

Study of exclusive two-photon production of W^+W^- pairs in pp collisions at 7 TeV, and constraints on anomalous quartic couplings in CMS

Krzysztof Piotrzkowski¹

*Centre for Cosmology, Particle Physics and Phenomenology, Université catholique de Louvain,
Chemin du Cyclotron 2, 1348 Louvain-la-Neuve, Belgium*

E-mail: krzysztof.piotrzkowski@uclouvain.be

A search for exclusive W^+W^- production by photon-photon fusion, $pp \rightarrow p W^+W^- p$, at $\sqrt{s} = 7$ TeV is reported using data collected by the CMS detector with an integrated luminosity of 5 fb^{-1} . Events are selected by requiring a $\mu^\pm e^\mp$ vertex with no additional associated charged tracks and dilepton transverse momentum $p_T(\mu^\pm e^\mp) > 30$ GeV. Two events passing all selection requirements are observed in the data, compared to a standard model expectation of 2.2 ± 0.4 signal events with 0.84 ± 0.15 background. The tail of the dilepton p_T distribution is studied for deviations from the standard model. No events are observed with $p_T > 100$ GeV. Model-independent upper limits are computed and compared to predictions involving anomalous quartic gauge couplings. The limits on the parameters $a_{0,C}^{W,C}/\Lambda^2$ with a dipole form factor and an energy cutoff $\Lambda_{\text{cutoff}} = 500$ GeV are of the order of 10^{-4} GeV^{-2} .

International Conference on the Structure and the Interactions of the Photon including the 20th
International Workshop on Photon-Photon Collisions and the International Workshop on High Energy
Photon Linear Colliders
May 20-24, 2013
Paris, France

¹Speaker, on behalf of the CMS collaboration

1. Introduction

The detection of high-energy photon interactions at the Large Hadron Collider (LHC) opens up the possibility of interesting and novel research [1, 2]. In particular, measurements of the two-photon production of a pair of W-bosons provide sensitivity to anomalous quartic gauge couplings of the gauge bosons. Exploratory studies [3, 4] showed potential for extending the experimental reach by several orders of magnitude with respect to the best limits so far obtained at the Tevatron [5] and LEP [6–13]. First measurements of the exclusive two-photon production of muon and electron pairs at $\sqrt{s} = 7$ TeV, $pp \rightarrow p l^+ l^- p$, were made using ~ 40 pb $^{-1}$ of data collected with the Compact Muon Solenoid (CMS) at the LHC in 2010 [14, 15]. The present analysis is based on the experimental technique developed in ref. [14] and uses the full data sample collected by the CMS experiment in 2011 [16].

In this analysis the $\mu^\pm e^\mp$ final state is used to search for fully exclusive (“elastic”) $pp \rightarrow pW^+W^-p$ production. Since both very forward-scattered protons escape detection, such a production process is characterized by a primary vertex formed from a $\mu^\pm e^\mp$ pair with no other tracks, with large transverse momentum, $p_T(\mu^\pm e^\mp)$, and large invariant mass, $m(\mu^\pm e^\mp)$. This signature is also accessible via quasi-exclusive (“inelastic” or “proton dissociative”) production, in which one or both of the incident protons dissociate into a low-mass system that escapes detection, denoted as p^* . The two-photon signal $\gamma\gamma \rightarrow W^+W^-$ is therefore comprised of both the elastic and inelastic contributions.

In the case of decays of the W^+W^- pair to same-flavor $\mu^+\mu^-$ or e^+e^- final states, the backgrounds are more than an order of magnitude larger than in the $\mu^\pm e^\mp$ final state. Therefore in the present analysis, only the $\mu^\pm e^\mp$ channel is used to search for a $pp \rightarrow p^{(*)}W^+W^-p^{(*)}$ signal. We use the $\mu^+\mu^-$ channel to select a control sample of high-mass $pp \rightarrow p^{(*)}\mu^+\mu^-p^{(*)}$ events originating mainly from direct $\gamma\gamma \rightarrow \mu^+\mu^-$ production. Final states containing a $\mu^\pm e^\mp$ pair may arise from direct decays of W^\pm bosons to electrons and muons or from $W \rightarrow \tau \nu$ decays, with the τ subsequently decaying to an electron or a muon. For brevity, we will refer to the full reaction as $pp \rightarrow p^{(*)}W^+W^-p^{(*)} \rightarrow p^{(*)}\mu^\pm e^\mp p^{(*)}$, where the final state is understood to contain between two and four undetected neutrinos, in addition to the charged $\mu^\pm e^\mp$ pair.

