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Gluon correlations in the transversely polarized nucleon at twist three

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We discuss the gluonic correlations in the transversely polarized nucleon, which are relevant to single and double spin asymmetries in various hard processes. We explain that the single transverse-spin asymmetry (SSA) to be observed in the D-meson production with large transverse-momentum in semi-inclusive deep inelastic scattering, $eP^{\uparrow} \to eDX$, is induced by the three-gluon correlation effects. We present the numerical calculations of the corresponding SSA at the kinematics relevant to a future Electron Ion Collider, based on the QCD factorization formula at twist three. We also clarify the independent degrees of freedom associated with three-gluon correlation effects, deriving the new exact twist-three relations between the multi-gluon correlators based on the operator product expansion and the QCD equations of motion. As a byproduct of our analysis, we mention the transverse-spin sum rule as the partonic decomposition of the transverse nucleon spin.

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We discuss the gluonic correlations in the transversely polarized nucleon, which are allowed at the twist-3 level and may be main source of single and double spin asymmetries in hard processes.

We first consider the D-meson production in the semi-inclusive deep inelastic scattering (SIDIS), $e(\ell)+P(p)\to e(\ell')+D(P_h)+X$, with the usual kinematic variables, $S_{ep}=(\ell+p)^2$, $q=\ell-\ell'$, $Q^2=-q^2$, $x_{bj}=Q^2/(2p\cdot q)$, and $z_f=p\cdot P_h/(p\cdot q)$. This process is induced by the photon-gluon fusion mechanism, $\gamma^* g\to c\bar{c}$, so that the cross section is given by the corresponding partonic hard cross section, $\hat{\sigma}^{\gamma^* g\to c\bar{c}}$, combined with the gluon distribution function in the nucleon, $G(x)$, and the charm quark fragmentation function into the D meson, $D_c(z)$, and reads, schematically,

$$d\sigma \sim G(x)\otimes\hat{\sigma}^{\gamma^* g\to c\bar{c}}(x,z,Q)\otimes D_c(z), \tag{1}$$

where '$\otimes$' represents the appropriate convolution with the relevant momentum fractions x, z integrated over. When the nucleon is transversely polarized, we would in principle obtain the single transverse-spin asymmetry (SSA), corresponding to the combination $d\sigma\sim\boldsymbol{S}\cdot(\boldsymbol{p}\times\boldsymbol{P}_h)$, associated with the nucleon's spin S. Its necessary condition is: (i) the nonzero transverse-momentum $P_{h\perp}$ originating from transverse motion of quark or gluon; (ii) the helicity flip by one unit in the cut diagrams for the cross section, corresponding to the transverse polarization; and (iii) the interaction beyond the Born level to produce the interfering phase between the LHS and the RHS of the cut in those cut diagrams, necessary for the naively T-odd SSA contribution. For large $P_{h\perp}(\gg\Lambda_{\rm QCD})$, the contribution (i) is provided as the recoil from the hard (unobserved) final-state partons. The contribution (iii) can be obtained by loop effects although the resultant phase might be small. On the other hand, the helicity flip by one unit of a gluon, for (ii) within the leading twist level, is impossible. However, if an additional nonperturbative gluon originating from the nucleon participate in the hard scattering, this gluon allows us to obtain the helicity flip of (ii) as well as the large interfering phase. This is the twist-3 mechanism for the SSA in the D-meson production, corresponding formally to the replacement,

$$G(x)\to\mathscr{M}_F(x_1,x_2), \qquad \hat{\sigma}^{\gamma^* g\to c\bar{c}}(x,z,Q)\to\delta\hat{\sigma}(x_1,x_2,z,Q), \tag{2}$$

in (1), where $\mathscr{M}_F(x_1,x_2)$ is the twist-3 three-gluon correlation function in the transversely polarized nucleon $p^\uparrow$, depending on two longitudinal momentum fractions x_1,x_2 of the gluons, and $\delta\hat{\sigma}(x_1,x_2,z,Q)$ corresponds to interference between the partonic processes, $\gamma^* g\to c\bar{c}$ and $\gamma^* gg\to c\bar{c}$. This factorization formula [1, 2] represents a pure gluonic analogue of the quark-gluon-correlation mechanism for the SSA at twist-3 in the high-$P_{h\perp}$ pion production (see [3]), and the latter mechanism is connected to the Sivers mechanism relevant for the low $P_{h\perp}$ region [4], observed in HERMES and COMPASS measurements. We calculate the three-gluon twist-3 mechanism for the SSA in the D-meson production, taking into account the masses for the D meson and the charm quark.

