

## Continuum limit of the axial anomaly and index for the staggered overlap Dirac operator: An overview

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Evaluation of the continuum limit of the axial anomaly and index is sketched for the staggered overlap Dirac operator. There are new complications compared to the usual overlap case due to the distribution of the spin and flavor components around lattice hypercubes in the staggered formalism. The index is found to correctly reproduce the continuum index, but for the axial anomaly this is only true after averaging over the sites of a lattice hypercube.

*31st International Symposium on Lattice Field Theory - LATTICE 2013*

*July 29 - August 3, 2013*

*Mainz, Germany*

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## 1. Introduction

Overlap fermions, as a massless lattice fermion formulation, have exact zero-modes with definite chirality and hence a well-defined index [1]. Furthermore, they satisfy [1] the Ginsparg-Wilson (GW) relation [2] and therefore have an exact chiral symmetry which can be identified with the axial  $U(1)$  symmetry of continuum massless Dirac fermions [3]. The latter is broken by quantum effects – specifically by the fermion measure á la Fujikawa [4] – so there is an axial anomaly. It is proportional to an index density for the index of the overlap Dirac operator, just like in the continuum. (The continuum anomaly is proportional to the topological charge density, whose integral – the topological charge of the background gauge field – is equal to the index by the Index Theorem.) An important test of the overlap lattice fermion formulation is then to show that the continuum anomaly and index are reproduced in the continuum limit in smooth gauge field backgrounds. This has been verified in a number of papers at various levels of rigor [2, 5, 6, 7], and also in a more general setting where the kernel operator in the overlap formulation is a more general hypercubic lattice Dirac operator [8].

The advantageous theoretical properties of overlap fermions are offset by the high cost of implementing them in numerical simulations of lattice QCD. Recently, a staggered version of the overlap fermions, describing 2 fermion species (flavors), was introduced in [9] as a further development of the spectral flow approach to the staggered fermion index in [10]. The fermion field in this case is a one-component field (no spinor indices) just like for staggered fermions. This offers the prospect of theoretical advantages of overlap fermions at a cheaper cost.<sup>1</sup>

To establish a secure theoretical foundation for staggered overlap fermions, one of the things that needs to be done is to verify that it reproduces the continuum anomaly and index in the continuum limit in smooth gauge field backgrounds, just like for usual overlap fermions. The purpose of this paper is to sketch the verification of this. Full details will be given in a later article.<sup>2</sup> There are new complications for evaluating the continuum limit of the anomaly and index in this case compared to the usual overlap case, due to the fact that the spin and flavor components are distributed around the lattice hypercubes in the staggered formalism. It turns out that, although the index correctly reproduces the continuum index, the axial anomaly only reproduces the continuum anomaly after averaging over the sites of a lattice hypercube. This is not surprising, since the sites around a lattice hypercube can be regarded as corresponding to the same spacetime point in the staggered formalism.

We focus on the original 2-flavor version of staggered overlap fermions introduced in [9]. A 1-flavor version of staggered Wilson fermions was later introduced in [14] and can be used as kernel to make a 1-flavor version of staggered overlap fermions. However, this formulation has the drawback of breaking lattice rotation symmetry; a new gluonic counterterm then arises which needs to be included in the bare action and fine-tuned to reproduce continuum QCD, thus reducing the attractiveness of this formulation for practical use. Nevertheless, it correctly reproduces the

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<sup>1</sup>An exploratory investigation of the cost of staggered overlap fermions was made in [11]. More recently, the cost of staggered Wilson fermions was investigated in [12]; the results there give positive indications for the cost effectiveness of staggered overlap fermions on larger lattices.

<sup>2</sup>Numerical checks that the staggered overlap Dirac operator has the correct index in smooth backgrounds have already been done in [13] (for lattice transcripts of instanton gauge fields) and in [11].

continuum axial anomaly and index; this can be shown by a straightforward modification of our arguments here for the 2-flavor case; the details will be given in the later article.

