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Proton structure functions and physical evolution kernels*

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Physical anomalous dimensions allow to describe the Q^2 dependence of proton structure functions in a factorization scheme independent manner. We perform a numerical implementation of these physical evolution kernels at both leading and next-to-leading order. A comparison to conventional DGLAP evolution based on factorization into coefficient and parton distribution functions is provided.

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1. Introduction

Collinear factorization provides today an amazingly accurate description of the proton structure function $F_2(x,Q^2)$, which has been measured with high accuracy at the HERA experiments [1] at DESY. Here x denotes the momentum fraction carried by the struck parton and Q^2 the resolution scale set by the momentum transfer squared. Within collinear factorization this observable is described as a convolution of perturbatively calculable Wilson coefficients and scale-dependent quark and gluon distribution functions $f_i(x,\mu_f^2)$, $i=q,\bar{q},g$, where the latter are obtained from global QCD fits [2]. μ_f denotes the factorization scale, which is generally of order of magnitude of Q. It is a direct consequence of the formulation of factorization, where it serves to separate hard from soft momenta. Independence of physical observables of the factorization scale can then be used to derive powerful renormalization group equations, known as the DGLAP equations, which describe the scale dependence of parton distribution functions. Furthermore, since factorization can be carried out in infinitely many different ways, one is left with an additional choice of the factorization scheme for which one usually adopts the $\overline{\mathrm{MS}}$ prescription. Quark and gluon distributions are therefore not physical observables, but a theory definitions.

Despite the success of collinear factorization there exists strong theoretical arguments that at sufficiently small momentum fractions x, deviations from DGLAP should manifest themselves in the data, driven by linear BFKL and/or its non-linear extensions such as BK-evolution. The study of this kinematic regime is also one of the main physics goals of a future electron-ion collider, such as the proposed EIC [3] and LHeC [4] projects. While successful descriptions of the HERA data at small x already exist, which use either next-to-leading order (NLO) BFKL resummation, see *e.g.* [5] or running coupling BK evolution, see *e.g.* [6], the same set of data is equally well described by DGLAP driven fits of parton distribution functions (pdfs) and no definite conclusion can be drawn.

Indeed, a direct manifestation of deviations from DGLAP evolutions is rather hard to achieve through a fits of parton distribution functions: given the usually limited range of Q^2 values at small x, combined with theoretical ambiguities due factorization scale and scheme choice and the rather large number of free parameters of initial pdfs renders this a very difficult task.

These problems can be however if circumvented if instead of the dependence of pdfs on the factorization scale, the dependence of structure functions themselves w.r.t the proton virtually is studied. This leads to the concept of physical anomalous dimensions [7, 8, 9, 10], which govern, in close analogy to the DGLAP anomalous dimensions, the evolution of structure functions. As they evolve directly physical observables, they are observables themselves and carry therefore neither scheme nor factorization scale dependence. In addition, analysis of DIS data requires in this case only a parametrization of structure functions, instead of an complete set of parton distribution functions, which allows to probe DGLAP evolution more efficiently. In the following we give some details on DGLAP evolution in terms of physical anomalous dimensions and first numerical results. For details we refer to the paper in preparation [11].

2. Physical evolution kernel

To define physical anomalous dimensions it is necessary to express convolutions in Bjorken x

space as products of their Mellin transforms defined as

$$a(n) \equiv \int_0^1 dx x^{n-1} a(x). \tag{2.1}$$

Within collinear factorization, the Mellin transforms of DIS structure functions $F_I(x,Q^2)$ are then given by

$$F_I(n,Q^2) = \sum_k C_{I,k}\left(n, \alpha_s(\mu^2), \frac{Q^2}{\mu^2}\right) f_k\left(n, \alpha_s(\mu^2), \frac{\mu^2}{Q_0^2}\right), \tag{2.2}$$

where the sum runs over all contributing quark flavors and the gluon, each represented by a pdf f_k. The coefficient functions $C_{I,k}$ can be calculated within perturbative QCD [7, 12, 13, 14] and exhibit the following expansion in α_s

$$C_{I,k}\left(n, \alpha_s(\mu^2), \frac{Q^2}{\mu^2}\right) = \sum_{m=0}\left(\frac{\alpha_s(\mu^2)}{4\pi}\right)^{m_0+m} C_{I,k}^{(m)}(n)\,, \tag{2.3}$$