We first use the $pp \rightarrow p^{(*)}\mu^+\mu^-p^{(*)}$ control sample to validate the selection by comparing the expected and observed numbers of events and to estimate from the data the proton dissociative contribution. The dominant backgrounds in the $\mu^\pm e^\mp$ channel, due to the inclusive production of W^+W^- and $\tau^+\tau^-$ pairs, are then constrained using control regions with low $p_T(\mu^\pm e^\mp)$ or a low-multiplicity requirement for extra tracks originating from the $\mu^\pm e^\mp$ vertex.

The data for the signal region are then compared to the standard model (SM) expectation for the backgrounds and the $\gamma\gamma \rightarrow W^+W^-$ signal. Finally, tails of the $p_T(\mu^\pm e^\mp)$ distribution, where the SM $\gamma\gamma \rightarrow W^+W^-$ contribution is expected to be small, are investigated to look for anomalous quartic gauge couplings [17].

The electroweak sector of the SM [18–20] predicts 3- and 4-point vertices with the gauge bosons, as a result, the diagrams that represent the $WW\gamma\gamma$ interaction at lowest order in the perturbation series consist of both quartic gauge coupling (figure 1(a)) and t- and u-channel W-boson exchange diagrams (figure 1(b,c)).

Measurements of the quartic $WW\gamma\gamma$ coupling can be used to look for any deviation from the SM predictions, which would reveal a sign of new physics [6]. One has to take into account more generic couplings in order to study the possibility of such deviations in high-energy collisions. Considering models with the anomalous triple gauge couplings, the quartic $WW\gamma\gamma$ and triple $WW\gamma$ couplings can be associated with a single anomalous dimension-six operator [17]. The genuine anomalous quartic gauge couplings considered here are instead introduced via an effective Lagrangian containing new terms respecting local $U(1)_{EM}$ and global custodial $SU(2)_C$ symmetry. Further imposing charge-conjugation and parity symmetries, C- and P, results in a minimum of two additional dimension-six terms, containing the anomalous couplings a^W_θ/Λ^2 and a^W_C/Λ^2 , where Λ is the energy scale of new physics [17].

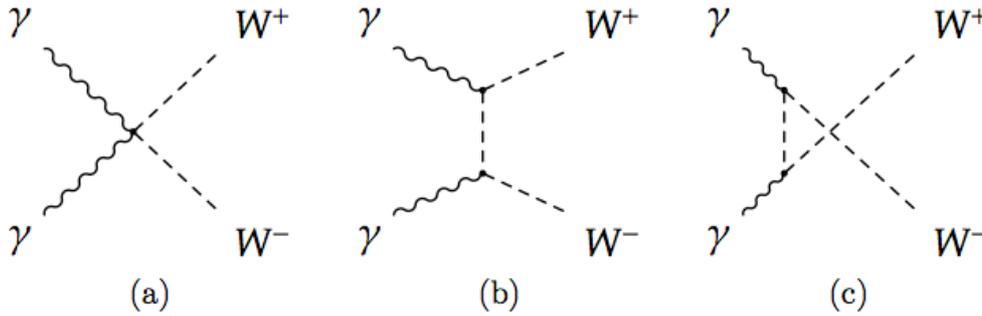


Figure 1: Quartic gauge coupling (a) and t- (b) and u-channel (c) W-boson exchange diagrams contributing to the $\gamma\gamma \rightarrow W^+W^-$ process at leading order in the SM.