We define the two azimuthal angles of the hadron plane and the nucleon's spin vector, ϕ_h and ϕ_S, measured from the lepton plane [2]. Then, the unpolarized cross section for the D-meson production, as given by the twist-2 factorization formula (1), receives the three independent azimuthal structures as, $d\sigma\sim\sigma_1^U+\sigma_2^U\cos\phi_h+\sigma_3^U\cos 2\phi_h$, while the twist-3 single-spin-dependent cross section $\Delta\sigma$ for $eP^\uparrow\to eDX$, given by (1) with the replacement (2), receives the five azimuthal structures, $d\Delta\sigma\sim\sin(\phi_h-\phi_S)\left(\mathscr{F}_1+\mathscr{F}_2\cos\phi_h+\mathscr{F}_3\cos 2\phi_h\right)+\cos(\phi_h-\phi_S)\left(\mathscr{F}_4\sin\phi_h+\mathscr{F}_5\sin 2\phi_h\right)$,

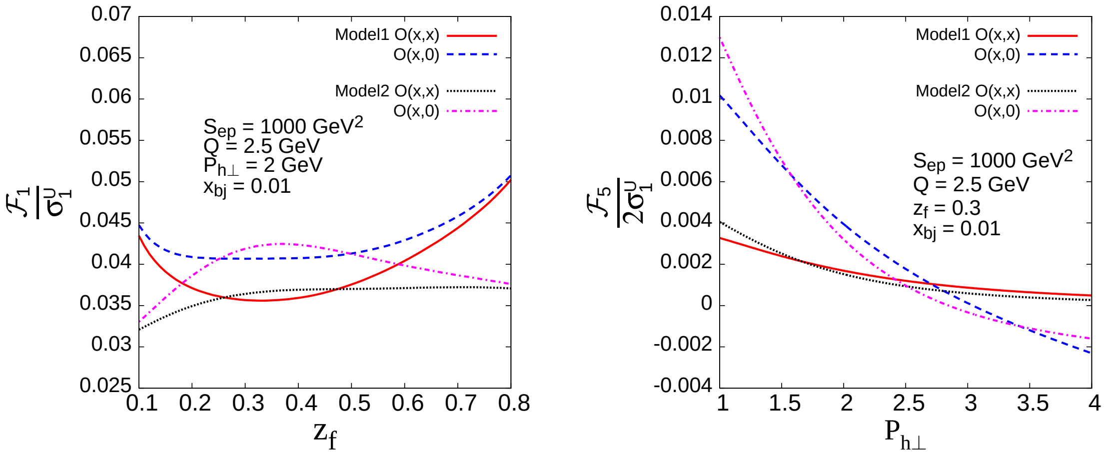

Figure 1: The three-gluon-correlation contributions to the SSA for D^0 production at EIC kinematics.

similarly as the SSA in the pion production from the quark-gluon twist-3 mechanism. This is a consequence of the general factorization formula (1) with (2), derived by the collinear expansion through the twist-3 accuracy[1]. The three-gluon correlation of twist-3, $\mathscr{M}_F(x_1,x_2)$, is given by correlator of the gluon-field-strength tensor $F^{\mu\nu}$ on the light-cone ($n^\mu = g^\mu_- n^-, p\cdot n = 1, p^2 = M_N^2$) [2]:

$$\mathscr{M}^{\alpha\beta\gamma}_{F(+)}(x_1,x_2) \equiv -gi^3\int\frac{d\lambda}{2\pi}\int\frac{d\zeta}{2\pi}e^{i\lambda x_1}e^{i\zeta(x_2-x_1)}\langle pS_\perp|if^{bca}F_b^{\beta n}(0)F_c^{\gamma n}(\zeta n)F_a^{\alpha n}(\lambda n)|pS_\perp\rangle$$
$$= 2iM_N\Big[N(x_1,x_2)g^{\alpha\beta}\varepsilon^{\gamma pnS_\perp} + N(-x_2,x_1-x_2)g^{\beta\gamma}\varepsilon^{\alpha pnS_\perp} + N(x_2-x_1,-x_1)g^{\gamma\alpha}\varepsilon^{\beta pnS_\perp}\Big], \quad (3)$$

for the color structure leading to the C-even component, and similarly for the C-odd component $\mathscr{M}^{\alpha\beta\gamma}_{F(-)}$ with the replacement $f^{bca}\to -id^{bca}$ and $N(x_1,x_2)\to O(x_1,x_2)$. Both $N(x_1,x_2)$ and $O(x_1,x_2)$ are symmetric under $x_1\leftrightarrow x_2$, and the RHS of (3) manifests permutation symmetry between the three gluons. In principle, we may obtain another type of three-gluon correlation function $\sim\langle pS_\perp|F^{+\perp}D^\perp F^{+\perp}|pS_\perp\rangle$ with the transverse covariant derivative $D^\perp$ instead of a field strength tensor; but this "D-type" correlation function does not arise actually in the cross section $d\Delta\sigma$, so we need only the "F-type' correlation function $\mathscr{M}_F(x_1,x_2)$. D-type correlation will be studied later.

In the leading-order QCD calculation of the partonic hard part, $\delta\hat{\sigma}(x_1,x_2,z,Q)$ of (2), the interfering phase of (iii) is provided by the pole contribution of the parton propagator, arising when the momentum of an external gluon is zero, and this "soft gluon pole" is associated with the four functions $O(x,x), O(x,0), N(x,x), N(x,0)$. The explicit form of the hard parts are derived in [2].

We discuss numerical estimate of the SSA for the D^0 meson production. We calculate the asymmetries, dividing the above-mentioned five structure functions in $d\Delta\sigma$ with the unpolarized twist-2 cross section, as $\mathscr{F}_1/\sigma_1^U$, $\mathscr{F}_{2,\cdots,5}/(2\sigma_1^U)$, corresponding to those types of angular averages, $\langle 1\rangle, \langle\cos\phi\rangle, \langle\cos 2\phi\rangle, \langle\sin\phi\rangle$, and $\langle\sin 2\phi\rangle$. We present the contributions due to the C-odd three-gluon functions $O(x,x), O(x,0)$, because the hard parts associated with the C-even functions $N(x,x), N(x,0)$ have similar structure as those for $O(x,x)$, $O(x,0)$. We assume the two types of functional forms of $O(x,x), O(x,0)$ with the different small-x behavior: $O(x,x) = O(x,0) =$

[1]For recent developments on the collinear twist-3 factorization formula, see [5].

$0.004xG(x)$ for 'Model 1' and $O(x,x) = O(x,0) = 0.001\sqrt{x}G(x)$ for 'Model 2', where the coefficients 0.004 and 0.001 are determined by comparing the three-gluon contribution to the SSA for $P^{\uparrow}P \to DX$ with the data observed at RHIC (see [7]). Figure 1 shows $\mathscr{F}_1/\sigma_1^U$ and $\mathscr{F}_5/(2\sigma_1^U)$ as functions of z_f and $P_{h\perp}$, respectively, at an EIC kinematics [6]. The red and blue curves show the contributions from $O(x,x)$ and $O(x,0)$, respectively, using Model 1, and similarly for the other two curves using Model 2. $\mathscr{F}_1/\sigma_1^U$ is several % level. In $\mathscr{F}_5/(2\sigma_1^U)$, we see the $1/P_{h\perp}$ behavior characteristic of the twist-3 mechanism. We note that the other asymmetries $\mathscr{F}_{2,\cdots,4}/(2\sigma_1^U)$ are small, but each shows the characteristic behavior [6]. The results for higher energy cases with $S_{ep} = 2500$ GeV^2 and 5000 GeV^2 are calculated [6], where the difference between the cases with Model 1 and Model 2 is more pronounced because the small x region plays more important role in high energy cases. For all cases, $\mathscr{F}_1/\sigma_1^U$ is several % level and is large, while the other asymmetries are small, except $\mathscr{F}_5/(2\sigma_1^U)$, which reaches a percent level for small $P_{h\perp}$.

