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Multiparton interactions: Theory and experimental findings

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I give an introduction to multiparton interactions in proton-proton collisions, with a focus on the perturbative regime. Recent experimental results are discussed, as well as progress and open questions in theory.

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1. Introduction

The theoretical basis for describing hard processes in proton-proton collisions is provided by factorization formulae, which express an observable cross section in terms of parton densities and cross sections for hard-scattering subprocesses at parton level. A textbook example is the process $pp \to Z + X \to \ell^+\ell^- + X$, for which a sketch is shown in figure 1a and a graph in figure 2a. It is essential to realize that such factorization formulae hold for *inclusive* cross sections: they contain all details of the particles produced in the hard-scattering subprocess (such as the momenta of the leptons ℓ^+ and ℓ^- in our example) but give no information about the other particles (denoted by X in the above formula). The physics picture suggested by figures 1a and 2a is deceptively simple, since it suggests that only the two partons annihilating into a Z boson interact. This is not the case: the other partons in the colliding protons interact with each other as well, but the effects of their interactions cancel in the inclusive cross section thanks to unitarity. If, however, we ask for details about the final state that concern the particles in X, these interactions do matter. An example is given in figures 1b and 2b.

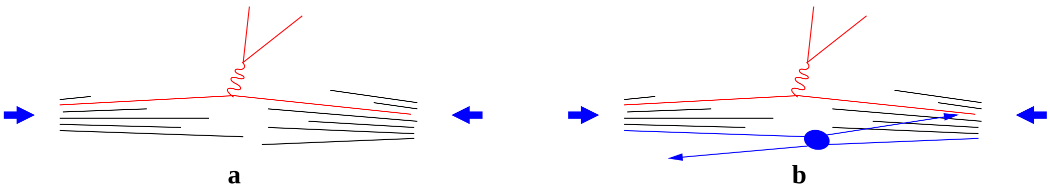

Figure 1: Sketch of the process $pp \to Z + X \to \ell^+\ell^- + X$ without (a) and with (b) scattering amongst the "spectator" partons.

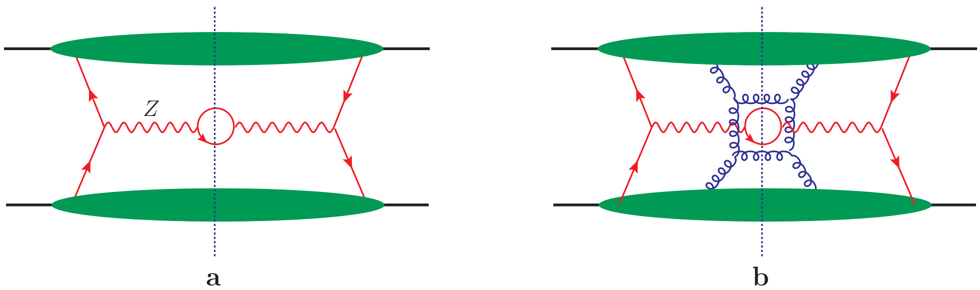

Figure 2: Cross section graphs corresponding to figure 1. The vertical line indicates the final state.

Most frequently, these additional interactions produce particles that have low transverse momentum and are part of the "underlying event" (UE), which is defined as "anything not produced in the hard-scattering subprocess". (Clearly, this definition is not based on observable features but depends on a specific theoretical description.) At high-energy colliders, especially the Tevatron and the LHC, the additional interactions can, however, also be hard and produce particles with large transverse momentum or large mass. One then speaks of "multiple hard scattering". This talk is focused on the case with one additional hard scattering, i.e. on "double parton scattering" (DPS). The terms "multiparton interactions" (MPI) and "multiple interactions" are sometimes restricted to the case where all interactions are hard and sometimes meant to include the case where the additional interactions contribute to the underlying event.

