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# Bottomonium and B results from full lattice QCD

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We have developed two methods for handling $b$ quarks in lattice QCD. One uses NRQCD (now improved to include radiative corrections) and the other uses Highly Improved Staggered Quarks (HISQ), extrapolating to the $b$ quark from lighter masses and using multiple lattice spacings to control discretisation errors. Comparison of results for the two different methods gives confidence in estimates of lattice QCD systematic errors, since they are very different in these two cases. Here we show results for heavyonium hyperfine splittings and vector current-current correlator moments using HISQ quarks, to add to earlier results testing the heavy HISQ method with pseudoscalar mesons. We also show the form factor for $B \to \pi l \nu$ decay at zero recoil using NRQCD $b$ quarks and $u/d$ quarks with physical masses. This allows us to test the soft pion theorem relation ($f_0(q^2_{max}) = f_B/f_\pi$) accurately and we find good agreement as $M_\pi \to 0$.

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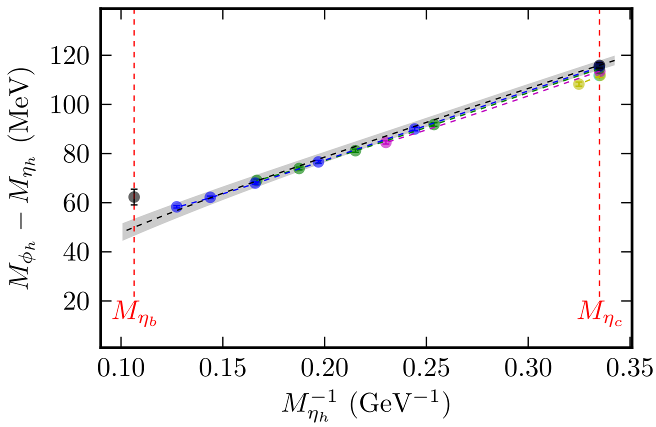


**Figure 1:** The heavyonium hyperfine splitting as a function of the inverse heavyonium mass. Results are given in different colours for coarse ($a$=0.12fm), fine ($a$=0.09fm), superfine ($a$=0.06fm) and ultrafine ($a$=0.045fm) lattices, including 2+1 flavours of sea quarks. The shaded band shows the fit described in the text and the black points at the $b$ and $c$ the current experimental averages [4]. Note that $\eta_b$ and $\eta_c$ annihilation effects are *not* included in the lattice results or the shaded fit.

## 1. Vector Heavyonium - hyperfine splitting

Fig. 1 shows the heavyonium vector-pseudoscalar (hyperfine) mass splitting as a function of inverse heavyonium mass for HISQ quarks [1] for a range of masses from charm upwards. We have used MILC gluon field configurations that include 2+1 flavours of asqtad sea quarks with lattice spacings ranging from 0.12 fm to 0.045 fm [2]. The finer lattice spacings allow a much larger reach in meson mass because a given meson mass corresponds to a smaller quark mass in lattice units. We have used quark masses in lattice units up to 0.8. Having so many values of the lattice spacing allows the discretisation errors to be well determined from the fit. The grey band shows the result of the physical heavy quark mass dependence determined at zero lattice spacing resulting from a fit function of form [3]:

$$F(M,a) = A\left(\frac{M}{M_0}\right)^b \sum_{i=0}^{7}\sum_{j=0}^{3} c_{ij}\left(\frac{M_0}{M}\right)^i (am)^{2j} \qquad (1.1)$$

where $M$ is the pseudoscalar meson mass and $m$ the quark mass and we also include terms allowing for sea quark mass dependence (which are then extrapolated to the physical point). Priors on the coefficients are generally taken as $0(1)$ and $M_0$ is taken as 1 GeV.