We turn to the issue on the D-type gluon correlation, which can be defined similarly as (3):

$$-i\int\frac{d\lambda}{2\pi}\int\frac{d\zeta}{2\pi}e^{i\lambda x_1}e^{i\zeta(x_2-x_1)}\langle pS_\perp|F^{\beta n}(0)D_\perp^{\gamma}(\zeta n)F^{\alpha n}(\lambda n)|pS_\perp\rangle$$
$$=2iM_N\Big[M_1(x_1,x_2)g^{\alpha\beta}\varepsilon^{\gamma pnS_\perp}+M_2(x_1,x_2)g^{\beta\gamma}\varepsilon^{\alpha pnS_\perp}-M_2(x_2,x_1)g^{\gamma\alpha}\varepsilon^{\beta pnS_\perp}\Big]. \tag{4}$$

This is C-even and may be related to the C-even F-type correlation function (3): working out the covariant derivative $D_\perp^\gamma(\zeta n)$ on the (suppressed) Wilson line to connect the points ζn and λn in (4), we immediately obtain [8], $M_1(x_1,x_2) = P\frac{N(x_1,x_2)}{x_2-x_1}$, $M_2(x_1,x_2) = P\frac{N(x_2,x_2-x_1)}{x_1-x_2} + \delta(x_1-x_2)\tilde{g}(x_1)$, where P denotes the principal value and $\tilde{g}(x_1)$ denotes a certain complicated matrix element. To reveal $\tilde{g}(x_1)$, we recall the twist decomposition of the correlator of two gluon-field-strength tensors [9],

$$\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle pS|F^{\mu n}(0)F^{\nu n}(\lambda n)|pS\rangle = -\frac{xM_N}{2}\left[\Delta G(x)i\varepsilon^{\mu\nu pn}(S\cdot n)+2\mathscr{G}_T(x)i\varepsilon^{\mu\nu\alpha n}S_{\perp\alpha}+\cdots\right], \tag{5}$$

where the ellipses denote the terms with twist higher than three and the terms independent of S_μ. This is analogous to the well-known decomposition of the quark correlator, $\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle pS|\bar{\psi}(0)\gamma^\sigma\gamma_5\,\psi(\lambda n)\,|pS\rangle = 2M_N\left[\Delta g(x)\,(S\cdot n)\,p^\sigma + g_T(x)S_\perp^\sigma+\cdots\right]$, and $\mathscr{G}_T(x)$ is the gluonic counterpart of the twist-3 transverse-spin quark distribution $g_T(x)$. Eq. (5) implies that $\mathscr{G}_T(x)$ is associated with F^{-+}, a 'bad' component in the light-cone quantization, which can be reexpressed by the 'good' components: using the equations of motion in the light-cone gauge, we obtain, $F^{-+} = -\partial^+A^- = (-1/\partial_-)\left(D_{\perp j}F^{j+}+g\bar{\psi}t^a\gamma^+\psi t^a\right)$, so that we find [8], $\mathscr{G}_T(x) = \frac{-1}{2x^2}\int dx'\,(8M_2(x,x')-4M_2(x',x)$ $-4M_1(x',x)+G_F(x'+x,x')+G_F(x'-x,x'))$, where the quark-gluon twist-3 correlation G_F is defined as [3], $\int\frac{d\lambda}{2\pi}\int\frac{d\mu}{2\pi}e^{i\lambda x_1}e^{i\mu(x_2-x_1)}\,\langle pS_\perp|\,\bar{\psi}(0)\gamma^\mu gF^{\alpha n}(\mu n)\psi(\lambda n)\,|pS_\perp\rangle = M_N p^\mu\varepsilon^{\alpha pnS_\perp}G_F(x_1,x_2)$.