Double parton scattering can be an important background in searches for new physics. In the process $pp \to HZ + X \to b\bar{b}Z + X$, one hard scatter can produce the Z and the other a continuum $b\bar{b}$ pair [1]. If the second scatter produces a Higgs boson, one has a contribution to the signal. The same holds for $pp \to HW + X \to b\bar{b}W + X$, where the DPS contribution to the background has been studied in detail for Tevatron kinematics in [2]. A process with DPS contributions of prominent size is W^+W^+ or W^-W^- production [3, 4], which is a search channel for supersymmetry [5].

A general statement about the relative size of single and double parton scattering for a given final state can be made on the basis of power counting [6–8]. Let Q be the typical hard scale in a process and $\mathbf{p}_{T1}$, $\mathbf{p}_{T2}$ the net transverse momenta of the particles produced in the first and the second interaction of the DPS process, respectively. (In the above example, $\mathbf{p}_{T1}$ is the transverse momentum of the $b\bar{b}$ pair and $\mathbf{p}_{T2}$ the transverse momentum of the Z.) One then finds

$$\frac{d\sigma_{\text{single}}}{d^2\mathbf{p}_{T1}\, d^2\mathbf{p}_{T2}} \sim \frac{d\sigma_{\text{double}}}{d^2\mathbf{p}_{T1}\, d^2\mathbf{p}_{T2}} \sim \frac{1}{\Lambda^2 Q^4}\,, \tag{1.1}$$

where Λ represents a typical hadronic scale. Double parton scattering is hence *not* power suppressed in the differential cross section. However, the phase space of DPS is limited to $p_{T1} \sim \Lambda$ and $p_{T2} \sim \Lambda$, whereas in single hard scattering we have the limitation $|\mathbf{p}_{T1} + \mathbf{p}_{T2}| \sim \Lambda$ but p_{T1} and p_{T2} can each be of order Q. Integrating over the transverse momenta, we thus obtain

$$\sigma_{\text{single}} \sim \frac{1}{Q^2} \;\gg\; \sigma_{\text{double}} \sim \frac{\Lambda^2}{Q^4}\,. \tag{1.2}$$

Double parton scattering is now power suppressed, in accordance with the usual factorization formulae that describe only single hard scattering. However, even in this situation DPS can be important. In particular, its contribution grows with the overall collision energy since for decreasing parton momentum fractions x the two-parton density grows faster than the one-parton density.

2. The cross section for double hard scattering

It is natural to assume that double hard scattering can be described by factorization formulae akin to those for single hard scattering. At tree level one then has

$$\frac{d\sigma_{\text{double}}}{dx_1\, d\bar{x}_1\, dx_2\, d\bar{x}_2} = \frac{1}{C}\, \hat{\sigma}_1(x_1\bar{x}_1)\, \hat{\sigma}_2(x_2\bar{x}_2) \int d^2\mathbf{b}\, F(x_1,x_2,\mathbf{b})\, F(\bar{x}_1,\bar{x}_2,\mathbf{b})\,, \tag{2.1}$$

where C is a combinatorial factor and $\hat{\sigma}_i$ the parton-level cross section for subprocess i $(= 1,2)$. For simplicity we have not displayed labels that specify the different partons. The momentum fractions x_i and $\bar{x}_i$ can be reconstructed from the final-state kinematics, whereas the transverse distance $\mathbf{b}$ between the two hard-scattering processes is unobservable. $F(x_1,x_2,\mathbf{b})$ is a double parton distribution (DPD) and describes the probability density for finding two partons with respective momentum fractions x_1 and x_2 at a transverse distance $\mathbf{b}$ inside the proton. It is sometimes useful to Fourier transform $F(x_1,x_2,\mathbf{b})$ w.r.t. $\mathbf{b}$. The resulting distributions have been termed "generalized two-parton distributions" in [8, 9], which should be distinguished from the generalized (single) parton distributions that appear in exclusive processes such as $ep \to ep\gamma$ [10].