The existing constraints on anomalous quartic gauge couplings from e^+e^- collisions at LEP are derived from $e^+e^- \rightarrow W^+W^-\gamma$ and $W^+W^- \rightarrow \gamma\gamma$ interactions in which the effective center-of-mass energy is limited to values well below the e^+e^- center-of-mass energy of $\sqrt{s} = 209$ GeV. In contrast, the spectrum of $\gamma\gamma$ interactions at the LHC and the Tevatron extends to much higher values, resulting in increased sensitivity to anomalous couplings.

The $\gamma\gamma \rightarrow W^+W^-$ cross section increases quadratically with anomalous coupling strength, and consequently unitarity is violated for high-energy $\gamma\gamma$ interactions. For anomalous couplings a^W_θ/Λ^2 , a^W_C/Λ^2 of order 10^{-5} GeV^{-2} , the unitarity bound is already reached for collisions with a $\gamma\gamma$ center-of-mass energy $W_{\gamma\gamma} \sim 1$ TeV [3, 4]. In order to tame this rising of the cross section, both a^W_θ/Λ^2 and a^W_C/Λ^2 parameters are multiplied by a form factor with a cutoff energy [17]. Because the new physics that enters to regulate the cross section has an energy scale Λ and a form that are a priori unknown, we consider both a scenario with a dipole form factor with energy cutoff scale $\Lambda_{\text{cutoff}} = 500$ GeV, and a scenario with no form factor (i.e., $\Lambda_{\text{cutoff}} \rightarrow \infty$).

2. Event selection

After trigger selection and lepton identification, a first preselection criterion is applied offline on the data by requiring a reconstructed muon and electron of opposite charge, each having $p_T > 20$ GeV and $|\eta| < 2.4$, matched to a common primary vertex with fewer than 15 additional tracks. After the trigger selection, the leptons are required to have an invariant mass $m(l^+l^-) > 20$ GeV in both the $\mu^\pm e^\mp$ and $\mu^+\mu^-$ channels. In the remainder of this paper we will use the notation $p_T(l^+, l^-)$ to indicate a p_T selection applied to each lepton of the pair, and $p_T(l^+l^-)$ to indicate the p_T of the pair.

In the $\mu^\pm e^\mp$ channel the SM signal region is defined to have zero extra tracks associated with the $\mu^\pm e^\mp$ vertex, and transverse momentum of the pair $p_T(\mu^\pm e^\mp) > 30$ GeV. The first requirement rejects backgrounds from inclusive production, while the second is chosen based on the simulated $p_T(\mu^\pm e^\mp)$ distribution of the signal and $\tau^+\tau^-$ background events. The efficiency for reconstruction of primary vertices with two or more tracks has been measured to be $\geq 98\%$ in simulation, and $\geq 99\%$ in data. In addition, events are only accepted as $\mu^\pm e^\mp$ events if they have failed to satisfy the $\mu^\pm \mu^\mp$ selection, in order to reject $\gamma\gamma \rightarrow \mu^+\mu^-$ events with the muon misidentified as an electron due to a bremsstrahlung photon overlapping with the muon track.

For the anomalous quartic gauge couplings search, a restricted region of $p_T(\mu^\pm e^\mp) > 100$ GeV is used. This is chosen to reduce the expected SM $\gamma\gamma \rightarrow W^+W^-$ contribution to approximately 0.1 events after all selection requirements, while retaining sensitivity to anomalous couplings of order 10^{-4} GeV⁻² for $\Lambda_{\text{cutoff}} = 500$ GeV or larger. This corresponds to values of the anomalous quartic gauge coupling parameters approximately two orders of magnitude smaller than the limits obtained at LEP [7, 8, 12, 13] and approximately one order of magnitude smaller than Tevatron limits [5].

At high luminosities, almost all signal events will have additional interactions within the same bunch crossing (“event pileup”) that produce extra charged tracks and extra activity in the calorimeters. During the 2011 LHC run the average number of interactions per crossing was approximately 9. In order to retain efficiency in high pileup conditions, a selection based only on the number of charged tracks originating from the same primary vertex as the l^+l^- pair is used, similar to the method in ref. [14].