where m_0 depends on the first non-vanishing order in α_s in the expansion for the observable under consideration, e.g., $m_0 = 0$ for F_2 and $m_0 = 1$ for F_L. The pdfs $f_k(n,\mu^2)$ obey the DGLAP evolution equations which read

$$\frac{d f_k(n,\mu^2)}{d\ln\mu^2} = \sum_l P_{kl}(n, \alpha_s(\mu^2), \frac{Q^2}{\mu^2}) f_l(n,\mu^2) \tag{2.4}$$

where the $l \to k$ splitting functions have a similar expansion [15, 16, 17] as the coefficient functions in Eq. (2.3):

$$P_{kl}(n, \alpha_s(\mu^2)) = \sum_{m=0}\left(\frac{\alpha_s(\mu^2)}{4\pi}\right)^{1+m} P_{kl}^{(m)}(n)\,. \tag{2.5}$$

The DGLAP evolution equation can then be formulated as $n_f - 1$ evolution equations for the different non-singlet quark flavor combinations and a 2×2 matrix valued evolution equation, which evolves the flavor singlet vector (Σ, g). Here $g(n,\mu^2)$ denotes the gluon distribution and

$$\Sigma(n,\mu^2) = \sum_f^{n_f}\left[q_f(n,\mu^2) + \bar{q}_f(n,\mu^2)\right] \tag{2.6}$$

the quark flavor singlet. The strong coupling itself obeys the following renormalization group equation, governed by the QCD beta function.

$$\frac{d\alpha_s(\mu)}{d\ln\mu^2} = 4\pi\beta(a_s) = -\alpha_s\sum_m\left(\frac{\alpha_s}{4\pi}\right)^{m+1}\beta_m \tag{2.7}$$

with $\beta_0 = 11 - 2n_f/3$ and $\beta_1 = 102 - 38n_f/3$ up to NLO accuracy. To formulate evolution in terms of physical anomalous dimensions we first restrict ourselves to the small x region where gluon and seaquark distribution dominate and the latter is well approximated by the flavor singlet Eq. (2.6). It is then possible to reexpress the flavor singlet doublet (Σ, g) through the doublet of structure functions (F_2, F_L). To this end we define, following [9], a rescaled version of the structure function F_L

$$\tilde{F}_L(n,Q^2) = 4\pi\frac{F_L(n,Q^2)}{\alpha_s(Q^2)C_{Lq}^{(1)}(n)} \tag{2.8}$$

which leads to a doublet $F(n,Q^2) = (F_2, \tilde{F}_L)$. Note that the coefficient function $C_{Lq}^{(1)}(n)$ is does not depend on the factorization scheme and therefore no scheme dependence is introduced through this rescaling. To define physical evolution kernels we introduce coefficient and evolution kernel matrices for the doublet $F(n,Q^2)$,

$$C = \begin{pmatrix} C_{2q} & C_{2g} \\ C_{\tilde{L}q} & C_{\tilde{L}g} \end{pmatrix} \qquad\qquad P = \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \tag{2.9}$$

and consider

$$\frac{dF(n,Q^2)}{d\ln Q^2} = \left(4\pi\beta \frac{dC}{d\alpha_s} C^{-1} + C \cdot P \cdot C^{-1}\right) F. \tag{2.10}$$

This yields physical evolution kernels as

$$K = \left(\beta \frac{dC}{da_s} C^{-1} + C \cdot P \cdot C^{-1}\right). \tag{2.11}$$

At LO one obtains

$$\begin{aligned} K_{22}^{(0)} &= P_{\mathrm{qq}}^{(0)} - \frac{C_{\mathrm{Lq}}^{(1)} P_{\mathrm{qg}}^{(0)}}{C_{\mathrm{Lg}}^{(1)}} & K_{2L}^{(0)} &= \frac{C_{\mathrm{Lq}}^{(1)} P_{\mathrm{qg}}^{(0)}}{C_{\mathrm{Lg}}^{(1)}} \\ K_{L2}^{(0)} &= \frac{C_{\mathrm{Lg}}^{(1)} P_{\mathrm{gq}}^{(0)}}{C_{\mathrm{Lq}}^{(1)}} - \frac{C_{\mathrm{Lq}}^{(1)} P_{\mathrm{qg}}^{(0)}}{C_{\mathrm{Lg}}^{(1)}} - P_{\mathrm{gg}}^{(0)} + P_{\mathrm{qq}}^{(0)} & K_{LL}^{(0)} &= \frac{C_{\mathrm{Lq}}^{(1)} P_{\mathrm{qg}}^{(0)}}{C_{\mathrm{Lg}}^{(1)}} + P_{\mathrm{gg}}^{(0)} \end{aligned} \tag{2.12}$$