## 2. The staggered overlap Dirac operator, its index, and the axial anomaly

The staggered overlap Dirac operator  $D_{so}$  is given by

$$D_{so} = \frac{1}{a} \left( 1 + A/\sqrt{A^\dagger A} \right), \quad A = D_{sW} - \frac{m}{a} \quad (2.1)$$

where  $D_{sW}$  is the staggered Wilson Dirac operator, obtained by adding a ‘‘Wilson term’’ to the staggered fermion action as discussed below (see (3.2)). The staggered Wilson term reduces the number of fermion species (flavors) described by the staggered fermion from 4 to 2 (the other 2 species get masses  $\sim 1/a$  and become doublers) [9]. Consequently, for suitable choice of  $m$  in (2.1), the staggered overlap fermion describes 2 physical fermion flavors. Specifically, the requirement is  $0 < m < 2$  just like for usual overlap fermions.

The role of unflavored  $\gamma_5$  in this setting is played not by the unflavored  $\gamma_5$  of the staggered fermion but by the *flavored*  $\gamma_5$  which gives the exact flavored chiral symmetry of the staggered fermion action, which we denote by  $\Gamma_{55}$ . This notation reflects the fact that it corresponds to  $\gamma_5 \otimes \xi_5$  in the spin-flavor interpretation of staggered fermions by Golterman and Smit [15], where  $\{\xi_\mu\}$  is a representation of the Dirac algebra in flavor space.<sup>3</sup> The use of  $\Gamma_{55}$  as the unflavored  $\gamma_5$  in the staggered versions of Wilson, domain wall and overlap fermions is justified by the fact that it acts in an unflavored way on the physical fermion species, since  $\xi_5 = 1$  on these [9].

With  $\Gamma_{55}$  playing the role of unflavored  $\gamma_5$ , the requirements  $\gamma_5^2 = 1$  and  $\gamma_5$  hermiticity of the staggered Wilson Dirac operator  $D_{sW}$  are satisfied, and hence the staggered overlap Dirac operator satisfies the the Ginsparg-Wilson (GW) relation [2], which in this case is

$$\Gamma_{55} D_{so} + D_{so} \Gamma_{55} = a D_{so} \Gamma_{55} D_{so}. \quad (2.2)$$

Moreover, (2.1) can be rewritten as

$$D_{so} = \frac{1}{a} \left( 1 + \Gamma_{55} H / \sqrt{H^2} \right) \quad (2.3)$$

where

$$H = H_{sW}(m) = \Gamma_{55} (D_{sW} - m/a) \quad (2.4)$$

is a hermitian operator.

The index of  $D_{so}$  is then determined by the spectral flow of  $H_{sW}(m)$  just like in the usual overlap case, and can be computed from the index formula in the 2nd paper of [2]:

$$\text{index } D_{so} = -\frac{1}{2} \text{Tr} \left( H / \sqrt{H^2} \right). \quad (2.5)$$

Here Tr is the operator trace for operators on the space of lattice staggered fermion fields, i.e. functions living on the lattice sites and taking values in the fundamental representation of the color gauge group SU(3) (color indices but no spinor indices).

<sup>3</sup> $\Gamma_{55}$  is denoted by  $\varepsilon$  in [15].

The lattice spacetime is the usual hypercubic discretization of a 4-dimensional box of fixed length  $L$ , with  $N$  sites along each axis so that the lattice spacing is  $a = L/N$ . The link variable associated with a lattice link  $[x, x + a\mu]$  is denoted  $U_\mu(x)$  or  $U(x, x + a\mu)$ . (The index  $\mu$  is also used to denote the unit vector along the positive  $\mu$ -axis.) The link variables are taken to be the lattice transcripts of a smooth continuum gauge field  $A_\mu(x)$  taking values in the Lie algebra of  $SU(3)$ . Boundary conditions are imposed as in [7] by requiring fields at opposite ends of the box to be related by gauge transformations in such a way that  $A_\mu(x)$  can be topologically nontrivial with topological charge  $Q \in \mathbb{Z}$  (see [7] for the details). The box contains finitely many lattice sites, so the vector space of lattice fermion fields is finite-dimensional and hence the index (2.5) is well-defined for all choices of  $m$  for which  $H_{sW}(m)$  does not have zero-modes. In the following we restrict for simplicity to the physical choices where  $0 < m < 2$ .