The result at the $b$, after an increase of 3(3) MeV for $\eta_b$ annihilation effects not included in our calculation (and also NOT included in the plot) is

$$M_\Upsilon - M_{\eta_b} = 53(5)\text{MeV}. \qquad (1.2)$$

This is in reasonable agreement with the current experimental average of 62.3(3.2) MeV [4]. The result we obtain at $c$ is 116.5(3.2) MeV [6], in good agreement with the experimental average of 113.2(7) MeV [4]. We are currently extending these calculations to the MILC 'second-generation' configurations that include 2+1+1 flavours of HISQ sea quarks [5].

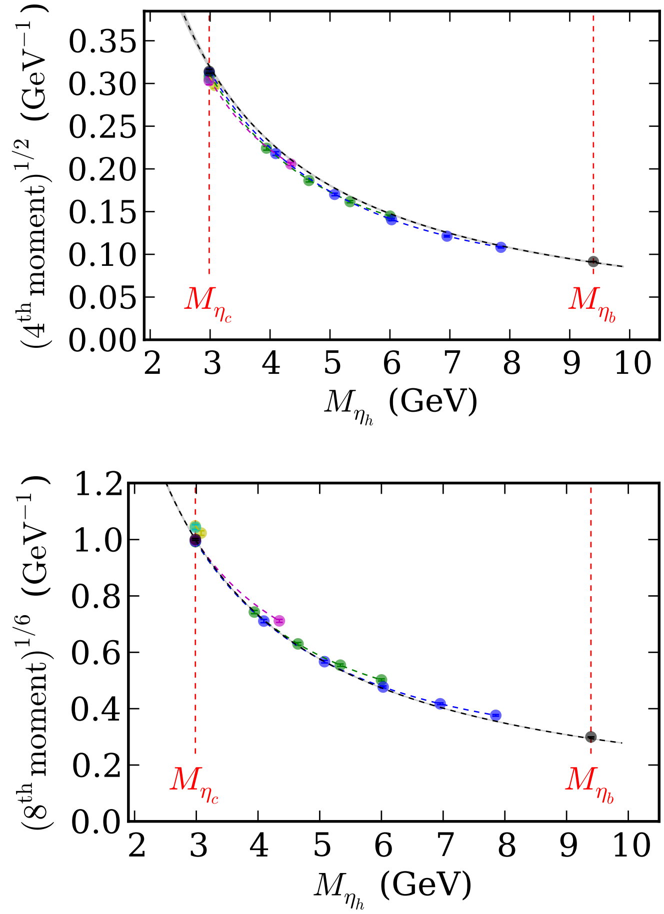


**Figure 2:** The 4th and 8th moments of the heavyonium vector current-current correlator as a function of heavyonium mass. Results are given for coarse ($a$=0.12fm), fine ($a$=0.09fm), superfine ($a$=0.06fm) and ultrafine ($a$=0.045fm) lattices, including 2+1 flavours of sea quarks. The shaded band shows the fit described in the text and the black points at the $b$ and $c$ are the results derived from experiment [8].

We have also calculated the bottomonium hyperfine splitting using NRQCD for the $b$ quark [7]. We have included spin-dependent relativistic corrections through $\mathcal{O}(v^6)$, radiative corrections to the leading spin-chromomagnetic coupling (at $\mathcal{O}(v^4)$) and, for the first time, non-perturbative 4-quark interactions. We use the MILC configurations that include 2+1+1 flavours of HISQ sea quarks. We obtain a splitting of 62.8(6.7) MeV [7], again in good agreement with experiment.

## 2. Vector Heavyonium - current-current correlator moments

Time moments of the vector heavyonium correlator are defined by:

$$G_n^V = Z^2 \sum_{\tilde{t}} \tilde{t}^n C_V(\tilde{t}) \tag{2.1}$$

where $\tilde{t}$ is the lattice time variable, symmetrised around the centre of the lattice and $Z$ is the current renormalisation factor, derived from continuum QCD perturbation theory for the 6th moment [6].