The formula (2.1) can be extended to include radiative corrections in $\hat{\sigma}_i$ (with convolution integrals instead of products between $\hat{\sigma}_i$ and F on the r.h.s.) and to be differential in the momenta of the final state particles produced by hard scattering. Notice that (2.1) still describes an *inclusive* cross section, i.e. it includes all further interactions between "spectator" partons, which are predominantly soft but may occasionally be hard. We will comment on the theoretical status of (2.1) in section 4.3.

To use (2.1) for estimating DPS cross sections, one needs an ansatz for the double parton distributions, which are essentially unknown. *If* one assumes that DPDs factorize as $F(x_1, x_2, \mathbf{b}) = f(x_1)\, f(x_2)\, G(\mathbf{b})$, where $f(x_i)$ are the usual parton densities, and *if* one assumes that the transverse distance distribution $G(\mathbf{b})$is the same for all parton types, then the cross section formula (2.1) (as well as its generalization to higher orders) turns into

$$\frac{d\sigma_{\text{double}}}{dx_1\, d\bar{x}_1\, dx_2\, d\bar{x}_2} = \frac{1}{C}\, \frac{d\sigma_1}{dx_1\, d\bar{x}_1}\, \frac{d\sigma_2}{dx_2\, d\bar{x}_2}\, \frac{1}{\sigma_{\text{eff}}} \tag{2.2}$$

with cross sections σ_i for single hard scattering and a universal factor $1/\sigma_{\text{eff}} = \int d^2\mathbf{b}\, \big[G(\mathbf{b})\big]^2$. With this "pocket formula" and a value for σ_{eff} in hand, one can conveniently estimate the rate for double parton scattering in any given process. One should, however, bear in mind that (2.2) relies on strong simplifications and must be expected to have a limited accuracy. Alternatively, one may use (2.2) to define σ_{eff} and extract it from experiment. A dependence of σ_{eff} on the process or on kinematic variables then indicates that the assumptions on $F(x_1, x_2, \mathbf{b})$ spelled out before (2.2) are too simple.

3. Recent experimental results

It has long been known that multiple interactions are indispensable for describing the underlying event in pp, $p\bar{p}$ and γp collisions (see [11] for an earlier review). It is hence not surprising that experimental studies of the underlying event can be used to tune those parameters in Monte Carlo event generators that describe the modeling of multiple interactions. We will not expand on this subject here but refer to the presentations [12 – 14] at this workshop.

In the left panel of figure 3 we collect determinations of σ_{eff} from several experimental studies at the Tevatron and the LHC. Within the uncertainties, there is no clear indication for a dependence on the process or the collision energy. The D0 analysis [17] has also performed a differential extraction in three bins for the p_T of the second hardest jet; the results are consistent with no dependence but also with a slight decrease of σ_{eff} with p_T.

It must be emphasized that these determinations of σ_{eff} are very difficult, since all considered processes have significant contributions from single hard scattering, which must be understood with high precision in order to determine the contribution from DPS. At this workshop, both ATLAS [18, 20] and CMS [21, 22] have presented studies of DPS in the production of a W associated with exactly two jets having $p_T > 20$ GeV. Among the variables used to exhibit DPS is the relative transverse momentum imbalance

$$\Delta^n_{\text{jets}}(\text{ATLAS}) = \Delta^{\text{rel}} p_T(\text{CMS}) = \frac{|\mathbf{p}_{T\,\text{jet1}} + \mathbf{p}_{T\,\text{jet2}}|}{|\mathbf{p}_{T\,\text{jet1}}| + |\mathbf{p}_{T\,\text{jet2}}|} \tag{3.1}$$

between the two jets, whose distribution in single hard scattering covers a broad range between 0 and 1 but is peaked at small values in DPS. The ATLAS analysis [18] extracted a fraction $f_{\text{DP}} =$