From (2.5) the index can be expressed as a sum over lattice sites (“lattice spacetime integral”) of a density  $q(x)$ :

$$\text{index } D_{so} = \sum_x a^4 q(x), \quad q(x) = -\frac{1}{2} \text{tr}_c \left( \frac{H}{\sqrt{H^2}} \right) (x, x) \quad (2.6)$$

where  $\text{tr}_c$  denotes the trace over color indices. Here we are using the concept of operator density  $\mathcal{O}(x, y)$ , defined by  $\mathcal{O}\chi(x) = \sum_y a^4 \mathcal{O}(x, y)\chi(y)$ , which can be used to express the operator trace as  $\text{Tr } \mathcal{O} = \sum_x a^4 \text{tr}_c \mathcal{O}(x, x)$ .

The index density  $q(x)$  is seen to be closely related to the axial anomaly in the same way as for usual overlap fermions [3]: The staggered overlap action  $\bar{\chi} D_{so} \chi$  is invariant under the axial  $U(1)$  transformations  $\delta\chi = \Gamma_{55}(1 - aD_{so})\chi$ ,  $\delta\bar{\chi} = \bar{\chi}\Gamma_{55}$ . The corresponding axial current fails to be conserved: its divergence fails to vanish, and is found in the massless limit to be  $\mathcal{A}(x) = 2iq(x)$ , which is by definition the anomaly.

In the usual overlap case, the main step in showing that both the index and axial anomaly have the correct continuum limits is to show that the index density  $q(x)$  reproduces the continuum topological charge density [6, 7]:

$$\lim_{a \rightarrow 0} q(x) = -\frac{1}{32\pi^2} \varepsilon_{\mu\nu\sigma\rho} \text{tr}_c (F_{\mu\nu}(x) F_{\sigma\rho}(x)) \quad (2.7)$$

Here  $x$  is fixed point in the spacetime box, and the  $a \rightarrow 0$  limit is taken by repeated subdivisions of the lattice such that  $x$  continues to be a lattice site of all the subdivided lattices. However, in the present case, it turns out that (2.7) only holds after  $q(x)$  is averaged over the sites of a lattice hypercube containing  $x$ . Note that the sites of the hypercube all converge to  $x$  in the continuum limit, so this situation is not unreasonable, and is moreover not surprising in the staggered formalism as mentioned in the Introduction.

Our goal in the remainder of this paper is to sketch the derivation of the modified version of (2.7), i.e. with  $q(x)$  averaged over the sites of a lattice hypercube containing  $x$ , and with an extra factor of 2 due to the two physical fermion flavors described by the staggered overlap fermion.

A further technical result is required to conclude that the index expression (2.6) converges to the continuum topological charge  $2Q$  (which equals the continuum Dirac index with 2 flavors by the continuum index theorem): It needs to be shown that the convergence in (2.7) is uniform in  $x$ . This can be shown in the same way as in the usual overlap case in [7]; we omit the details here.

### 3. Continuum limit of the lattice index density

By the same arguments as in the usual overlap case [7] we have

$$\lim_{a \rightarrow 0} \frac{H}{\sqrt{H^2}}(x, x) = \lim_{a \rightarrow 0} \int_{-\pi/2a}^{3\pi/2a} d^4 p e^{-ipx} \frac{H}{\sqrt{H^2}} e^{ipx}. \quad (3.1)$$

