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About the EW contribution to the relation between pole and $\overline{\text{MS}}$ masses of the top-quark in the Standard Model

Fred Jegerlehner
Humboldt-Universität zu Berlin, Institut für Physik,
Newtonstrasse 15, D-12489 Berlin, Germany
Deutsches Elektronen-Synchrotron (DESY), Platanenallee 6, D-15738 Zeuthen, Germany

Mikhail Kalmykov[*†]
II. Institut für Theoretische Physik, Universität Hamburg,
Luruper Chaussee 149, D-22761 Hamburg, Germany
E-mail: kalmykov.mikhail@gmail.com

Bernd A. Kniehl
II. Institut für Theoretische Physik, Universität Hamburg,
Luruper Chaussee 149, D-22761 Hamburg, Germany

Results of our recent re-analysis of the electroweak contribution to the relation between pole and running masses of top-quark within the Standard Model is reviewed. We argue, that if vacuum of SM is stable, then there exists an optimal value of renormalization group scale (IR-point), at which the radiative corrections to the matching condition between parameters of Higgs sector and pole masses is minimal or equal to zero. Within the available accuracy, we find the IR-point to lie in an interval between value of Z-boson mass and twice the value of W-boson mass. The value of scale is relevant for extraction of Higgs self-coupling from cross-section as well as for construction of effective Lagrangian.

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Motivation. After the discovery of the Higgs boson [1] - the last important building block of the Standard Model (SM) required by its renormalizability [2]- and the still missing direct detection of new physics beyond SM at the LHC, the self-consistency of the SM has attracted a lot of notice. The key question is the stability of the Higgs potential when extending SM physics to such high scales as the Planck mass, where we know gravity must come into play and SM physics alone cannot lead further. It would be interesting to determine the scale at which SM may break down [3]. One of the approaches to answer this question is based on the renormalization group (RG) analysis of the SM running couplings, specifically of the Higgs self-coupling λ and the question whether it stays positive up to the Planck scale which would imply the vacuum to remain stable. The recent re-analysis of the stability of vacuum performed in [4, 5], with 3-loop RG functions evaluated in [6, 7, 8], revealed a surprisingly small "critical value" of Higgs boson mass (i.e. the stability bound), $M_H^{\rm cr.} \sim 129$ GeV. The main difference between [4] and [5] was related to the uncertainties adopted for the input parameters $\delta M_H^{\rm cr.} = \pm 6$ GeV in [4] versus $\delta M_H^{\rm cr.} = \pm 1.5$ GeV in [5]. These stability bounds sensitively depend on the value of pole mass of top-quark, M_t and/or its $\overline{\rm MS}$ version m_t and therefore a careful evaluation of the relationship between M_t and m_t is mandatory. An updated analysis of the determination of M_t and its uncertainties has been presented in [9]. It has been pointed out, that $M_t = 173.1 \pm 0.7$ GeV, which has been used in [5] as an input, does not relate directly to the value of pole mass of the top-quark (see also the discussion in [10]). Adopting the updated value [9], $M_t = 173.3 \pm 2.8$ GeV, (an indirect determination yielded $M_t = 175.7^{+3.0}_{-2.2}$ GeV [11] or $M_t = 175.8^{+2.7}_{-2.4}$ GeV [12]) the results of [4] and [5] are close to each other: $\delta M_H^{\rm cr.} = \pm 5.6$ GeV (see also the detailed discussion of [4, 5, 9] in [13]). In the case of stability of vacuum [4, 5, 14, 15], the SM works up to the Planck scale and new physics is not necessary to cure for the instability of the vacuum [1]. The interrelation between the SM and gravity/cosmology have been discussed in [17, 18].

The value of the pole-mass of top-quark has been extracted from measurements of the hadronic cross section $\sigma_{pp\to t\bar{t}}$ at the Tevatron and the LHC: for example, $M_t = 173.18 \pm 0.94$ GeV [19], $M_t = 173.3 \pm 1.4$ GeV [20], $M_t = 172.31 \pm 1.55$ GeV [21], $M_t = 176.7^{+3.8}_{-3.4}$ GeV [22]. Recent theoretical QCD predictions, like the results of the direct perturbative evaluation of QCD NNLO corrections to the total inclusive top-quark pair production cross-section at hadronic colliders [23], can be improved by inclusion of the electroweak (EW) corrections [24]. Numerically, the EW contribution to the hadronic $t\bar{t}$ production is of the order of a few percent. Differential cross sections can be affected substantially by EW corrections especially with increasing LHC energies [25].