3. Cross-check with $\mu+\mu-$ events

The dimuon sample with zero extra tracks is divided into two kinematic regions based on the p_T balance ($|\Delta p_T(\mu^+\mu^-)|$) and acoplanarity ($1 - |\Delta\phi(\mu^+\mu^-)/\pi|$) of the pair. The first region with $1 - |\Delta\phi(\mu^+\mu^-)/\pi| < 0.1$ and $|\Delta p_T(\mu^+\mu^-)| < 1$ GeV is defined as the “elastic” region, where the dimuon kinematic requirements are consistent with elastic $pp \rightarrow p \mu^+\mu^- p$ events where both protons remain intact [14]. The second region with $1 - |\Delta\phi(\mu^+\mu^-)/\pi| > 0.1$ or $|\Delta p_T(\mu^+\mu^-)| > 1$ GeV (“dissociation” selection) is dominated by $\gamma\gamma \rightarrow \mu^+\mu^-$ in which one or both protons dissociate. The latter process is less well-known theoretically, and subject to corrections from re-scattering, in which strong interactions between the protons produce additional hadronic activity.

In figure 2 the invariant mass distribution in the elastic-enhanced region is shown, with the marked Z-peak region defined as $70 \text{ GeV} < m(\mu^+\mu^-) < 106 \text{ GeV}$. In figure 3 the dimuon kinematic distributions for events having zero extra tracks are shown. The distributions are plotted separately for the Z-peak region, which is expected to include a large inclusive Drell-Yan component, and for the region outside the Z peak, which is expected to be dominated by two-photon interactions.

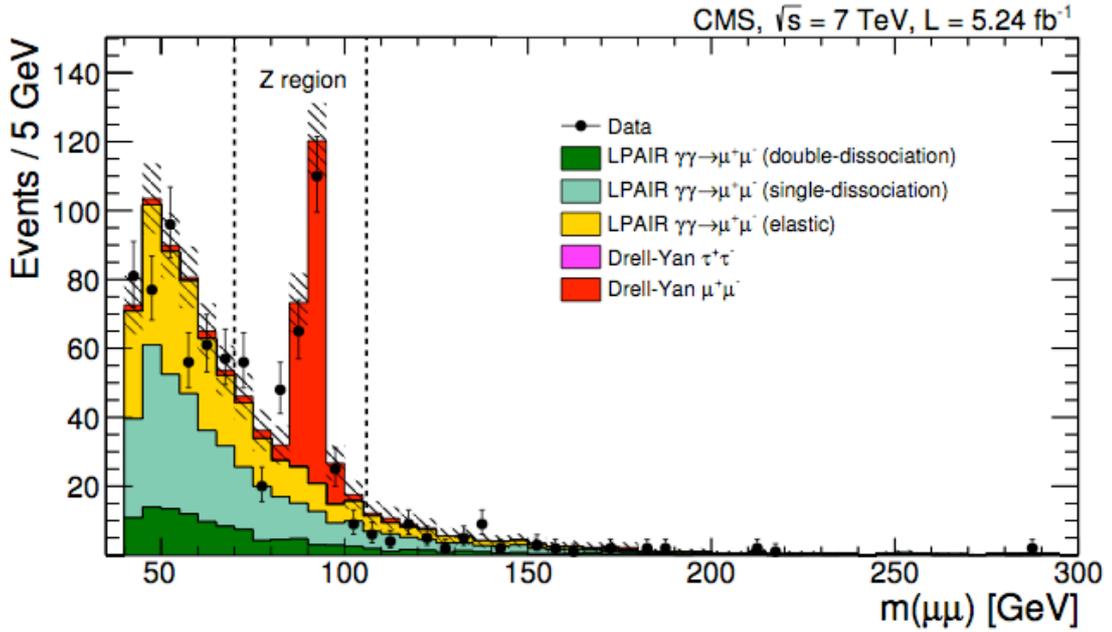


Figure 2: Invariant mass distribution of the muon pairs for the elastic selection with no additional track on the dimuon vertex. The dashed lines indicate the Z-peak region. The hatched bands indicate the statistical uncertainty in the simulation.

