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’t Hooft loop and the phases of SU(2) LGT

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We analyze the vacuum structure of SU(2) lattice gauge theories in $D = 2,3,4$, concentrating on the stability of ’t Hooft loops. High precision calculations have been performed in D=3; similar results hold also for D=4 and D=2. We discuss the impact of our findings on the continuum limit of Yang-Mills theories.

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1. Introduction

Most ideas aimed at solving confinement in $SU(N)$ Yang-Mills theories involve topological degrees of freedom of some sort. Among these, $\mathbb{Z}_N$ center vortices have received much attention in the literature in general and in lattice investigations in particular.

According to 't Hooft's original idea [1], since the gauge group of pure Yang-Mills $SU(N)/\mathbb{Z}_N$ posseses a non trivial first homotopy class corresponding to its center, $\pi_1(SU(N)/\mathbb{Z}_N) = \mathbb{Z}_N$, a superselection rule will arise for the physical Hilbert space of gauge invariant states [2], with sectors labelled by a vortex topological index $n \in \mathbb{Z}_N$. The low temperature confinement phase will correspond to a superposition of all topological sectors, while above the deconfinement transition vortex symmetry gets broken to the trivial sector $n = 0$; the 't Hooft loop H, dual to the Wilson loop W, is the natural observable to describe the transition, "counting" the number of topological vortices piercing it. From H one can reconstruct the free energy for vortex creation, $F = \Delta U - T\Delta S$; the monitoring of the F behaviour across the deconfinement transition has received broad attention in the literature [3, 4, 5, 6, 7].

The natural choice to investigate F upon lattice discretization of Yang-Mills theories would be to define the partition function through the adjoint Wilson action $Z \sim e^{\beta_A \mathrm{Tr}_A(U)}$; in this case all topological sectors are dynamically included [8]. Universality should of course allow the equivalent use of the standard Wilson plaquette action $S \sim \mathrm{Tr}_F(U)$. In this case topology must however be introduced "by hand" summing over all twisted boundary conditions[1]. The partition function $\tilde{Z}$ will then be defined through the weighted sum of partition functions with fixed twisted BC[2]. Since each of them must be determined by independent simulations, their relative weights can only be calculated through indirect means [4, 5, 9].

There is however a caveat in such argument. The two partition functions Z and $\tilde{Z}$ can only be shown to be equivalent when $\mathbb{Z}_N$ magnetic monopoles are absent [8, 10]. Taking the explicit case of $SU(2)/\mathbb{Z}_2 = SO(3)$, this translates into the constraint:

$$\sigma_c = \prod_{\overline{P}\in\partial c} \mathrm{sign}(\mathrm{Tr}_F\, U_{\overline{P}}) = 1 \tag{1.1}$$

being satisfied for every elementary 3-cube c, where $U_{\overline{P}}$ denotes the plaquettes belonging to the cube surface ∂c. This ensures that $\mathbb{Z}_2$ magnetic monopoles are suppressed and only closed $\mathbb{Z}_2$ center vortices winding around the boundaries are allowed.

The above condition is usually quoted when claiming that the bulk transition separating the strong and weak coupling regime along β_A [11, 12, 13, 14, 15] constitutes an obstacle in defining the continuum limit for the adjoint Wilson action. This however assumes that topological sectors along the fundamental coupling β are always well defined. We will show this *not* to be the case. Together with a set of established results demonstrating that above the adjoint bulk transition topological sectors are well defined and a physical continuum limit of the theory exists [6, 7, 16, 17, 18, 19, 20, 21, 22, 23], we can turn the argument around, casting doubts that investigations of vortex topology for the fundamental Wilson are well defined. Preliminary results had been presented in Ref. [24].

[1] Such topological boundary conditions play a rôle in lattice investigations of string spectrum or large N reduction.

[2] See Ref. [9], Chapt. 3.

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2. Setup

We will investigate as a test case the $SU(2)$ fundamental Wilson action with periodic BC:

$$S = \beta \sum_{x,\mu>\nu} [1 - \mathsf{Tr}_F(U_{\mu\nu}(x)] \tag{2.1}$$

in $D = 2,3,4$ dimensions. Different groups or BC can be considered as well and won't change the main results given below.

The twist operator, measuring the number of topological vortices piercing the 't Hooft loop in the μ,ν planes, can be constructed via [3]:

$$z_{\mu\nu} = \frac{1}{L^{D-2}} \sum_{\vec{\rho}\perp\mu\nu} \prod_{x\in\mu,\nu\text{plane}} \mathsf{sign}(\mathsf{Tr}_F U_{\mu\nu}(x)) \,. \tag{2.2}$$

