

Heavy quark impact factor and single bottom quark production at the LHC

Grigorios Chachamis

Instituto de Física Corpuscular, Universitat de València – Consejo Superior de Investigaciones Científicas, Parc Científic, E-46980 Paterna (Valencia), Spain

E-mail: grigorios.chachamis@ific.uv.es

Michal Deák*

Instituto de Física Corpuscular, Universitat de València – Consejo Superior de Investigaciones Científicas, Parc Científic, E-46980 Paterna (Valencia), Spain

E-mail: michal.deak@ific.uv.es

Germán Rodrigo

Instituto de Física Corpuscular, Universitat de València – Consejo Superior de Investigaciones Científicas, Parc Científic, E-46980 Paterna (Valencia), Spain

E-mail: german.rodrigo@csic.es

We calculate the finite part of the heavy quark impact factor at next-to-leading logarithmic accuracy in a form suitable for phenomenological studies such as the calculation of the cross-section for single bottom quark production at the LHC.

*XXI International Workshop on Deep-Inelastic Scattering and Related Subjects - DIS2013,
22-26 April 2013
Marseille, France*

*Speaker.

1. Introduction

Major developments in the last two decades in small- x physics made possible the phenomenological analysis of deep inelastic scattering (DIS) processes within the k_T factorization scheme. They were mainly driven by the Balitsky-Fadin-Kuraev-Lipatov (BFKL) framework for the resummation of high center-of-mass energy logarithms at leading [1] and next-to-leading [2] logarithmic accuracy.

Indeed, a lot of studies were dedicated on setting up the stage for studying DIS processes within the BFKL formalism [3] and recently there were successful attempts for the detailed description of the Q^2 and x dependence of the structure functions F_2 and F_L by making use of a collinearly-improved BFKL equation at next-to-leading logarithmic accuracy [4].

Apart from the gluon density, another key ingredients for studying DIS processes are the impact factors which are process dependent objects. The impact factors for gluons and massless quarks have been calculated in Ref. [5], at next-to-leading logarithmic accuracy (NL x). This allows for the calculation of various DIS and ‘double DIS’ processes with massless quarks and gluons in the initial state. The generalization to hadron-hadron collisions has also been established [6–8].

The NL x impact factor for a massive quark in the initial state has been calculated in Ref. [9]. However, the result was written in the form of a sum of an infinite number of terms. To make that result of Ref. [9] available for phenomenological studies we recalculate the next-to-leading order heavy quark impact factor in a compact and resummed form which is more suitable for numerical applications.

2. k_T -factorization

We start with some useful definitions. The differential cross-section of the high-energy scattering of two partons, a and b, can be written in a factorized form in terms of the gluon Green’s function \mathcal{G}_ω and the impact factors of the two partons, h_a and h_b respectively:

$$\frac{d\sigma_{ab}}{d[\mathbf{k}_1]d[\mathbf{k}_2]} = \int \frac{d\omega}{2\pi i\omega} h_a(\mathbf{k}_1) \mathcal{G}_\omega(\mathbf{k}_1, \mathbf{k}_2) h_b(\mathbf{k}_2) \left(\frac{s}{s_0(\mathbf{k}_1, \mathbf{k}_2)} \right)^\omega. \quad (2.1)$$

We adopt $d[\mathbf{k}] = d^{2+2\varepsilon}\mathbf{k}/\pi^{1+\varepsilon}$ as the transverse phase-space measure. The leading log x impact factor, $h^{(0)}$, can be written as

$$h^{(0)}(\mathbf{k}) = \sqrt{\frac{\pi}{N_c^2 - 1}} \frac{2C_F \alpha_s N_\varepsilon}{\mathbf{k}^2 \mu^{2\varepsilon}}, \quad \text{where} \quad N_\varepsilon = \frac{(4\pi)^{\varepsilon/2}}{\Gamma(1 - \varepsilon)}, \quad (2.2)$$

and has the same form for quarks and gluons, whereas μ is the renormalization scale, and

$$\bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}, \quad \alpha_s = \frac{g^2 \Gamma(1 - \varepsilon) \mu^{2\varepsilon}}{(4\pi)^{1+\varepsilon}}, \quad (2.3)$$

is the dimensionless strong coupling constant. We also define

$$A_\varepsilon = \mathbf{k}^2 h^{(0)}(\mathbf{k}) \frac{\bar{\alpha}_s}{\Gamma(1 - \varepsilon) \mu^{2\varepsilon}}, \quad (2.4)$$

to factor out the dependence on the strong coupling constant and color factors.