The standard parametrization of cross-sections is based on the on-shell scheme with pole-masses of particles as input parameters. However, the pole mass of a quark suffers from renormalon contributions [26], which give rise to a slow convergence of the perturbative expansion for any physical observable parametrized in terms of on-shell parameters. Moreover, due to the confinement of quark and gluons, the pole mass of a quark does not have an evident physical meaning. Early in the history of perturbative QCD calculations it has been noticed that the $\overline{\rm MS}$ parametrization has better convergence properties and in addition is much simpler (mass independent) in describing the scale dependence governed by the renormalization group equations. The advantage of utilizing the $\overline{\rm MS}$ mass (running mass) m_t of the top-quark as input parameter for parametrization

[1]The new analysis is presented in [16]

of top-pair production has been explored in [27]. It has been shown in by direct calculations, that there is an essential reduction of the scale dependence as well as a faster convergence of the perturbative expansion for the $t\bar{t}$ production cross section. If fact, the NLO and NNLO corrections are much smaller in the $\overline{\text{MS}}$ parametrization. The same properties - stability of scale dependence and fast convergence of perturbative series - are valid also for differential distributions [28]. To include the EW contribution [24] in a corresponding way as performed in [27], the EW corrections in the relation between pole and $\overline{\text{MS}}$ masses should be also included.

The EW contribution to the running mass of the top-quark. For the low energy experiments at LEP energies, the splitting of full SM corrections into QED-, weak- and QCD-contributions has been quite reasonable - within NLO accuracy, the mixing effects do not play an essential role. Moreover, the QED and QCD corrections are “universal”, in the sense that they depend mainly on the number of fermions and on their masses. However, in order to achieve percent level precision theoretical predictions not only QCD NNLO radiative corrections should be included. The EW part as well as mixing EW $\times$ QCD corrections have to be included in a systematic way. For example, the QCD interaction is not responsible for the non-zero width of top-quark, which can be understood precisely only by inclusion of EW interaction. In any case, this non-zero width (EW effect) should be included in QCD corrections [23, 27, 24] (which has not yet been done) and can modify the prediction up to 1% (the technique of [29] can be directly applied in this case).

In [14] we have evaluated the EW contribution to the relation between pole- and running-mass of the top quark within the SM. The main effect is due to the matching conditions between M_t and m_t at the scale M_t. In addition, the running of parameters has to be taken into account. The quark mass anomalous dimension $\mu^2 \frac{d}{d\mu^2} \ln m_q^2(\mu^2) = \gamma_q(\alpha_s, \alpha)$, has two parts: the QCD and the EW contribution, $\gamma_q(\alpha_s, \alpha) = \gamma_q^{\text{QCD}} + \gamma_q^{\text{EW}}$, where γ_q^{QCD} includes all terms which are proportional to powers of α_s only and γ_q^{EW} includes all other terms proportional to at least one power of α, and beyond one loop multiplied by further powers of α and/or α_s. It has been shown in Ref. [30], that γ_t^{EW} in the $\overline{\text{MS}}$ scheme may be written in terms of RG functions of parameters in the unbroken phase of the SM [6, 8], $\gamma_t^{\text{EW}} = \gamma_{y_t} + \frac{1}{2}\gamma_{m^2} - \frac{1}{2}\frac{\beta_\lambda}{\lambda}$, where y_t is the Yukawa coupling of the top-quark, $\gamma_{y_t} \equiv \mu^2 \frac{d}{d\mu^2} \ln y_t$, and m^2 and λ are the parameters of the scalar potential[2] $V = \frac{m^2}{2}\phi^2 + \frac{\lambda}{24}\phi^4$. The values of γ_{y_t} and γ_{m^2} are bounded under scale variations $M_W < \mu < M_{\text{Planck}}$ [14, 33, 15], β_λ is negative, such that λ is decreasing with increasing value of μ. According to estimates in [14, 15] β_λ has a zero somewhat below the Planck scale while λ is still positive and then slightly increases before μ reaches M_{Planck}.