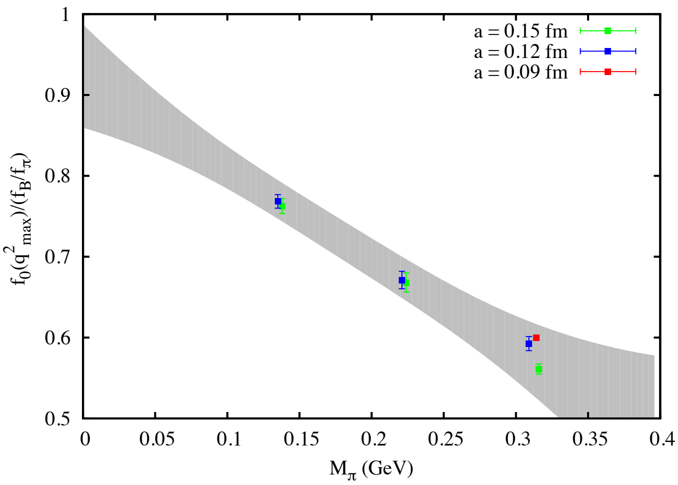


**Figure 3:** The ratio of $f_0(q^2_{max})$ for $B \to \pi l \nu$ decay to $f_B/f_\pi$ as a function of $M_\pi$. The points are lattice results, at 3 values of the lattice spacing, with the leftmost points being at the physical value of $M_\pi$. The shaded band shows a simple extrapolation through the two most chiral sets of points to $M_\pi = 0$ and $a = 0$.

The lattice time-moments can be compared to $q^2$-derivative moments of the heavy quark vacuum polarisation. Values for these can be extracted [8] from experimental results in the $c$ and $b$ regions for $R_{e^+e^-}$ where

$$R_{e^+e^-} = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma_{\text{point}}}. \tag{2.2}$$

The two plots in Fig. 2 show our results for the 4th and 8th moments, calculated with HISQ quarks, as a function of heavyonium mass. Again we have results for a wide range of lattice spacing values, from 0.12 fm to 0.045 fm, on the MILC gluon field configurations that include 2+1 flavours of asqtad sea quarks.

The shaded bands show the results of fits to the form given above for the hyperfine splitting (Eq. 1.1) but with leading power, $b$=-1. The results show excellent agreement with experiment, both at $c$ [6] and at $b$. The experimental values [8] are shown as the black points. This is a stringent test of lattice QCD since experimental errors are small.

## 3. $B \to \pi l \nu$ decay and soft pion theorems

The matrix element of the temporal vector current for $B \to \pi$ decay at zero recoil (both mesons at rest) is given by:

$$\langle \pi | V^4 | B \rangle = f_0(q^2_{max})(M_B + M_\pi). \tag{3.1}$$

Soft pion theorems relate this to decay constants. At leading order, for $M_\pi \to 0$,

$$f_0(q^2_{max}) = \frac{f_B}{f_\pi}. \tag{3.2}$$

This result seemed not to hold well in the quenched approximation (see [9] for a review), but large uncertainties arose from large pion masses (along with the absence of significant pion mass dependence), quenching and current renormalisation.

Here we have small uncertainties because we are using improved NRQCD $b$ quarks combined with HISQ light quarks. We go down to physical pion masses on the MILC configurations that include 2+1+1 flavours of HISQ sea quarks. $Z$ factors cancel between $f_0$ and $f_B$ from staggered chiral symmetry. We include $\mathscr{O}(\Lambda/m_b)$ relativistic corrections to axial and vector currents.

The plot in Fig. 3 shows the ratio of $f_0$ to the decay constant ratio as a function of $M_\pi$. The decay constants are the current lattice state-of-the-art results obtained in [10] and [11]. We see the ratio is around 0.75, quite far from 1, for physical $\pi$ masses. However, there is clearly strong dependence on $M_\pi$. Using an extrapolation form that includes linear terms in $M_\pi$ (coming from the dependence of $q^2_{max}$ on $M_\pi$ [12]) readily gives a result in agreement with 1 in the $M_\pi \to 0$ limit.

We are currently calculating form factors for $B/B_s$ semileptonic decays using HISQ quarks and extrapolating to the $b$ as described above. This will give us another opportunity to test systematic errors by comparing the two formalisms.

## References

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