For the kinematic distributions with zero extra tracks originating from the $\mu^+\mu^-$ production vertex, good agreement is observed between data and simulation. This confirms that pileup effects and low-multiplicity fluctuations of the inclusive Drell-Yan processes are well modeled. The suppression due to rescattering corrections is particularly significant in the case of quasi-exclusive production when one or both incident protons dissociate. This suppression is practically impossible to calculate from first principles as it involves very soft interactions and only phenomenological models are available. Therefore, using the low-background sample of dimuons produced via two-photon interactions, we use the data to determine an effective, observed “luminosity” of two-photon interactions at high energies relevant for W-pair production. For this purpose, the number of detected dimuon events with invariant mass over 160 GeV, corrected for the DY contribution, is divided by the prediction for the fully exclusive, elastic production predicted by LPAIR,

$$F = \frac{N_{\mu\mu \text{ data}} - N_{\text{DY}}}{N_{\text{elastic}}} \Big|_{m(\mu^+\mu^-) > 160 \text{ GeV}},$$

$$F = 3.23 \pm 0.53.$$

This factor F is then be applied to scale the CalcHEP signal prediction and obtain the total cross section for two-photon W^+W^- production including elastic and proton dissociative contributions. This assumes the dilepton kinematics are the same in elastic and proton dissociative production, with the difference in efficiency arising from the requirement of zero extra tracks originating from the W^+W^- production vertex.