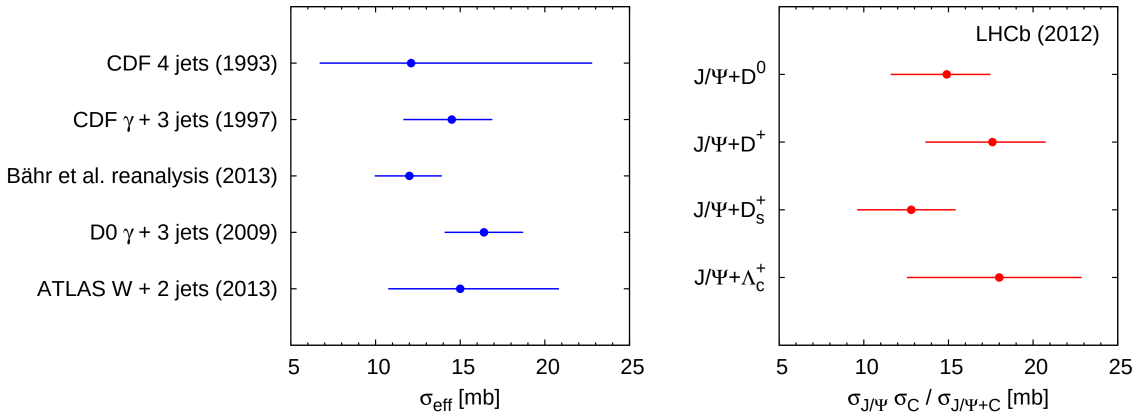

Figure 3: Left: experimental determinations of σ_{eff} by CDF [15, 16], D0 [17] and ATLAS [18], as well as a reanalysis of the CDF data [16] by Bähr et al. [19]. Right: cross section ratios measured by LHCb [30] for inclusive production of a J/Ψ, a charmed hadron C, and a pair $J/\Psi + C$. In both panels, statistical and systematic uncertainties have been added in quadrature.

$0.08 \pm 0.01 \pm 0.02$ of double parton scattering in their event sample, which is very significant for a process that is formally a power correction to single hard scattering.

There are several investigations of multiparton interactions in the charm sector. Among those is an ALICE study of particle multiplicities in J/Ψ production [23]. An earlier LHCb measurement [24] has raised significant interest since it suggests that double parton parton scattering may play a prominent role in the production of J/Ψ pairs [25 – 29]. Even more spectacularly, the cross sections measured by LHCb for the production of a J/Ψ associated with a charmed hadron C are more than one order of magnitude larger than predictions based on single hard scattering [30]. If this process is dominated by double parton scattering, the pocket formula (2.2) gives $\sigma_{\text{eff}} = \sigma_{J/\Psi}\, \sigma_C \,/ \sigma_{J/\Psi+C}$. The values for this cross section ratio determined by LHCb are shown in the right panel of figure 3. Their size suggests that DPS may indeed dominate the production channels in question. The situation is, however, not so simple if one looks at differential distributions. The p_T slopes of the charmed hadron in $J/\Psi + C$ production are comparable to those in prompt C production, but the p_T spectrum of the J/Ψ in the $J/\Psi + D^0$ and $J/\Psi + D^+$ channels is significantly less steep than in inclusive J/Ψ production (in the $J/\Psi + D_s$ and $J/\Psi + \Lambda_c$ channels, larger errors prevent a similarly strong statement). This does not support the simplest picture of DPS where there is no correlation between the two partons in each proton, so that the two hard scatters are completely independent of each other. It will be interesting to follow up on these issues, also for double open charm production, which has been measured by LHCb as well [30]. Theoretical analyses of these channels can be found in [31 – 33].

4. A closer look at theory

The dynamics of double parton scattering is far more involved than then pocket formula (2.2) suggests, and we will now discuss a few theoretical aspects that have been investigated in the recent literature.