The evaluation of the matching conditions. Since masses are generated by the Higgs mechanism, the decoupling theorem [34] is not valid in the week sector of the SM, such that a top quark-mass dependence cannot be renormalized away even at such low scales as $\mu \sim M_W$. As a consequence, the complete set of particles and the corresponding diagrams must be evaluated, especially for the top-quark. The numerical value of the EW contribution to the relation between pole and $\overline{\text{MS}}$ mass

[2]The bridge between the UV mass counterterms of the masses m_H^2, m_t^2, m_W^2 evaluated in [30, 31, 4] in the broken phase of the SM and the RG equations for the SM parameters m^2, λ, y_t^2 in the unbroken phase [32], gives rise to the following parametrization, in terms of the effective Higgs vacuum expectation value $v(\mu^2)$:
$\sqrt{2}G_F = 1/v^2 = 1/(246.22)^2 \text{ GeV}^{-2}, \quad 1/v^2(\mu^2) = \sqrt{2}G_F^{\overline{MS}}(\mu^2),$
$m_H^2(\mu^2) = 2m^2(\mu^2)\,, \quad \lambda(\mu^2) = \frac{3m_H^2(\mu^2)}{v^2(\mu^2)}\,, \quad y_t^2(\mu^2) = \frac{2m_t^2(\mu^2)}{v^2(\mu^2)}\,, \quad g^2(\mu^2) = \frac{4m_W^2(\mu^2)}{v^2(\mu^2)}.$

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of a quark may be extracted from the $\overline{\text{MS}}$ renormalized propagator [35] and have been evaluated analytically[3] in [36, 31]. For the finite part of the $O(\alpha\alpha_s)$ mixing-correction, numerical agreement with the semi-analytic result of [38] has been established. Intermediate results of [31] have been cross-checked in [39]. The complete $O(\alpha^2)$ EW contribution is not yet available, and results presented in [40] and [41] are not in agreement. Using an indirect estimation of the $O(\alpha^2)$ corrections, we evaluated in [14] that for $124 < M_H < 126$ GeV the EW contribution is large and has opposite sign relative to the QCD contributions, so that the total SM correction is small and approximately equal to $m_t(M_t) - M_t \sim 1 \pm O(1)$ GeV.[4]

An optimal value for the RG matching scale. At the Planck scale perturbative SM RG-evolution stops to make sense, since in any case gravity contributions would come into play [18]. When considering matching conditions for a fixed pole mass M versus a running $\overline{\text{MS}}$ mass $M(\mu^2)$ it is evident that at some scale, we will call it "IR scale", we must have $m(\mu^2_{\text{IR}}) = M$. Depending of the sign of the correction $m(M) - M$ and the sign of $\mu^2 dm(\mu^2)/d\mu^2$ at scale M one finds $\mu_{\text{IR}} < M$ or $\mu_{\text{IR}} > M$. An interesting question then is whether there exists a preferable μ_{IR} and how to define it? An optimal value for μ_{IR} could be defined as value of μ where the radiative corrections to the some (or all) matching conditions between running coupling and pole masses are minimal, close or equal to zero. Since the main interest of our RG-analysis concerns the Higgs sector of the SM, we will analyze the matching conditions for $\lambda(\mu^2)$, $G_F(\mu^2)$ and $m_H(\mu^2)$. At the one-loop level, the relation between the running and the physical Fermi constant is (see Eq.(A.3) in [4]):

$$\frac{G_F(\mu^2)}{G_F} - 1 = \frac{g^2(\mu^2)}{16\pi^2} f^{(1)}_{G,\alpha}(\mu^2) = \frac{g^2(\mu^2)}{16\pi^2}\left[\gamma_{G_F,\alpha} L - \Delta X^{(1)}_{G_F,\alpha}(\mu^2, M_t, M_H)\right] ,$$

where $L = \ln\frac{\mu^2}{M_X^2}$ and $\gamma_{G_F,\alpha}$ is defined[5] in [30] as $\gamma_{G_F} = \mu^2 \frac{d}{d\mu^2} \ln G_F(\mu^2) = \frac{\beta_\lambda}{\lambda} - \gamma_m^2 = \frac{g^2}{16\pi^2}\gamma_{G_F,\alpha} + \cdots$. From the condition $f^{(1)}_{G,\alpha}(\mu^2_{\text{IR}}) = 0$, we obtain

$$\frac{\mu^2_{\text{IR}}}{M_X^2} = \exp\left\{\frac{\Delta X^{(1)}_{G_F,\alpha}(\mu^2, M_t, M_H)}{\gamma_{G_F,\alpha}}\right\} = \exp\left\{-\frac{f^{(1)}_{G,\alpha}(\mu^2 = M_X^2)}{\gamma_{G_F,\alpha}}\right\} .$$