Now decompose the lattice momentum  $p \in [-\pi/2a, 3\pi/2a]^4$  as  $p = \frac{\pi}{a}A + q$ , where  $q \in [-\pi/2a, \pi/2a]^4$  and  $A = (A_1, A_2, A_3, A_4)$  with  $A_\mu \in \{0, 1\}$ . In the following, if  $A$  and  $B$  are two such vectors, we also consider  $A + B$  to be a vector of this type with the components  $(A + B)_\mu \in \{0, 1\} \bmod 2$ . Let  $V$  be the vectorspace spanned by the 16 plane waves  $\{e_A(x) \equiv e^{i\frac{\pi}{a}Ax}\}$ . In [15] two commuting representations of the Dirac algebra are defined on  $V$ , namely  $\{\hat{\Gamma}_\mu\}$  and  $\{\hat{\Xi}_\nu\}$ , given by  $(\hat{\Gamma}_\mu)_{AB} = (-1)^{A_\mu} \delta_{A, B + \eta_\mu}$  and  $(\hat{\Xi}_\nu)_{AB} = (-1)^{A_\nu} \delta_{A, B + \zeta_\nu}$  where  $\eta_\mu$  and  $\zeta_\nu$  are vectors whose components are all zero except  $(\eta_\mu)_\sigma = 1$  for  $\sigma < \mu$  and  $(\zeta_\nu)_\sigma = 1$  for  $\sigma > \nu$ .<sup>4</sup>

The staggered Wilson Dirac operator [9] is  $D_{SW} = D_{st} + W_{st}$  where  $D_{st}$  is the usual (massless) staggered Dirac operator and  $W_{st} = \frac{r}{a}(1 - \Xi_5)$  is the Wilson term. Here  $\Xi_5 (= \Gamma_{55}\Gamma_5)$  is an extension of  $\hat{\Xi}_5$  from  $V$  to the space of staggered lattice fermion fields  $\chi(x)$ , described in [9]. For present purposes it suffices to note that the action of the staggered Wilson Dirac operator on a plane wave  $e^{ipx}$  can be expressed as follows. Let  $T_{+\mu}$  denote the parallel transporter given by  $T_{+\mu}\chi(x) = U(x, x + a\mu)\chi(x + a\mu)$ , and let  $T_{-\mu}$  denote its inverse. The symmetrized covariant derivative is  $\nabla_\mu = \frac{1}{2}(T_{+\mu} - T_{-\mu})$ , and we define  $C_\mu = \frac{1}{2}(T_{+\mu} + T_{-\mu})$ . We will also need the symmetrized product of the  $C_\mu$ s, namely  $C_5 \equiv \frac{1}{4!} \sum C_\mu C_\nu C_\sigma C_\rho$  where the sum is over all permutations  $\{\mu, \nu, \sigma, \rho\}$  of  $\{1, 2, 3, 4\}$ . Then, writing the plane wave as  $e^{ipx} = e_B(x)e^{iqx}$ , the action of  $D_{SW}$  on it can be expressed as

$$\begin{aligned} D_{SW}(e_B(x)e^{iqx}) &= e_A(x) \left( (\hat{\Gamma}_\mu)_{AB} \nabla_\mu + \frac{r}{a} \left( \delta_{AB} - (\hat{\Xi}_5)_{AB} C_5 \right) \right) e^{iqx} \\ &= e_A(x) \left( \hat{\Gamma}_\mu \nabla_\mu + \frac{r}{a} \left( \mathbf{1} - \hat{\Xi}_5 C_5 \right) \right)_{AB} e^{iqx} \equiv e_A(x) (\tilde{D}_{SW})_{AB} e^{iqx} \end{aligned} \quad (3.2)$$

Here and in the following there is an implicit sum over repeated indices. This includes the vector  $A$  which is regarded as an index with 16 possible values.