for the NLO kernels we refer for this particular convention to the paper in preparation [11].

3. Numerical analysis

In this section we provide some first numerical results, where we compare an implementation of physical anomalous dimensions against DGLAP evolution of parton distribution functions. To this end we construct structure functions F_2 and F_L from a set of toy pdfs, for which we use the default input of the evolution code PEGASUS [18], taken at an initial scale $Q_0^2 = 2\text{GeV}^2$ with $\alpha_s(Q_0^2) = 0.35$.

The evolution equations, which we treat at NLO accuracy, are solved analytically, as provided *e.g.* by Eq. (2.32) of [18]. For details we refer to [11]. While we find exact agreement between evolution of structure functions through physical anomalous dimensions and evolution of pdfs at leading order, there exist sizeable deviations at next-to-leading order. They can be traced to the evaluation of coefficient functions at initial scale Q_0 (physical anomalous dimensions) and final scale Q (pdf style evolution). For a detailed discussion and an extensive analysis we refer the reader to [11].

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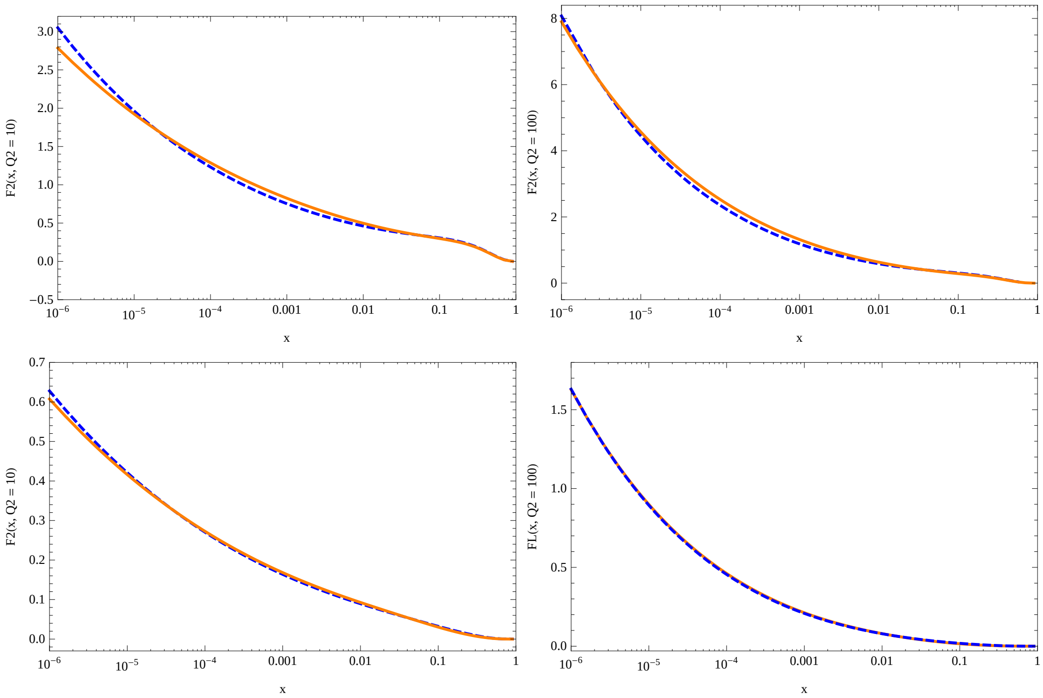

Figure 1: The doublet (F_2, F_L) at $Q^2 = 10\text{GeV}^2$ (first line) and $Q^2 = 100\text{GeV}^2$ (second line). Straight orange lines denote evolution with physical kernels, while dotted blue lines denotes pdf style evolution.

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