

The Vincia Parton Shower

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We summarize recent developments in the VINCIA parton shower. After a brief review of the basics of the formalism, the extension of VINCIA to hadron collisions is sketched. We then turn to improvements of the efficiency of tree-level matching by making the shower history unique and by incorporating identified helicities. We conclude with an overview of matching to one-loop matrix elements.

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1. Introduction

The VINCIA parton shower [1, 2, 3, 4, 5, 6, 7] is a plugin to the PYTHIA 8 [8] event generator based on the dipole-antenna picture of QCD radiation [9]. It emphasizes comprehensive uncertainty estimates and efficient matching to fixed-order matrix elements (obtained from [10] in the present implementation).

2. Formalism

We give a very condensed overview of the VINCIA formalism as presented in [1, 2]. Like every parton shower, VINCIA implements the unitary evolution of a parton configuration from a scale Q_{start} on the order of the hard scale of the partonic interaction to a scale Q_{had} on the order of the hadron masses. Working in the large N_C limit of QCD, a configuration is organized into pairs of color-connected partons, called antennæ, which emit independently. The algorithm can then be characterized by the non-emission probability when evolving an antenna from Q_{start} to Q_{emit} , also referred to as the Sudakov factor:

$$\Delta(Q_{\text{start}}^2, Q_{\text{emit}}^2) = \exp \left[- \int_{Q_{\text{emit}}^2}^{Q_{\text{start}}^2} a_c d\Phi_{\text{ant}} \right], \quad a_c = g_s^2(Q^2) C \bar{a}, \quad (2.1)$$

where a_c is the antenna function composed of the coupling constant, the color factor and the color-and coupling-stripped antenna function \bar{a} . It encodes the singular unresolved limits of tree-level matrix elements¹ such that

$$\frac{|M_n^0|^2}{\sum_{ijk \rightarrow IK} a_c(i, j, k) |M_{n-1}^0(\dots, I, K, \dots)|^2} \xrightarrow{\text{single unresolved limit}} 1. \quad (2.2)$$

The antenna is integrated over the antenna phase space $d\Phi_{\text{ant}}$ which satisfies

$$d\Phi_n(\dots, i, j, k, \dots) = d\Phi_{n-1}(\dots, I, K, \dots) d\Phi_{\text{ant}}(I, K \rightarrow i, j, k) \quad (2.3)$$

where $p_i + p_j + p_k = p_I + p_K$ and all momenta are on-shell. In terms of invariants it is given by

$$d\Phi_{\text{ant}} = \frac{1}{16\pi^2} \frac{1}{m_{IK}^2} ds_{ij} ds_{jk} \frac{d\phi}{2\pi} \quad (2.4)$$

where $s_{ij} = 2 p_i \cdot p_j$ and ϕ parametrizes rotations around \mathbf{p}_I in the center-of-mass frame of $p_I + p_K$. Several choices for the definition of Q are implemented in VINCIA, among them $Q^2 \propto s_{ij} s_{jk} / m_{IK}^2$ and $Q^2 \propto \min(s_{ij}, s_{jk})$.

The direct inversion of the Sudakov factor for the emission scale Q_{emit} is cumbersome. Therefore, VINCIA uses the veto algorithm. It uses a trial Sudakov factor

$$\Delta^{\text{trial}}(Q_{\text{start}}^2, Q_{\text{emit}}^2) = \exp \left[- \int_{Q_{\text{emit}}^2}^{Q_{\text{start}}^2} a_c^{\text{trial}} d\Phi_{\text{ant}}^{\text{trial}} \right] \quad (2.5)$$

¹Neglecting spin correlations and color-subleading contributions.

to generate trial emissions, where a_c^{trial} is an overestimate of the physical antenna function and $\Phi_{\text{ant}}^{\text{trial}}$ is a hull of the physical antenna phase space, both chosen such that the inversion of eq. 2.5 is simple. The trial emissions are then accepted with probability

$$P_{\text{accept}} = \frac{a_c}{a_c^{\text{trial}}} [\{p\} \in \Phi_{\text{ant}}] \quad (2.6)$$

where the angle bracket denotes unity if the condition is true and zero otherwise.