Since  $H$  is built from  $D_{SW}$  and  $\Gamma_{55}$ , and the latter acts on plane waves by  $\Gamma_{55}(e_B(x)e^{iqx}) = e_A(x)(\hat{\Gamma}_5 \hat{\Xi}_5)_{AB} e^{iqx}$ , it follows that the action of  $H/\sqrt{H^2}$  on a plane wave has the same structure as in (3.2):

$$\frac{H}{\sqrt{H^2}}(e_B(x)e^{iqx}) = e_A(x) \left( \frac{\tilde{H}}{\sqrt{\tilde{H}^2}} \right)_{AB} e^{iqx} \quad (3.3)$$

where  $\tilde{H}$  is obtained from  $H$  by replacing  $D_{SW} \rightarrow \tilde{D}_{SW}$  and  $\Gamma_{55} \rightarrow \hat{\Gamma}_5 \hat{\Xi}_5$ .

Using the preceding to evaluate (3.1), we get

$$\begin{aligned} \lim_{a \rightarrow 0} \frac{H}{\sqrt{H^2}}(x, x) &= \lim_{a \rightarrow 0} \sum_B \int_{-\pi/2a}^{\pi/2a} d^4 q e^{-iqx} e_B(x) \frac{H}{\sqrt{H^2}} e_B(x) e^{iqx} \\ &= \lim_{a \rightarrow 0} \sum_{A, B} e^{i\frac{\pi}{a}(A-B)x} \int_{-\pi/2a}^{\pi/2a} d^4 q e^{-iqx} \left( \frac{\tilde{H}}{\sqrt{\tilde{H}^2}} \right)_{AB} e^{iqx} \end{aligned} \quad (3.4)$$

<sup>4</sup>The hats on  $\hat{\Gamma}_\mu$  and  $\hat{\Xi}_\nu$  are not present in the notation of [15]. We include them here, since we use the unhatted versions to denote the extensions of these operators from  $V$  to the space of one-component lattice spinor fields  $\chi(x)$ .

We will show below that (i) the contribution to  $q(x)$  from the terms in (3.4) with  $A = B$  reproduces the continuum topological charge density (2.7), and (ii) the contributions from the terms with  $A \neq B$  vanish after averaging  $q(x')$  over the sites  $x'$  in a lattice hypercube containing  $x$ . This will complete the derivation of the averaged version of (2.7).

The sum over  $A, B$  with  $A = B$  in (3.4) gives

$$\lim_{a \rightarrow 0} \int_{-\pi/2a}^{\pi/2a} d^4 q e^{-iqx} \text{Tr} \left( \frac{\tilde{H}}{\sqrt{\tilde{H}^2}} \right) e^{iqx} \quad (3.5)$$

where  $\text{Tr}$  denotes the trace of a linear operator (matrix) on the vectorspace  $V$  spanned by the  $e_A(x)$ 's. As shown in [15], there is a (non-unique) isomorphism  $V \simeq \mathbb{C}^4 \otimes \mathbb{C}^4$  such that  $\hat{\Gamma}_\mu \simeq \gamma_\mu \otimes \mathbf{1}$  and  $\hat{\Xi}_v \simeq \mathbf{1} \otimes \xi_v$ , where  $\{\gamma_\mu\}$  and  $\{\xi_v\}$  are Dirac matrices on spinor space and flavor space, respectively. After choosing a basis for flavor  $\mathbb{C}^4$  such that  $\xi_5$  is diagonal, (3.2) gives

$$\tilde{D}_{sW} = (\gamma_\mu \otimes \mathbf{1}) \nabla_\mu + (\mathbf{1} \otimes \mathbf{1}) \frac{r}{a} (1 \mp C_5) \quad (3.6)$$

with sign  $\mp$  for the flavors on which  $\xi_5 = \pm \mathbf{1}$ . Thus, on each of the two 2-dimensional flavor subspaces on which  $\xi_5 = \pm \mathbf{1}$ ,  $\tilde{D}_{sW}$  is a hypercubic lattice Dirac operator of the form considered in [8] (note that  $C_5$  couples opposite corners of lattice hypercubes). Also,  $\Gamma_{55}$  becomes  $\pm(\gamma_5 \otimes \mathbf{1})$  in this case.

