

## Diffractive vector meson production at HERA using holographic AdS/QCD wavefunctions

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We demonstrate another success of the AdS/QCD correspondence by showing [1, 2] that an AdS/QCD holographic light-front wavefunction for the  $\rho$  meson generates predictions for the cross-sections of diffractive  $\rho$  production that are in agreement with data collected at the HERA electron-proton collider [3, 4].

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## 1. Introduction

The AdS/QCD correspondence [5, 6, 7, 8] refers to the connection between QCD in physical spacetime and string theory in a higher dimensional anti-de Sitter (AdS) space. The precise nature of this connection has not yet been elucidated but there is growing evidence, to which we add here, that there exists such a connection. One particular realization of this connection is light-front holography [9] proposed by Brodsky and de Téramond. In light-front holography, the confining QCD potential at equal light-front time between a quark and antiquark in a meson is determined by the profile of the dilaton field which breaks conformal invariance of the higher dimensional AdS space in which strings propagate.

In a semi-classical approximation to light-front QCD, Brodsky and de Téramond derived a Schroedinger-like equation for mesons:

$$\left( -\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right) \Phi(\zeta) = M^2 \Phi(\zeta), \quad (1.1)$$

where  $\zeta = \sqrt{x(1-x)}r$  is the transverse separation between the quark and antiquark at equal light-front time<sup>1</sup>,  $L$  is the orbital quantum number,  $M$  is the mass of the meson and  $\Phi(\zeta)$  is the transverse mode of the light-front wavefunction which is itself given by

$$\phi(x, \zeta, \varphi) = \frac{\Phi(\zeta)}{\sqrt{2\pi\zeta}} f(x) e^{iL\varphi}. \quad (1.2)$$

It remains a challenge to derive the confining potential  $U(\zeta)$  from first-principles QCD but after identifying  $\zeta$  with the co-ordinate in the fifth dimension and angular momentum with the fifth dimensional mass<sup>2</sup>, equation (1.1) describes the propagation of spin- $J$  string modes, in which case  $U(\zeta)$  is determined by the choice for the dilaton field. Remarkably, it can be shown [10] that the dilaton profile is constrained to be quadratic so that the resulting confining potential is given by

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(J-1). \quad (1.3)$$

Solving equation (1.1) with this confining potential yields the eigenfunctions

$$\Phi_{nL}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n}{(n+L)}} \zeta^{1/2+L} \exp(-\kappa^2 \zeta^2 / 2) L_n^L(\kappa^2 \zeta^2) \quad (1.4)$$

with the corresponding eigenvalues

$$M_{nL,S}^2 = 4\kappa^2 \left( n + L + \frac{S}{2} \right). \quad (1.5)$$

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<sup>1</sup> $x$  is the fraction of light-front momentum carried by the quark and  $r$  is the transverse separation between the quark and the antiquark at equal ordinary time

<sup>2</sup> $(m_5 R)^2 = -(2-J)^2 + L^2$  where  $R$  is the radius of curvature in AdS space.

## 2. The $\rho$ meson wavefunction

For the  $\rho$  meson,  $n = 1$ ,  $L = 0$  and  $J = 1$  so that  $\kappa = M_\rho/\sqrt{2} = 0.54$  GeV. In equation (1.2),  $f(x)$  is fixed by comparing the expressions for the pion EM form factor in light-front QCD and in AdS space. This yields  $f(x) = \sqrt{x(1-x)}$  so that the resulting AdS/QCD for the  $\rho$  is then given by

$$\phi(x, \zeta) \propto \sqrt{x(1-x)} \exp\left(-\frac{\kappa^2 \zeta^2}{2}\right) \exp\left(-\frac{m_f^2}{2\kappa^2 x(1-x)}\right) \quad (2.1)$$

where the dependence on the quark mass has been introduced according to the prescription by Brodsky and de Téramond [11]. Here we use a light quark mass  $m_f = 0,14$  GeV [1].