For $M_t \sim 173 - 173.5$ GeV and 124 GeV $\leq M_H \leq 126$ GeV, we then find, $\mu^2_{\text{IR}} \sim M_t^2 \times 0.3425 \Rightarrow \mu_{\text{IR}} \sim 101.5$ GeV . The inclusion of the $O(\alpha\alpha_s)$ correction moves the value of the IR scale to μ^2_{IR,G_F}:

$$\mu^2_{\text{IR},G_F} \sim M_t^2 \left[0.3075 - 0.0005\left(\frac{M_H}{\text{GeV}} - 125\right)\right] \Rightarrow \mu_{\text{IR},G_F} = \left[96.2 - 0.01\left(\frac{M_H}{\text{GeV}} - 125\right) \pm 5.\,3\right] \text{GeV} , \tag{1}$$

which for $M_t = 173.5$ and 124 GeV $\leq M_H \leq 126$ GeV and theoretical uncertainties ± 5.3 GeV, defines the central value of the IR scale, extracted with $O(\alpha)/O(\alpha\alpha_s)$ accuracy. At scale μ_{IR,G_F}, which is close to value of the Z-boson mass, the running v.e.v. is: $v\left(\mu^2_{\text{IR},G_F}\right) = v \equiv 246.22$ GeV

[3]Two terms on the second line of Eq. (12) in [37] are to be modified: "$-\frac{m_t^4}{m_H^4}\ln(1+y) + \frac{m_t^2}{m_H^2}\frac{3+y^2}{1+y}\ln y$" should read "$-\frac{m_H^4}{m_t^4}\ln(1+y) + \frac{m_H^2}{m_t^2}\frac{3+y^2}{1+y}\ln y$".

[4]The results of [42] are not relevant for our analysis.

[5]In [14], the r.h.s. of Eq. (8) should be multiply by the overall factor "$2 \times G_F^{\overline{\text{MS}}}$".

and the value of the Yukawa coupling $y_q(\mu^2_{\mathrm{IR},G_F})$ is proportional to value of the running mass of quark evaluated at this scale: $y_q(\mu^2_{\mathrm{IR},G_F}) = \frac{\sqrt{2}}{246.22}\frac{m_q(\mu^2_{\mathrm{IR},G_F})}{\mathrm{GeV}}$. Another relation valid at this scale is:

$$\lambda(\mu^2_{\mathrm{IR},G_F}) = 3\left(\frac{m_H(\mu^2_{\mathrm{IR},G_F})}{246.22\ \mathrm{GeV}}\right)^2 , \qquad (2)$$

where $m_H^2(\mu^2)$ is the $\overline{\mathrm{MS}}$ mass of the Higgs propagator.

On the same ground we defined the IR scale (we denote it as μ_{IR,m_H^2}) from the ratio between the running mass m^2 and the pole mass of the Higgs boson (see Eq. (A.26) in [4], and the discussion in [33]). With $O(\alpha)$ accuracy, $\mu_{\mathrm{IR},m_H^2} \sim 4$ GeV $O(\alpha\alpha_s)$-order corrections shifts this number to 11 GeV. The last ingredient of the Higgs potential is the Higgs self-coupling λ. The corresponding value of the IR scale we denote as $\mu_{\mathrm{IR},\lambda}$. With $O(\alpha)+O(\alpha\alpha_s)$ accuracy, $\mu_{\mathrm{IR},\lambda} \sim 164.55$ GeV, and $\mu_{\mathrm{IR},\lambda} \sim 152.65$ GeV, correspondingly. The inclusion the leading $O(\alpha^2)$ order corrections from [5] stabilizes the $\mu_{\mathrm{IR},\lambda}$ around ~ 154 GeV. At this scale,

$$\lambda(\mu_{\mathrm{IR},\lambda}) = \sqrt{2}G_F 3M_H^2 = 3\left(\frac{M_H}{246.22\ \mathrm{GeV}}\right)^2 . \qquad (3)$$

The IR-scale $\mu_{\mathrm{IR},EW}$ follows from the minimization of the values of the matching conditions for the parameters of the Higgs potential and lies in the interval $M_Z \le \mu_{\mathrm{IR},EW} \le 2M_W$. At the boundary points of this interval, the quantity $\delta\lambda(\mu^2) = \frac{\lambda(\mu^2)}{\Lambda_0} - 1$, with $\Lambda_0 \equiv 3\left(\frac{M_H}{246.22\ \mathrm{GeV}}\right)^2$, changes from

$$\delta\lambda(\mu^2)\big|_{\mu\sim M_Z} \approx \frac{m_H^2(\mu^2_{\mathrm{IR},G_F})}{M_H^2} - 1 \Longrightarrow \delta\lambda(\mu^2)\big|_{\mu\sim 2M_W} \sim 0 .$$

The existence of such an IR scale, numerically close to vector boson masses, may be relevant for the extraction of the Higgs self-coupling from cross-sections [45] as well as for construction of effective Lagrangian [46].

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