The free field momentum representation of  $C_5$  is  $C_5(aq) = \prod_\mu \cos(aq_\mu) = 1 + O(a^2)$ . Using this, we see from (3.6) that  $\tilde{D}_{sW}$  describes one physical fermion species for each of the two flavors of the flavor subspace with  $\xi_5 = \mathbf{1}$ . Now note that in this case (3.5) is precisely  $\text{tr}(H/\sqrt{H^2})(x, x)$  for  $H = (\gamma_5 \otimes \mathbf{1})(\tilde{D}_{sW} - m)$ . Hence it is given by the general result of [8] on the continuum limit of the anomaly and index for lattice hypercube fermions. In (3.5) the part of the trace  $\text{Tr}$  over the flavor subspace is trivial and just produces a factor 2, leaving the trace over spinor space. The general result of [8] then implies that this contribution of (3.5) to  $q(x)$  gives the continuum topological charge density (2.7) as claimed, with an extra factor of 2 for the two physical fermion flavors.

On the other hand, the contribution of (3.5) coming from the other 2-dimensional flavor subspace on which  $\xi_5 = -\mathbf{1}$  vanishes. This can also be inferred from the general result of [8], since in this case the sign  $\mp$  is  $+$  in (3.6) and hence there are no physical fermion species.

It remains to show that the contributions to (3.4) from the terms in the sum over  $A, B$  with  $A \neq B$  vanish when averaged over  $x \in \{\text{a lattice hypercube}\}$ . Writing  $x = an$ ,  $n \in \mathbb{Z}^4$ , we have  $e^{i\frac{\pi}{a}(A-B)x} = (-1)^{(A-B)n}$ . It is easy to see that summing this over the sites  $x$  of a lattice hypercube gives zero if at least one of the components of  $A - B$  is nonzero (mod 2). Consequently, the problem of showing that the hypercube-averaged terms with  $A \neq B$  in (3.4) vanish is reduced to showing a property of the  $x$ -dependence of the integral there,

$$\int_{-\pi/2a}^{\pi/2a} d^4 q e^{-iqx} \left( \frac{\tilde{H}}{\sqrt{\tilde{H}^2}} \right)_{AB} e^{iqx}, \quad (3.7)$$

namely, that this integral changes by  $O(a)$  as  $x$  is varied among the sites of a lattice hypercube. The argument for this is as follows. Formally, (3.7) diverges  $\sim 1/a^4$  for  $a \rightarrow 0$ . However, the integrand can be expanded in powers of the continuum gauge field just like in the usual overlap case

[6, 7]<sup>5</sup> and it can be shown that only the terms of mass-dimension 4 or higher in this expansion are nonvanishing.<sup>6</sup> For each mass-dimension of the expansion terms there is an accompanying power of  $a$ , so the nonvanishing terms contain at least a factor  $a^4$  which balances the divergence  $\sim 1/a^4$  in (3.7). The fact that (3.7) changes by  $O(a)$  as  $x$  is varied among neighboring lattice sites then follows from the fact that the smooth continuum gauge field has this property.

**Acknowledgments.** D.A. is supported by AcRF grant RG61/10 and a start-up grant from NTU.

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<sup>5</sup>This relies on a certain bound on the spectrum of the hermitian operator  $H$  when the plaquette variable of the lattice gauge field satisfies an approximate smoothness condition. It was established in the usual overlap case in [16]. An analogous bound holds in the present staggered overlap case; the proof will be given elsewhere.

<sup>6</sup>This relies on two things: First, exponential locality of the integrand of (3.7) in the gauge field (shown in the same way as in the usual overlap case [16]), which implies that the terms in the expansion become local functionals of the gauge field in the  $a \rightarrow 0$  limit. Second, the gauge invariance and lattice rotation invariance of staggered overlap fermions imply that the local functionals of the continuum gauge field that arise in the expansion of (3.7) must have mass-dimension  $\geq 4$ . (There are no such terms with mass-dimension  $\leq 3$ , while for mass-dimension 4 there are the Yang-Mills action functional and topological charge density.)