$\mathscr{G}_T(x)$ can be treated in another way using the exact operator identity that reexpresses the light-cone limit $z^2\to 0$ of $\Theta^\rho \equiv z^\nu\left[(\partial/\partial z^\nu)F^{\mu z}(0)\tilde{F}_\mu^{\ \rho}(z)-(\nu\leftrightarrow\rho)\right]$ in terms of the F-type three-gluon correlation and the equations of motion (see [8]); because matrix element of Θ^ρ itself is given by the helicity distribution $\Delta G(x)$ and the derivative of $\mathscr{G}_T(x)$ based on (5), we obtain ($\varepsilon(x) = x/|x|$)

$$\mathscr{G}_T(x) = \int_x^{\varepsilon(x)}dx'\frac{\Delta G(x')}{2x'} - \int_x^{\varepsilon(x)}\frac{dx'}{2x'^2}P\int\frac{dx''}{x'-x''}\left[2\left(\frac{\partial}{\partial x'}-\frac{\partial}{\partial x''}\right)\left(N(x',x'-x'')-N(x'',x''-x')\right)\right.$$
$$+\left(\frac{\partial}{\partial x'}+\frac{\partial}{\partial x''}\right)\left(4N(x',x'')-6N(x'',x''-x')-6N(x',x'-x'')\right)+\frac{8}{x'}\left(N(x'',x''-x')\right.$$

$$+N(x',x'-x''))\Big]-\int_x^{\varepsilon(x)}\frac{dx'}{2x'^2}\int dx''\left(\frac{1}{x'}-\frac{\partial}{\partial x'}\right)\left[G_F(x''+x',x'')+G_F(x''-x',x'')\right]. \tag{6}$$

Comparing this with another form of $\mathscr{G}_T(x)$ derived above, the function $\tilde{g}(x_1)$, mentioned below (4), can be expressed in terms of the F-type correlation $N(x_1,x_2)$, the quark-gluon correlation $G_F(x_1,x_2)$, and the gluon helicity distribution $\Delta G(x)$. Thus, we find that the D-type correlation function (4) is completely expressed by $N(x_1,x_2)$, $G_F(x_1,x_2)$, and $\Delta G(x)$; similar relation has been known between the D-type and F-type quark-gluon twist-3 correlation functions, as derived in [3].

It is worth noting that (6) is the gluonic analogue of the well-known decomposition of the quark distribution $g_T(x)$ into the Wandzura-Wilczek (WW) part and the genuine twist-3 part (see [3]). Integrating (6) over x, we find [8] that the contribution from the genuine twist-3 effect vanishes, and, from the WW part, the first term in the RHS, we get, $\int dx\mathscr{G}_T(x)=\int dx\Delta G(x)\equiv\Delta G$, similarly to $\int dx g_T(x)=\int dx\Delta q(x)\equiv\Delta q$ in the quark case. On the other hand, dividing the both sides of (5) by x and integrating the result over x, we find, $\langle p\,S_\perp|-\int d\lambda\,\varepsilon(\lambda)F^{\beta n}(0)F^{\alpha n}(\lambda n)|p\,S_\perp\rangle=2\varepsilon^{n\alpha\beta\sigma}S_{\perp\sigma}\int dx\mathscr{G}_T(x)$, indicating that $\int dx\mathscr{G}_T(x)$ is given by matrix element of $\mathscr{M}^{\mu\alpha\beta}_{g\text{-spin}}=F^{\mu\beta}A^\alpha-F^{\mu\alpha}A^\beta$, the gluon-spin contribution to the angular momentum tensor, where $A^\alpha(0)=\int d\lambda\,\varepsilon(\lambda)F^{\alpha n}(\lambda n)/2$ in the light-cone gauge. Combining these results for $\int dx\mathscr{G}_T(x)$, the gluon-spin contribution to the *transverse* nucleon spin equals the value of the total gluon helicity ΔG. Similarly, the quark-spin contribution to the *transverse* nucleon spin equals the value of the total quark helicity. The remaining contribution is the orbital angular momentum contribution, L; thus, the *transverse* spin sum rule reads [8],