3. The NLO heavy quark impact factor

According to Ref. [9], the NLO correction to the heavy quark impact factor can be written as a sum of three contributions:

$$h_q^{(1)}(\mathbf{k}_2) = h_{q,m=0}^{(1)}(\mathbf{k}_2) + \int_0^1 dz_1 \int d[\mathbf{k}_1] \Delta F_q(z_1, \mathbf{k}_1, \mathbf{k}_2) + \int d[\mathbf{k}_1] \bar{\alpha}_s h_q^{(0)}(\mathbf{k}_1) K_0(\mathbf{k}_1, \mathbf{k}_2) \log \frac{m}{k_1} \Theta_{mk_1}, \quad (3.1)$$

where $k_1 = |\mathbf{k}_1|$, $\Theta_{mk_1} = \theta(m - k_1)$ and

$$\bar{\alpha}_s K_0(\mathbf{k}_1, \mathbf{k}_2) = \frac{\bar{\alpha}_s}{\mathbf{q}^2 \Gamma(1 - \varepsilon) \mu^{2\varepsilon}} + 2\omega^{(1)}(\mathbf{k}_1^2) \delta[\mathbf{q}], \quad \text{where } \delta[\mathbf{q}] = \pi^{1+\varepsilon} \delta^{2+2\varepsilon}(\mathbf{q}), \quad (3.2)$$

is the leading order BFKL kernel. $\mathbf{q} = \mathbf{k}_1 + \mathbf{k}_2$ and

$$\omega^{(1)}(\mathbf{k}^2) = -\frac{g^2 N_c \mathbf{k}^2}{(4\pi)^{2+\varepsilon}} \int \frac{d[\mathbf{p}]}{\mathbf{p}^2 (\mathbf{k} - \mathbf{p})^2} = -\frac{\bar{\alpha}_s \Gamma^2(1 + \varepsilon)}{2\varepsilon \Gamma(1 + 2\varepsilon)} \left(\frac{\mathbf{k}^2}{\mu^2} \right)^\varepsilon, \quad (3.3)$$

is the gluon Regge trajectory. The first term on the right-hand side of Eq. (3.1) is the massless NLO correction to the impact factor which was computed in Ref. [5]. Here, we discuss the last two terms in the right-hand side of Eq. (3.1).

3.1 The ΔF_q term

The second term in the right-hand side of Eq. (3.1) reads in momentum space:

$$\begin{aligned} \Delta F_q(\mathbf{k}_2) &= \Delta F_{q,real}(\mathbf{k}_2) + \Delta F_{q,virt}(\mathbf{k}_2) \\ &= A_\varepsilon \left[\frac{\Gamma(-\varepsilon)}{2(1+2\varepsilon)} \frac{(m^2)^\varepsilon}{\mathbf{k}_2^2} + \frac{\Gamma(1-\varepsilon)}{2} \left\{ \int_0^1 \int_0^1 dz_1 dx \left(\frac{1-z_1}{z_1} + \frac{1+\varepsilon}{2} z_1 \right) \right. \right. \\ &\quad \times \left[\frac{1}{[x(1-x)\mathbf{k}_2^2 + m^2 z_1^2]^{1-\varepsilon}} - \frac{1}{[x(1-x)\mathbf{k}_2^2]^{1-\varepsilon}} \right] \\ &\quad \left. \left. + \frac{2m^2}{\mathbf{k}_2^2} \int_0^1 \int_0^1 \frac{z_1(1-z_1) dz_1 dx}{[x(1-x)\mathbf{k}_2^2 + m^2 z_1^2]^{1-\varepsilon}} \right\} \right]. \end{aligned} \quad (3.4)$$