3. Extension to Hadron Collisions

To extend the VINCIA formalism to incoming hadrons, as described in more detail in [5], we have to include the ratio of parton distribution functions before and after the branching in the integrated antenna [11]:

$$\Delta(Q_{\text{start}}^2, Q_{\text{emit}}^2) = \exp \left[- \int_{Q_{\text{emit}}^2}^{Q_{\text{start}}^2} a_c \frac{f_a(x_a, Q^2)}{f_A(x_A, Q^2)} \frac{f_b(x_b, Q^2)}{f_B(x_B, Q^2)} d\Phi_{\text{ant}} \right], \quad (3.1)$$

where a, b are the generic labels we use for incoming partons after branching and A, B denote their pre-branching counterparts. The resolution is defined in [5] as the crossing of $Q^2 \propto |s_{ij}| |s_{jk}| / (|s_{ij}| + |s_{jk}| + |s_{ik}|)$. This amounts to an evolution variable which does not change if an outgoing parton is replaced by an incoming one with the same momentum. Alternative definitions of Q for incoming partons are left to future investigations.

The antenna functions a_c still obey eq. 2.2, but since incoming partons cannot become soft, they are not the crossings of their final-final counterparts in general.

We also need to specify the phase space factorization used if incoming partons are involved. At variance with the final-final case, only the integral of the phase space over the momentum fraction(s) of the incoming emitter(s) factorizes. For initial–final antennæ we use a variant which does only rescale the incoming emitter [12]:

$$\int \frac{dx_a}{x_a} d\Phi_3(-a, -b; j, k, R) = \int \frac{dx_A}{x_A} d\Phi_2(-A, -b; K, R) d\Phi_{\text{ant}}^{if}(-A; K \rightarrow -a; j, k) \quad (3.2)$$

where R denotes one or more outgoing particles and incoming partons are indicated by a minus sign. The pre- and post-branching momentum fractions are related by $x_A/x_a = s_{AK}/(s_{AK} + s_{jk})$. The initial–final antenna phase space in terms of invariants is

$$d\Phi_{\text{ant}}^{if}(-A; K \rightarrow -a; j, k) = \frac{1}{16\pi^2} \frac{s_{AK}}{(s_{AK} + s_{jk})^2} ds_{jk} ds_{aj} \quad (3.3)$$

with the boundaries $0 \leq s_{jk} \leq s_{AK}(1 - x_A)/x_A$, $0 \leq s_{aj} \leq s_{AK} + s_{jk}$. The third coordinate of the initial–final phase space has been suppressed above since the antenna function does not depend on it.

Initial–initial branchings add a parton with momentum transverse to the beam to the event. In the phase space factorization we use, this transverse momentum is compensated by all the other particles in the event (both colored and colorless). This is implemented by replacing p_A, p_B by

p_a, p_b, p_j , with $p_A + p_B = p_a + p_b - p_j$, followed by applying a Lorentz boost such that p_a and p_b are aligned with the beams again. The corresponding phase space factorization is [13]

$$\int \frac{dx_a}{x_a} \frac{dx_b}{x_b} d\Phi_2(-a, -b; j, R) = \int \frac{dx_A}{x_A} \frac{dx_B}{x_B} d\Phi_1(-A', -B'; R') d\Phi_{\text{ant}}^{ii} \quad (3.4)$$

with the initial–initial antenna phase space in terms of invariants given by

$$d\Phi_{\text{ant}}^{ii}(-A', -B' \rightarrow -a, -b; j) = \frac{1}{16\pi^2} \frac{s_{AB}}{s_{ab}^2} \theta(1-x_a) \theta(1-x_b) ds_{aj} ds_{jb} \quad (3.5)$$

where we have suppressed the integration over the angle ϕ parametrizing rotations around the beam. Momenta X' are Lorentz transformed with respect to the frame in which p_a and p_b are aligned with the beam directions.