An earlier procedure to obtain the meson wavefunction is by boosting a non relativistic gaussian Schroedinger wavefunction [12, 13] which results in the so-called Boosted Gaussian (BG):

$$\phi^{\text{BG}}(x, \zeta) \propto x(1-x) \exp\left(\frac{m_f^2 R^2}{2}\right) \exp\left(-\frac{m_f^2 R^2}{8x(1-x)}\right) \exp\left(-\frac{2\zeta^2}{R^2}\right). \quad (2.2)$$

If  $R^2 = 4/\kappa^2$  then the two wavefunctions differ only by a factor of  $\sqrt{x(1-x)}$ , which is not surprising given that in both cases confinement is modelled by a harmonic oscillator [1]. In what follows we shall consider a parameterization that accommodates both the AdS/QCD and the BG wavefunctions:

$$\phi(x, \zeta) \propto [x(1-x)]^\beta \exp\left(-\frac{\kappa^2 \zeta^2}{2}\right) \exp\left(-\frac{m_f^2}{2\kappa^2 x(1-x)}\right). \quad (2.3)$$

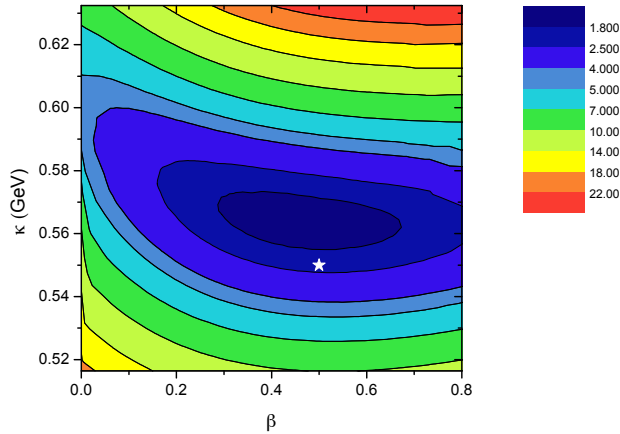
The AdS/QCD wavefunction is obtained by fixing  $\beta = 0.5$  and  $\kappa = 0.55$  GeV where as the BG wavefunction is obtained by fixing  $\beta = 1$  and treating  $\kappa$  as a free parameter.

## 3. Results and conclusions

To compute the rate for diffractive  $\rho$  production, we use the dipole model of high-energy scattering [14, 15, 16, 17] in which the scattering amplitude for diffractive  $\rho$  meson production is a convolution of the photon and vector meson  $q\bar{q}$  light-front wavefunctions with the total cross-section to scatter a  $q\bar{q}$  dipole off a proton. QED is used to determine the photon wavefunction and the dipole cross-section can be extracted from the precise data on the deep-inelastic structure function  $F_2$  [18, 19]. This formalism can then be used to predict rates for vector meson production and diffractive DIS [13, 20] or to extract information on the  $\rho$  meson wavefunction using the HERA data on diffractive  $\rho$  production [21, 22]. Here we use it to test whether the HERA data prefer the AdS/QCD wavefunction given by equation (2.1). To do so, we compute the  $\chi^2$  per data point in the  $(\beta, \kappa)$  parameter space using the parametrization (2.3) for the  $\rho$  wavefunction.<sup>3</sup>

Figure 1 confirms that the AdS/QCD prediction lies impressively close to the minimum in  $\chi^2$ . The best fit has a  $\chi^2$  per data point equal to 114/76 and is achieved with  $\kappa = 0.56$  GeV and  $\beta = 0.47$  which should be compared with the AdS/QCD prediction:  $\kappa = 0.54$  and  $\beta = 0.5$  shown

<sup>3</sup>We include the electroproduction data and decay width datum in the fit.



**Figure 1:** The  $\chi^2$  distribution in the  $(\beta, \kappa)$  parameter space. The AdS/QCD prediction is the white star.

as the white star on figure 1. Note that the BG prediction i.e.  $\beta = 1, \forall \kappa$ , is clearly further away from the minimum in  $\chi^2$ .

Finally, we note that these results are produced using a particular Color Glass Condensate dipole model [18] but that similar results are obtained by using other forward dipole models [19] that fit the  $F_2$  structure function data. It remains to be seen how the  $\chi^2$  distribution changes if a more sophisticated dipole model, such as the recent impact parameter saturation model [23] which fits the combined HERA  $F_2$  data, is used.