$$\frac{1}{2}=L+\frac{1}{2}\Delta\Sigma+\Delta G, \tag{7}$$

which is formally similar as the longitudinal spin sum rule.

Now the remaining question is whether the orbital angular momentum contribution, L, in the transverse spin sum rule (7) can be further decomposed into the quark and gluon contributions or not. The authors in [10] recently calculated the matrix elements of the quark and gluon contributions to the Pauli-Lubansky vector, $-\frac{1}{2}\varepsilon^{\mu}_{\ \nu\rho\sigma}p^\nu\int d^3x\mathscr{M}^{+\rho\sigma}\equiv W^\mu_q+W^\mu_g$, with the transversely polarized nucleon state, as $\langle pS_\perp|W^j_{q,g}|pS_\perp\rangle/[2p^+(2\pi)^3\delta^3(0)]=J_{q,g}S^j_\perp$, and argued that the result $J_{q,g}=(A_{q,g}+B_{q,g})/2$ holds, based on $\mathscr{M}^{\lambda\mu\nu}_{q,g}=x^\mu T^{\lambda\nu}_{q,g}-x^\nu T^{\lambda\mu}_{q,g}$, combined with the well-known parameterization of the off-forward matrix element of the (Belinfante-improved) energy-momentum tensor of quarks/gluons,

$$\langle p'S'|T^{\mu\nu}_{q,g}|pS\rangle=\bar{u}(p',S')\left[A_{q,g}\gamma^{(\mu}\bar{p}^{\nu)}+B_{q,g}\frac{\bar{p}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M_N}+C_{q,g}\frac{\Delta^\mu\Delta^\nu-g^{\mu\nu}\Delta^2}{M_N}+\bar{C}_{q,g}M_N g^{\mu\nu}\right]u(p,S), \tag{8}$$

with $\bar{p}^\mu=\frac{1}{2}(p^\mu+p'^\mu)$, $\Delta^\mu=p'^\mu-p^\mu$; this suggests affirmative answer to the above question. However, we find that the corresponding result receives the additional term as [8],

$$J_{q,g}=\frac{1}{2}(A_{q,g}+B_{q,g})+\frac{p^3}{2(p^0+M_N)}\bar{C}_{q,g}, \tag{9}$$

which is associated with the twist-4, trace term in (8). Thus, our conclusion is that the decomposition of the total nor orbital angular momentum of the transversely polarized nucleon into the quark and gluon contributions is frame dependent.

We have discussed the twist-3 gluonic correlations in the transversely polarized nucleon which are relevant to single- and double-spin asymmetries in various hard processes. We discussed the SSA in SIDIS for high-$P_{h\perp}$ D-meson production, induced by the twist-3 mechanism from the three-gluon correlation inside the nucleon, and by the photon-gluon fusion. Based on the factorization formula for the twist-3 SSA with the convolution with the F-type three-gluon correlation functions, we find the five different azimuthal dependences and our numerical estimates of the corresponding asymmetries show that the SSAs corresponding to $\langle 1 \rangle$ and $\langle \sin 2\phi \rangle$ are large, demonstrating good chance to access multi-gluon effects at a future EIC. We have also derived the exact relation between the F-type and D-type three-gluon twist-3 correlation functions, through giving the decomposition of the twist-3 transverse-spin gluon distribution $\mathscr{G}_T(x)$ into the WW and genuine twist-3 parts. Those relations imply that the gluon and quark spin contributions to the *transverse* nucleon spin equal the total gluon and quark helicities, respectively, and this fact allow us to obtain explicit form of the *transverse* spin sum rule.

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