If the topological sectors are well defined $z_{\mu\nu}$ should take values ± 1 for fixed μ and ν; e.g. in the case at hand, i.e. periodic BC, the topological sector must be trivial and one should always have $z_{\mu\nu} = 1\ \forall\mu,\nu$. It is now easy to define an order parameter z such that $z = 1$ if, whatever the BC, the vortex topology takes the correct value expected in the continuum theory, while $z = 0$ when $\mathbb{Z}_2$ monopoles are still present and open $\mathbb{Z}_2$ center vortices dominate the vacuum, making the identification of topological sectors ill defined at best:

$$z = 1 - |z_{12} - \langle z_{12}\rangle| \qquad D = 2 \tag{2.3}$$

$$z = \frac{2}{D(D-1)} \sum_{\mu>\nu=1}^{D} \langle |z_{\mu\nu}|\rangle \qquad D \geq 3\,. \tag{2.4}$$

3. Results

The $D = 2$ case offers an interesting cross-check of numerical results, since here everything can be calculated analytically. For the order parameter z and its susceptibility χ we have, for fixed volume $V = L^2$:

$$\langle z\rangle_L = e^{-4L^2 p(\beta)}\,; \qquad \chi_L = L^2 \left[e^{-4L^2 p(\beta)} - e^{-8L^2 p(\beta)}\right] \tag{3.1}$$

$$p(\beta) = \frac{1}{2}\left[1 - \frac{L_1(\beta)}{I_1(\beta)}\right] \simeq \sqrt{\frac{2\beta}{\pi}} e^{-\beta}(1 + \mathscr{O}(\frac{1}{\beta}))\,, \tag{3.2}$$

where L and I denote the modified Struve and Bessel functions, respectively. Plotting the above functions (see Fig. (1)) we can clearly distinguish a "strong" coupling regime, where the topology is ill defined, and a "weak" coupling one, where z takes the correct value it should have in the continuum theory. The finite size scaling analysis can be performed exactly, giving for the susceptibility peaks and the corresponding pseudo-critical coupling:

$$\beta_c(L) = \ln L^2 + \frac{1}{2}\ln\ln L^2 + O(1)\,; \qquad \chi_L(\beta_c(L)) = \frac{L^2}{4}\,. \tag{3.3}$$

From the above equation it is easy to extract the critical behaviour of the correlation lenght around the critical coupling $\beta_c = \infty$, including logarithmic corrections:

$$\xi \sim \sqrt[4]{\frac{\pi \ln^2 2}{2\beta}} \cdot e^{\frac{1}{2}\beta} \tag{3.4}$$

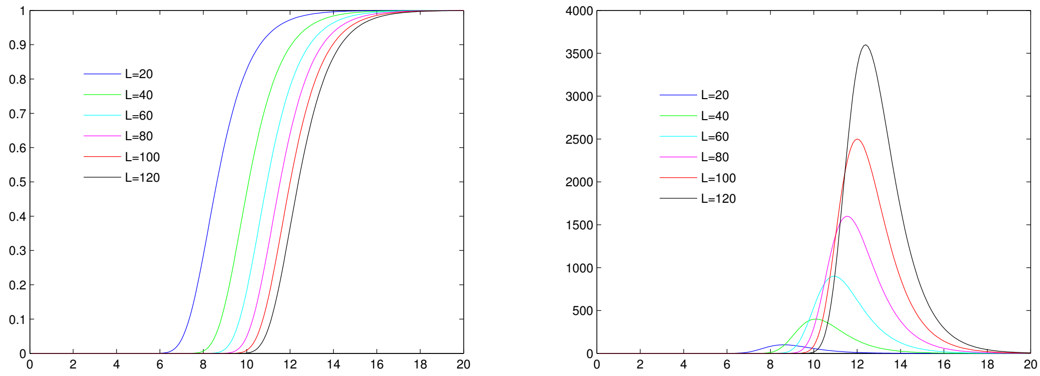

Figure 1: Order parameter z in $D = 2$ (left) and its susceptibility χ (right).

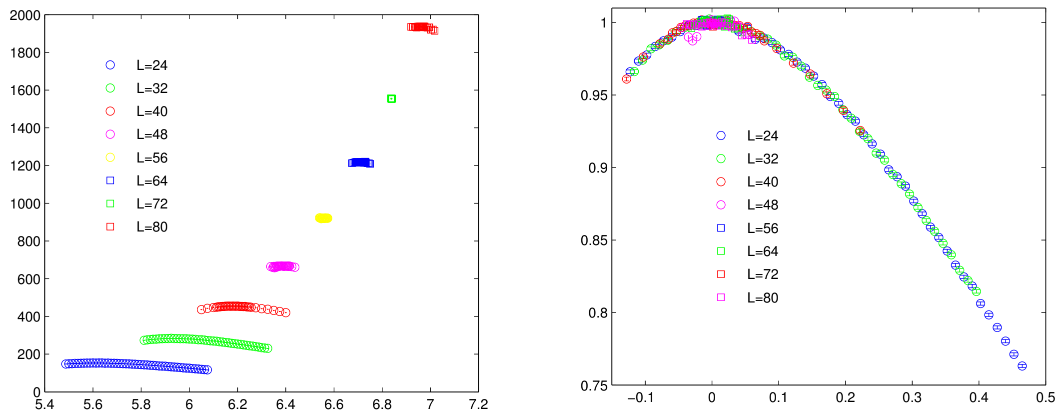

Figure 2: Left: susceptibility χ of the order parameter z in $D = 3$. Right: same with FSS as in Eq. (3.5).

i.e. an essential scaling[3] with critical exponent $\nu = 1$. Summarizing, although for any fixed volume L^2 one can always find a coupling above which the topology corresponds to that dictated by the boundary conditions, taking the thermodynamic limit first, as one should, the "strong" coupling regime extends to $\beta = \infty$ and the system is always in the disordered phase. A vortex topology cannot be defined.