We cannot calculate the integral directly but we can calculate its Mellin transform,

$$\Delta \tilde{F}_q(\gamma, \varepsilon) = \Gamma(1 + \varepsilon) (m^2)^{-\varepsilon} \int d[\mathbf{k}_2] \left(\frac{\mathbf{k}_2^2}{m^2} \right)^{\gamma-1} \Delta F_q(\mathbf{k}_2), \quad (3.5)$$

to get finally in γ -space:

$$\begin{aligned} \Delta \tilde{F}_q(\gamma, \varepsilon) &= A_\varepsilon (m^2)^\varepsilon \frac{\Gamma(\gamma + \varepsilon) \Gamma(1 - \gamma - 2\varepsilon) \Gamma^2(1 - \gamma - \varepsilon)}{8\Gamma(2 - 2\gamma - 2\varepsilon)} \\ &\quad \times \left[\frac{1 + \varepsilon}{\gamma + 2\varepsilon} + \frac{2}{1 - 2\gamma - 4\varepsilon} \left(\frac{1}{1 - \gamma - 2\varepsilon} - \frac{1}{3 - 2\gamma - 2\varepsilon} \right) \right]. \end{aligned} \quad (3.6)$$

In Ref. [9], the residua of Eq. (3.6) at the poles $\gamma = 1 - \varepsilon$ and $\gamma = 1 - 2\varepsilon$ were studied and the singular terms in ε were isolated. For the remaining poles, the limit $\varepsilon \rightarrow 0$ was taken and the residua

were summed. It turns out, however, that the resulting infinite sums¹ are not suitable for further studies. Namely, the sum of residua to the left of the real axis for $\gamma > 1$ does not converge in the region $4m^2/\mathbf{k}_2^2 > 1$, which makes it impossible to obtain values of the impact factor in that region. We find that this obstacle can be overcome by keeping the full ε -dependence in the resummation of the pole-contributions and isolating the singular and finite terms after resumming the infinite sum. In the following, we keep the full ε -dependence, we perform the inverse Mellin transform by summing the residua of the poles appearing for $\gamma \geq 1 - \varepsilon$ and $\gamma \leq 1 - 2\varepsilon$ and then we expand in ε to resolve $1/\varepsilon^2$, $1/\varepsilon$ and finite terms.

The inverse Mellin transform is given by

$$\Delta F_q(\mathbf{k}_2) = \frac{1}{m^2} \int_{1-2\varepsilon < \text{Re } \gamma < 1-\varepsilon} \frac{d\gamma}{2\pi i} \left(\frac{\mathbf{k}_2^2}{m^2} \right)^{-\gamma-\varepsilon} \Delta \tilde{F}_q(\gamma, \varepsilon). \quad (3.7)$$

The integral in Eq. (3.7) can be calculated by either closing the integration contour at infinity to the left of the pole at $\gamma = 1 - 2\varepsilon$ or to the right of the pole at $\gamma = 1 - \varepsilon$ and summing the residua within the contour. The respective contributions are given by h_1^- and h_1^+ (for convenience we define $R = \frac{\mathbf{k}_2^2}{4m^2}$):

$$h_1^-(R) = \sum_{\gamma \leq 1-2\varepsilon} \text{Res} \left\{ (4R)^{1-\gamma-\varepsilon} \Delta \tilde{F}_q(\gamma, \varepsilon) \right\}, \quad (3.8)$$

$$h_1^+(R) = - \sum_{\gamma \geq 1-\varepsilon} \text{Res} \left\{ (4R)^{1-\gamma-\varepsilon} \Delta \tilde{F}_q(\gamma, \varepsilon) \right\}. \quad (3.9)$$

We choose to close the contour to the left and resum all the pole contributions. These are distinct contributions from the poles located at $\gamma = 1 - 2\varepsilon$, $\frac{1}{2} - 2\varepsilon$, $-\varepsilon$, -2ε and finally from the poles at $\gamma = -n + \varepsilon$ with n being positive integer.

3.2 The $K_0(\mathbf{k}_1, \mathbf{k}_2)$ term

Let us now turn to the final ingredient in order to have the full NLx heavy quark impact factor with mass corrections. For the real emission contribution to K_0 in Eq. (3.1) we define the integral

$$I_m = \int d[\mathbf{k}_1] \frac{\bar{\alpha}_s h_q^{(0)}(\mathbf{k}_1)}{\mathbf{q}^2 \Gamma(1-\varepsilon) \mu^{2\varepsilon}} \log \frac{m}{k_1} \Theta_{mk_1}. \quad (3.10)$$

