the case of the tensor form factor we adopt a slightly different procedure and consider the ratio $R_{T/V}(q^2) = f_T(q^2)/f_+(q^2)$, since the nearest pole contributing to the both form factors is the same vector meson D^*, and since that contributions cancels in the ratio to a large extent it is interesting to check for the flatness of $R_{T/V}(q^2)$.

4. Preliminary results

In fig. 3 we show the comparison of the form factors $f_{+,0}(q^2)$ as obtained by using the two methods discussed above, for both $D\to\pi$ and $D\to K$ form factors. The agreement between the two methods is good, and the $f_+^{D\to\pi}$ agrees well with experimental data reported by CLEO [1]. The z parametrization allows to cover the whole kinematical range available in the continuum at the physical mass of the pion. Similarly, one can use any model to parameterize the form factors obtained at various fixed q^2's in the continuum limit. We believe we are also in a unique position to check for the difference between the $f_+^{D\to\pi}(q^2)$ and the contribution of the first pole. To that end we remind the reader that the first pole contribution to the vector form factor reads,

$$f_+^{\text{pole}}(q^2) = \frac{1}{2}\frac{f_{D^*}m_{D^*}g_{D^*D\pi}}{m_{D^*}^2-q^2}\,. \tag{4.1}$$

More specifically, in ref. [16] we found $f_{D^*} = 278(16)$ MeV, while in ref. [17] we obtained $g_{D^*D\pi^\pm} = 15.8(8)$. In the left panel of fig. 4 we show a comparison of the results of the vector form factor $f_+^{D\to\pi}(q^2)$ with with its nearest pole contribution (vector meson dominance, VMD). We see that

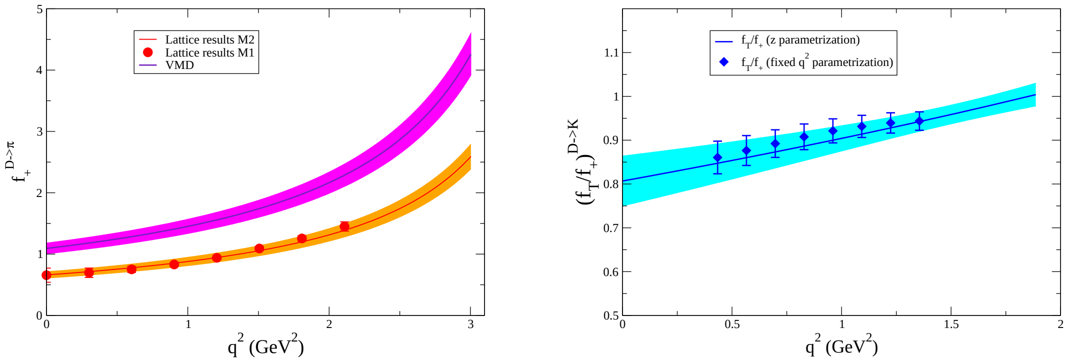

Figure 4: Left panel: comparison of the $D \to \pi$ vectorial form factor with the prediction of Vector Meson Dominance model. Right panel: result of the ratio between tensorial and vectorial form factor for $D \to K$ decay.

the first pole does not saturate the form factor at all, and that the contributions coming from other singularities are significant and, in total, negative.

We present in fig. 4 a comparison of the ratio $R_{T/V}(q^2)^{D\to\pi}$ as obtained with two methods discussed above. The determination of the tensor form factor (not presented in any lattice studies of D-decays before) indicates a slight slope of the ratio $R_{T/V}(q^2)^{D\to\pi}$, which is a departure from the flat behavior that can be deduced from the consideration in the heavy quark effective theory.

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