Turning now to the $D = 3$ case and using MonteCarlo simulations to calculate z we basically get the same picture. In Fig. (2) we show the susceptibility χ and its FSS with an essential scaling Ansatz, again with $\beta_c = \infty$:

$$\beta_c(L) \simeq A \ln L^2 + B \ln\ln L^2 ; \qquad \chi_L(\beta_c(L)) \simeq C L^2 \tag{3.5}$$

The result is the same, i.e. in the continuum limit the theory is always in disordered phase and

[3] Compare with the critical behaviour of the XY model, $\xi \sim \exp\left(bt^{-\nu}\right)$ and $\chi \sim L^{2-\eta} \ln^{-2r} L$ with $\nu = 1/2$, $\eta = 1/4$ and $r = -1/16$ [25].

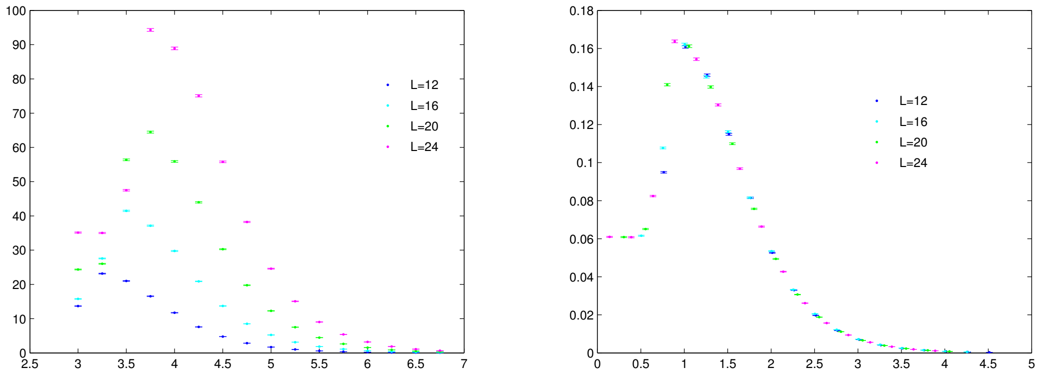

Figure 3: Left: susceptibility χ of the order parameter z in $D = 4$. Right: same with FSS as in Eq. (3.5).

no vortex topology can be defined. The values obtained for β_c are well within the scaling region and for fixed volumes they are always lower than the pseudo-critical coupling at which the finite temperature transition is measured.

The situation is inversed at $D = 4$. Here we still get the same result as above, i.e. an essential scaling with the same critical exponents for our order paramter z. The values of the pseudo-critical coupling β_c are however higher than the values measured for the deconfinement scale, cfr. Fig. (3).

4. Conclusions

We have shown that $\forall\, D \leq 4$ the vortex topology for the standard Wilson action is always ill defined in the continuum limit. This discretization artefact is driven by the too weak fall off of the $\mathbb{Z}_2$ magnetic monopole density, Eq. (1.1). For any fixed β there always exists a lattice size L for which enough open $\mathbb{Z}_2$ vortices can form, spoiling the superselection rule in the thermodinamic limit. The separation among the regimes in $D = 3$ and $D = 4$ are substantially different. While in $D = 3$ the deconfinement transition for fixed length L always lies in the spurious phase above the pseudoctitical coupling $\beta_c(L)$ given in Eq. (3.5), for $D = 4$ the latter is always way above the physical scale, making the physical volumes quite small.

For $D = 4$ one could define the theory above the the bulk transition along β_A, corresponding to the phase where topology is well defined, as in Ref. [6, 7]. There it was however found that $F \neq 0$ in the confined phase, casting some doubt on the standard center vortices symmetry breaking argument as a model for confinement. Further analysis in this case would be welcome, although simulations in this case are technically quite difficult.

There is however an easy way out of the problem: just define the discretized theory through a Positive Plaquette Model [26]. In this c ase the $\mathbb{Z}_2$ magnetic monopole contraint is always satisfied while the order paramer $z \equiv 1$ by construction. This model should then deliver the correct results if one is interested in investigating the role of center vortices.

We would like to stress that none of the above results contradicts universality. First, universality hold as long as no discretization artifacts obstacle the continuum limit. In our case, center monopoles and the associated open center vortices spoil the equivalence among different discretizations. Second, all physical properties measurable in "experiments", like glueball masses and the critical exponents at the transition, should be reflected by physical observables which can be defined *irrespective* of the discretization chosen, meaning that not everything that can be defined in a given discretization should immediately acquire a physical meaning.

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