Eq. (3.10) can then be rewritten as:

$$\begin{aligned} I_m &= \frac{A_\varepsilon}{2} \lim_{\alpha \rightarrow 0^+} \int_{-i\infty}^{+i\infty} \frac{d\lambda}{2\pi i} \frac{1}{(\lambda + \alpha)^2} (m^2)^\lambda \int \frac{d[\mathbf{k}_1]}{\mathbf{q}^2 (\mathbf{k}_1^2)^{1+\lambda}} \\ &= \frac{A_\varepsilon}{2} \lim_{\alpha \rightarrow 0^+} \int_{-i\infty}^{+i\infty} \frac{d\lambda}{2\pi i} \frac{1}{(\lambda + \alpha)^2} \frac{\Gamma(1 + \lambda - \varepsilon) \Gamma(\varepsilon) \Gamma(\varepsilon - \lambda)}{\Gamma(1 + \lambda) \Gamma(2\varepsilon - \lambda)} (m^2)^\lambda (\mathbf{k}_2^2)^{-1-\lambda+\varepsilon}, \end{aligned} \quad (3.11)$$

where $\alpha > 0$. To recover I_m the limit $\alpha \rightarrow 0$ has to be taken. Eq. (3.11) has a form similar to Eq. (3.7). A similar procedure to the one in Section 3.1 can be applied in order to calculate the integral on the right-hand side of Eq. (3.7). The leading poles containing the singular terms in ε

¹See Ref. [9]

are now at $\lambda = -\alpha$, ($\alpha \rightarrow 0^+$), if the integration contour is closed to the left and at $\lambda = \varepsilon$ if it is closed to the right.

In analogy with $h_1^\pm(R)$ we define $h_2^\pm(R)$. After summing the residues of the integrand of Eq. (3.7) and expanding in ε we obtain for $\mathbf{k}_2^2 < m^2$:

$$h_2^-(R) = h^{(0)}(\mathbf{k}_2) \omega^{(1)}(\mathbf{k}_2) \left(-\varepsilon^{-1} + \log(4R) - \varepsilon \text{Li}_2(4R) \right), \quad (3.12)$$

and for $\mathbf{k}_2^2 > m^2$:

$$h_2^+(R) = h^{(0)}(\mathbf{k}_2) \omega^{(1)}(\mathbf{k}_2) \left[-\varepsilon^{-1} + \log(4R) - \varepsilon \left(\frac{1}{2} \log^2(4R) + \text{Li}_2\left(\frac{1}{4R}\right) \right) \right]. \quad (3.13)$$

It is now noteworthy to explain why $h_2^-(R) \neq h_2^+(R)$ in contrast to $h_1^-(R) = h_1^+(R)$. The original integral definition of I_m in Eq. (3.10) contains the θ -function Θ_{mk_1} . After rewriting it into the form of Eq. (3.11) we see that the θ -function generates after integration a discontinuity in the first derivative of I_m .

For $\mathbf{k}_2^2 < m^2$ we have to include also the virtual term of K_0 , which leads to the contribution

$$h_V^+(\mathbf{k}_2) = -h^{(0)}(\mathbf{k}_2) \omega^{(1)}(\mathbf{k}_2) \log\left(\frac{\mathbf{k}_2^2}{m^2}\right) \Theta_{mk_2}. \quad (3.14)$$

3.3 Final compact expression for the heavy quark impact factor

The contributions discussed in the previous sections can be put together into a very compact formula. The NLx heavy quark impact factor can be expressed as the sum

$$h_q(\mathbf{k}_2) = h_q^{(1)}(\mathbf{k}_2)|_{\text{sing}} + h_q(\mathbf{k}_2)|_{\text{finite}}, \quad (3.15)$$

where the singular term $h_q^{(1)}(\mathbf{k}_2)|_{\text{sing}}$ has been calculated in Ref. [9], and the finite contribution is given by

$$h_q(\mathbf{k}_2)|_{\text{finite}} = h_q^{(0)}(\alpha_s(\mathbf{k}_2)) \left\{ 1 + \frac{\alpha_s N_c}{2\pi} \left[\mathcal{K} - \frac{\pi^2}{6} + 1 - \log(R_1) \left((1+2R) \sqrt{\frac{1+R}{R}} + 2 \log(R_1) \right) \right. \right. \\ \left. \left. - 3\sqrt{R} \left(\text{Li}_2(R_1) - \text{Li}_2(-R_1) + \log(R_1) \log\left(\frac{1-R_1}{1+R_1}\right) \right) \right. \right. \\ \left. \left. + \text{Li}_2(4R) \Theta_{mk_2} + \left(\frac{1}{2} \log(4R) + \frac{1}{2} \log^2(4R) + \text{Li}_2\left(\frac{1}{4R}\right) \right) \Theta_{k_2 m} \right] \right\}, \quad (3.16)$$