## References

- [1] J. R. Forshaw and R. Sandapen, *An AdS/QCD holographic wavefunction for the rho meson and diffractive rho meson electroproduction*, *Phys.Rev.Lett.* **109** (2012) 081601, [[arXiv:1203.6088](#)].
- [2] J. R. Forshaw and R. Sandapen, *Diffractive rho production with an AdS/QCD holographic wavefunction for the rho meson*, *AIP Conf.Proc.* **1523** (2012) 87–90, [[arXiv:1211.4729](#)].
- [3] ZEUS Collaboration, S. Chekanov *et. al.*, *Exclusive  $\rho^0$  production in deep inelastic scattering at HERA*, *PMC Phys.* **A1** (2007) 6, [[arXiv:0708.1478](#)].
- [4] The H1 Collaboration, *Diffractive Electroproduction of  $\rho$  and  $\phi$  Mesons at HERA*, *JHEP* **05** (2010) 032, [[arXiv:0910.5831](#)].
- [5] J. Erdmenger, N. Evans, I. Kirsch, and E. Threlfall, *Mesons in Gauge/Gravity Duals - A Review*, *Eur.Phys.J.* **A35** (2008) 81–133, [[arXiv:0711.4467](#)].
- [6] J. Casalderrey-Solana, H. Liu, D. Mateos, K. Rajagopal, and U. A. Wiedemann, *Gauge/String Duality, Hot QCD and Heavy Ion Collisions*, [arXiv:1101.0618](#).
- [7] M. S. Costa and M. Djuric, *Deeply Virtual Compton Scattering from Gauge/Gravity Duality*, [arXiv:1201.1307](#).
- [8] G. F. de Teramond and S. J. Brodsky, *Hadronic Form Factor Models and Spectroscopy Within the Gauge/Gravity Correspondence*, [arXiv:1203.4025](#).

- [9] G. F. de Teramond and S. J. Brodsky, *Light-Front Holography: A First Approximation to QCD*, *Phys.Rev.Lett.* **102** (2009) 081601, [[arXiv:0809.4899](#)].
- [10] S. J. Brodsky, G. F. de Teramond, and H. G. Dosch, *Conformal Symmetry, Confinement, and Light-Front Holographic QCD*, [arXiv:1302.5399](#).
- [11] S. J. Brodsky and G. F. de Teramond, *Light-Front Holography and Novel Effects in QCD*, *AIP Conf.Proc.* **1116** (2009) 311–326, [[arXiv:0812.3192](#)].
- [12] J. Nemchik, N. N. Nikolaev, E. Predazzi, and B. G. Zakharov, *Color dipole phenomenology of diffractive electroproduction of light vector mesons at HERA*, *Z. Phys.* **C75** (1997) 71–87, [[hep-ph/9605231](#)].
- [13] J. R. Forshaw, R. Sandapen, and G. Shaw, *Colour dipoles and  $\rho$ ,  $\phi$  electroproduction*, *Phys. Rev.* **D69** (2004) 094013, [[hep-ph/0312172](#)].
- [14] N. N. Nikolaev and B. G. Zakharov, *Colour transparency and scaling properties of nuclear shadowing in deep inelastic scattering*, *Z. Phys.* **C49** (1991) 607–618.
- [15] N. N. Nikolaev and B. G. Zakharov, *Pomeron structure function and diffraction dissociation of virtual photons in perturbative QCD*, *Z. Phys.* **C53** (1992) 331–346.
- [16] A. H. Mueller, *Soft gluons in the infinite momentum wave function and the BFKL pomeron*, *Nucl. Phys.* **B415** (1994) 373–385.
- [17] A. H. Mueller and B. Patel, *Single and double BFKL pomeron exchange and a dipole picture of high-energy hard processes*, *Nucl. Phys.* **B425** (1994) 471–488, [[hep-ph/9403256](#)].
- [18] G. Soyez, *Saturation QCD predictions with heavy quarks at HERA*, *Phys. Lett.* **B655** (2007) 32–38, [[arXiv:0705.3672](#)].
- [19] J. R. Forshaw and G. Shaw, *Gluon saturation in the colour dipole model?*, *JHEP* **12** (2004) 052, [[hep-ph/0411337](#)].
- [20] J. R. Forshaw, R. Sandapen, and G. Shaw, *Further success of the colour dipole model*, *JHEP* **11** (2006) 025, [[hep-ph/0608161](#)].
- [21] J. R. Forshaw and R. Sandapen, *Extracting the rho meson wavefunction from HERA data*, *JHEP* **11** (2010) 037, [[arXiv:1007.1990](#)].
- [22] J. R. Forshaw and R. Sandapen, *Extracting the Distribution Amplitudes of the rho meson from the Color Glass Condensate*, *JHEP* **1110** (2011) 093, [[arXiv:1104.4753](#)].
- [23] A. H. Rezaeian, M. Siddikov, M. Van de Klundert, and R. Venugopalan, *Analysis of combined HERA data in the Impact-Parameter dependent Saturation model*, *Phys.Rev.* **D87** (2013) 034002, [[arXiv:1212.2974](#)].