4.1 Parton correlations

Beyond a certain precision, one cannot expect that the distribution $F(x_1,x_2,\mathbf{b})$ of two partons in a proton is given by a product $f(x_1)\,f(x_2)$ of usual PDFs times a universal factor $G(\mathbf{b})$. Two-parton correlations involving x_1,x_2 and $\mathbf{b}$ lead to a more complicated form of $F(x_1,x_2,\mathbf{b})$ and have been considered in several studies. Moreover, there can be correlations involving the quantum numbers of the two partons, namely their polarization, color and flavor. This requires the addition of further distributions in the cross section formula (2.1). More detail is given in [34].

4.2 Parton splitting

While the typical transverse distance $\mathbf{b}$ between two partons in a proton is of hadronic size, the cross section formula (2.1) involves all values of $\mathbf{b}$ down to zero. For sufficiently small $\mathbf{b}$, one can compute DPDs in terms of PDFs and the perturbative splitting of one parton into two [7]. At leading order in α_s, this gives

$$F(x_1,x_2,\mathbf{b}) = \frac{1}{\pi\mathbf{b}^2}\,P\left(\frac{x_1}{x_1+x_2}\right)\frac{f(x_1+x_2)}{x_1+x_2}\,, \tag{4.1}$$

where P is the familiar DGLAP splitting function for the relevant process (again we omit labels for the parton species for simplicity). This mechanism, shown in figure 4a, generates substantial correlations between x_1 and x_2 in the DPD, as well as between the polarization and the color of the two partons. It also plays a role in the scale dependence of the DPDs. One contribution to their evolution equation describes the separate parton cascades radiated from parton 1 and from parton 2, as shown in figure 4b. Depending on how exactly DPDs are defined, their evolution equation contains or does not contain an additional term originating from the splitting (4.1). The evolution equation including this term has been studied intensively [35–40]. It remains, however, to be established which version of evolution is actually relevant in the factorization formula for DPS.

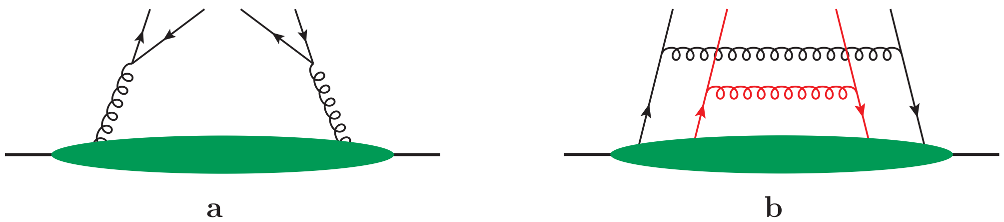

Figure 4: (a) A graph for the splitting of a single parton into two in a double parton distribution. (b) A ladder graph for the separate DGLAP evolution of partons 1 and 2 in a double parton distribution.

The $1/\mathbf{b}^2$ short-distance singularity in (4.1) gives an integral diverging like $\int d\mathbf{b}^2/\mathbf{b}^4$ when inserted in the cross section formula (2.1). This is obviously inappropriate and indicates a deeper problem. The graph in figure 5a can be read as a contribution to DPS, with each DPD being replaced by its leading contribution at small $\mathbf{b}$. The same graph is, however, a higher-order graph for the production of two electroweak gauge bosons by gluon fusion. To even define what is meant by double parton scattering (and to derive a corresponding cross section formula) requires a careful