with $R_1 = (\sqrt{R} + \sqrt{1+R})^{-1}$, and \mathcal{K} being

$$\mathcal{K} = \frac{67}{18} - \frac{\pi^2}{6} - \frac{5n_f}{9N_c}. \quad (3.17)$$

As in Ref. [9], we have absorbed the singularities proportional to the beta function into the running of the strong coupling constant, $\alpha_s(\mathbf{k}_2)$ [10].

4. Conclusions and outlook

We discussed in some detail how, based on the results in Ref. [9], we obtained expressions suitable for the numerical implementation of the heavy quark impact factor in momentum space. We have also showed, that the first derivative in the t -channel transverse momentum \mathbf{k}_2 of the finite part of the heavy quark impact factor exhibits a discontinuity at the point where the transverse momentum is equal to the mass of the heavy quark, $|\mathbf{k}_2| = m$. We will present further results of the actual numerical implementation [11] of the heavy quark impact factor in momentum and γ -space [12] for phenomenological studies —such as the single bottom quark production at the LHC— elsewhere.

Acknowledgments

This work has been supported by the Research Executive Agency (REA) of the European Union under the Grant Agreement number PITN-GA-2010-264564 (LHCPhenoNet), by the Spanish Government and EU ERDF funds (grants FPA2007-60323, FPA2011-23778 and CSD2007-00042 Consolider Project CPAN) and by GV (PROMETEUII/2013/007). GC acknowledges support from Marie Curie actions (PIEF-GA-2011-298582). MD acknowledges support from Juan de la Cierva programme (JCI-2011-11382).

References

- [1] L. N. Lipatov, *Sov. J. Nucl. Phys.* **23** (1976) 338; E. A. Kuraev, L. N. Lipatov, V. S. Fadin, *Phys. Lett. B* **60** (1975) 50, *Sov. Phys. JETP* **44** (1976) 443, *Sov. Phys. JETP* **45** (1977) 199; Ia. Ia. Balitsky, L. N. Lipatov, *Sov. J. Nucl. Phys.* **28** (1978) 822.
- [2] V. S. Fadin, L. N. Lipatov, *Phys. Lett. B* **429** (1998) 127 [hep-ph/9802290]; M. Ciafaloni, G. Camici, *Phys. Lett. B* **430** (1998) 349 [hep-ph/9803389].
- [3] G. P. Salam, *JHEP* **9807**, 019 (1998) [hep-ph/9806482]; M. Ciafaloni, D. Colferai and G. P. Salam, *Phys. Rev. D* **60**, 114036 (1999) [hep-ph/9905566].
- [4] M. Hentschinski, A. S. Vera and C. Salas, arXiv:1301.5283 [hep-ph].
- [5] M. Ciafaloni and D. Colferai, *Nucl. Phys. B* **538**, 187 (1999) [hep-ph/9806350].
- [6] A. H. Mueller and H. Navelet, *Nucl. Phys. B* **282** (1987) 727.
- [7] A. Sabio Vera and F. Schwennsen, *Nucl. Phys. B* **776** (2007) 170 [hep-ph/0702158 [HEP-PH]].
- [8] J. Kwiecinski, A. D. Martin, L. Motyka and J. Outhwaite, *Phys. Lett. B* **514** (2001) 355 [hep-ph/0105039].
- [9] M. Ciafaloni and G. Rodrigo, *JHEP* **0005** (2000) 042 [hep-ph/0004033].
- [10] G. Rodrigo and A. Santamaria, *Phys. Lett. B* **313** (1993) 441 [hep-ph/9305305]; G. Rodrigo, A. Pich and A. Santamaria, *Phys. Lett. B* **424** (1998) 367 [hep-ph/9707474].
- [11] G. Chachamis, M. Deak and G. Rodrigo, work in progress.
- [12] G. Chachamis, M. Deak, M. Hentschinski, G. Rodrigo, C. Salas and A. S. Vera, work in progress.