4. Uncertainties

VINCIA offers variations of many of its parameters to enable an assessment of the uncertainty its predictions have. For example, it allows changing the antenna functions \bar{a} by terms which do not contribute in single unresolved limits or switching between color factors which differ by terms suppressed by N_C^{-2} . As elaborated upon in [2], it also provides the option to explore these uncertainty variations at a small computational cost. For this purpose, a weight corresponding to each uncertainty variation is produced along with each (unweighted) event. Applying these weights (which average to unity by construction), one obtains the distribution of events as it would have been produced when running VINCIA with the modified parameters. This procedure generalizes naturally to parton distribution function uncertainties for the ratio occurring in eq. 3.1.

5. Tree-level Matching

VINCIA implements a unitary matching to consecutive tree-level matrix elements over all of phase space². This means that it produces unweighted events such that the expansion of the probability density for n -jet events to Born level agrees with the fixed-order description. This is achieved by changing the accept probability in eq. 2.6 to

$$P_{\text{accept}} = \frac{a_c}{a_{\text{trial}}} [\{p\} \in \Phi_{\text{ant}}] P_{\text{ME}}, \quad P_{\text{ME}} = \frac{|M_n^0|^2}{\sum_{ijk \rightarrow IK} a_c(i, j, k) |M_{n-1}^0(\dots, I, K, \dots)|^2}. \quad (5.1)$$

The sum in the denominator of the matching factor P_{ME} runs over all $n-1$ -parton configurations from which the shower could have arrived at the n -parton configuration in question. The formula is valid under the assumption that tree-level matching has already been performed at multiplicity $n-1$ and that the shower algorithm does not depend on how we have arrived at a configuration. By virtue of eq. 2.2, the matching factor tends to unity in single unresolved limits.

Using a shower algorithm in which all antennæ contribute over all of phase space, the cost of calculating the denominator of P_{ME} grows linearly with n . In [4], a variant of the VINCIA

²Technically, there is still a matching scale, starting from the third emission by default. However, this scale it is not a defining feature of the matching. It is needed because the shower misses some subleading logarithms.

algorithm was presented in which there is always exactly one shower history for each phase space point, enabling substantially faster running times for the generation of tree-level matched samples. A further extension of the VINCIA formalism to identified parton helicities has been described in [6], building on earlier work in [14]. It enables to perform the matching by evaluating only individual helicity amplitudes, which is particularly helpful at high multiplicities.

6. One-loop Matching

In [7], matching of VINCIA to next-to-leading order has been described and demonstrated explicitly for to $Z \rightarrow 3$ jets. The paradigm is the same as for tree-level matching: A differential matching factor is applied to the trial accept probability such that the expansion of the parton shower agrees with the fixed-order result.

In the example of $Z \rightarrow 3$ jets, we want to reproduce the exclusive 3-jet rate:

$$R_3^{\text{ex}} = |M_3^0|^2 + 2\Re(M_3^0 M_3^{1*}) + \int_0^{Q_{\text{had}}^2} \frac{d\Phi_4}{d\Phi_3} |M_4^0|^2 \quad (6.1)$$

where it is required that additional radiation is unresolved at the scale Q_{had} at which the shower is stopped. For this purpose, a fully differential matching factor is required such that

$$(1 + V_3) \text{Approximate} = R_3^{\text{ex}} + \mathcal{O}(\alpha_S^3) \quad (6.2)$$

where ‘‘Approximate’’ stands for the tree-level matched shower approximation. V_3 is written almost completely in terms of integrals which are calculated analytically. The only integrals entering V_3 which have to be computed numerically are related to the fact that due to the tree-level matching at the four-parton level, $|M_4^0|^2$ enters into the calculation of V_3 . However, these integrals are well-behaved and give small contributions for almost all phase space points.

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