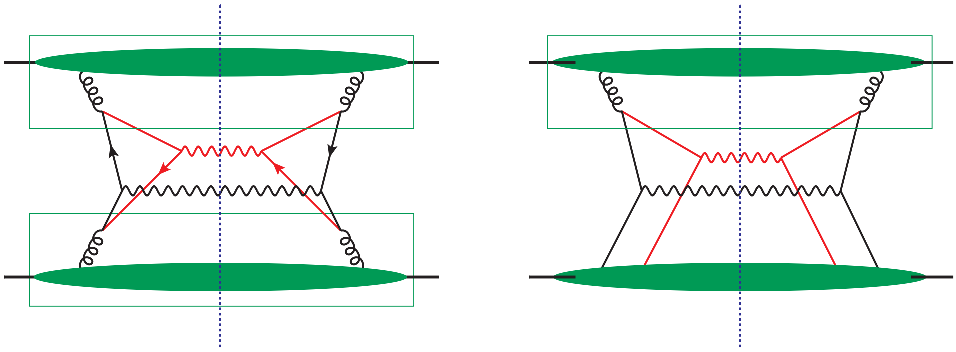

Figure 5: Graphs for the production of two electroweak gauge bosons (γ^*, Z, W) involving the splitting of one parton into two in (a) both protons or (b) in one of the protons only. The boxes indicate the small **b** limit of a DPD as shown in figure 4a.

discussion of this double counting problem, which was pointed out for multijet production already in [41]. Currently there is no consensus regarding its solution [6 – 8, 42 – 44].

A similar discussion applies to the graph in figure 5b, which can be read either as a hybrid of double and single parton scattering, or as a contribution to DPS with the small **b** limit of the DPD taken for the upper proton. Detailed investigations of this mechanism can be found in [9, 45, 46] and in [47].

4.3 Factorization

The factorization formula (2.1), which forms the theory basis for most estimates of double parton scattering, does not have the same theoretical status as factorization formulae for single hard scattering in hadron-hadron collisions. At lowest perturbative order, i.e. for graphs like those in figure 2b, the formula can be derived using standard techniques for approximating Feynman graphs [48, 49, 7]. In particular, the transverse distance **b** between the scattering partons naturally appears in the quantum-level calculation, without any semiclassical approximation. To establish factorization requires many further elements, regarding in particular the exchange of soft gluons between the different parts of the graph. Several ingredients to a possible factorization proof have been given in [7] and [50]. In particular, one can show that soft-gluon exchange (see figure 6) can be arranged into Sudakov factors if the gluon momentum is in the kinematic region where the so-called eikonal approximation can be made.

Among the questions that remain unsolved is whether soft gluon exchange breaks factorization when the gluon momentum is in the so-called Glauber region. This is one of the most difficult parts in any factorization proof for hadron-hadron collisions, and even for single hard scattering a detailed argument only exists for a small number of processes.

5. Conclusions

This is a good time for gaining a deeper understanding of multiparton interactions, and there is significant activity in the field [51 – 53]. The theoretical analysis has made important progress

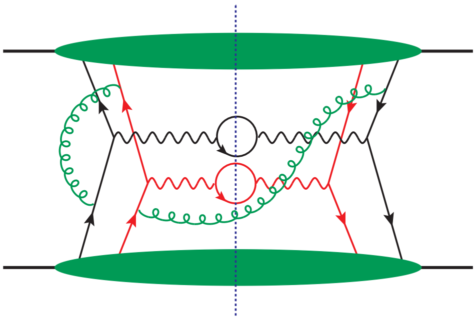

Figure 6: Graphs for the production of two electroweak gauge bosons by double parton scattering with additional gluon exchange. It is nontrivial to show that the effect of such graphs is properly taken into account in a factorization formula.

in the last few years, but difficult questions still await an answer. On the experimental side, the jump in energy from the Tevatron to LHC has opened many new possibilities for a detailed study of processes where double parton scattering is expected to play a role. First results from the LHC have demonstrated that corresponding analyses are feasible even if they are not easy. Input from experiment will be essential to guide the development of phenomenology.

Many recent results on multiparton interactions were not discussed in this talk for reasons of time. Among them are estimates of DPS contributions to a variety of processes [54 – 59], multiparton interactions in proton-nucleus and nucleus-nucleus collisions [60 – 65], and the small-x approach [66 – 68]. The latter provides in particular a connection between multiparton interactions and diffraction [69], as well as an explanation of the "ridge-effect" in pp and pA collisions, which has been observed by CMS, ATLAS and ALICE [70 – 74].

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