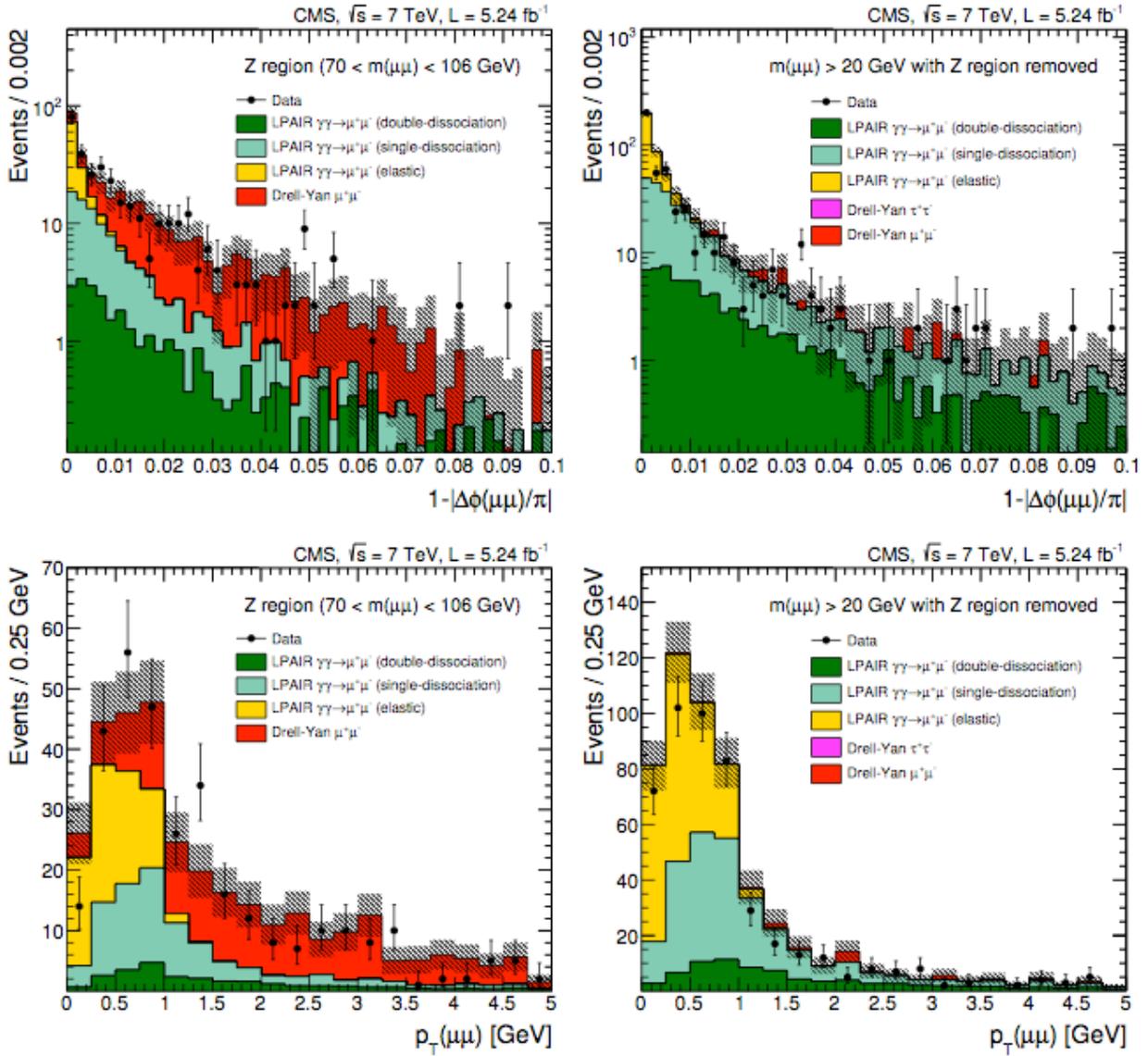


Figure 3: Kinematic distributions for the elastic selection, for the Z region only ($70 \text{ GeV} < m(\mu^+\mu^-) < 106 \text{ GeV}$, left panel) and with the Z region removed (right panel). The acoplanarity (above) and p_T of $\mu^+\mu^-$ pairs with zero extra tracks (below) are shown. The hatched bands indicate the statistical uncertainty in the simulation.

4. The $W^+W^- \rightarrow \mu^\pm e^\mp$ signal

The SM cross section for the purely elastic process $pp \rightarrow pW^+W^-p$ is predicted to be 40 fb using CalcHEP, or 1.2 fb for the cross section times branching fraction to $\mu^\pm e^\mp$ final states. Using the scale factor F extracted from the high-mass $\gamma\gamma \rightarrow \mu^+\mu^-$ sample to account for the additional proton dissociation contribution, the total predicted cross section times branching fraction is:

$$\sigma_{\text{theory}}(pp \rightarrow p^{(*)}W^+W^-p^{(*)} \rightarrow p^{(*)}\mu^\pm e^\mp p^{(*)}) = 4.0 \pm 0.7 \text{ fb.}$$

The acceptance for the SM signal in the fiducial region $|\eta(\mu, e)| < 2.4$, $p_T(\mu, e) > 20$ GeV is determined to be 55% using the CalcHEP generator. We use the simulated background sample, corrected for trigger and lepton identification efficiencies, to estimate the backgrounds in the W signal region, $p_T(\mu^\pm e^\mp) > 30$ GeV. The W+jets contribution to the background is estimated from the control sample of events with lepton identification inverted, while the $\gamma\gamma \rightarrow \tau^+\tau^-$ contribution is normalized using the factor derived from the high-mass $\gamma\gamma \rightarrow \mu^+\mu^-$ data sample. No simulated Drell-Yan $\tau^+\tau^-$ events survive all selection criteria, The estimated total background is 0.84 ± 0.15 events, including the systematic uncertainty on the backgrounds.

5. Results

Examining the SM $\gamma\gamma \rightarrow W^+W^-$ signal region, we find two events passing all the selection criteria, compared to the expectation of 2.2 ± 0.4 signal events. In figure 4, the CMS event displays are shown for these two, first ever, candidates of the two-photon production of W boson pairs.



Figure 4: Two exclusive μe events – the candidates of two-photon produced W pairs in CMS.

The observed upper limit is estimated using the Feldman-Cousins method to be 2.6 times the expected SM yield at 95% CL. The dilepton invariant mass, acoplanarity, and missing

transverse energy in the two selected events are consistent with the expectation for the sum of backgrounds and SM $\gamma\gamma \rightarrow W^+W^-$ signal (figure 5).

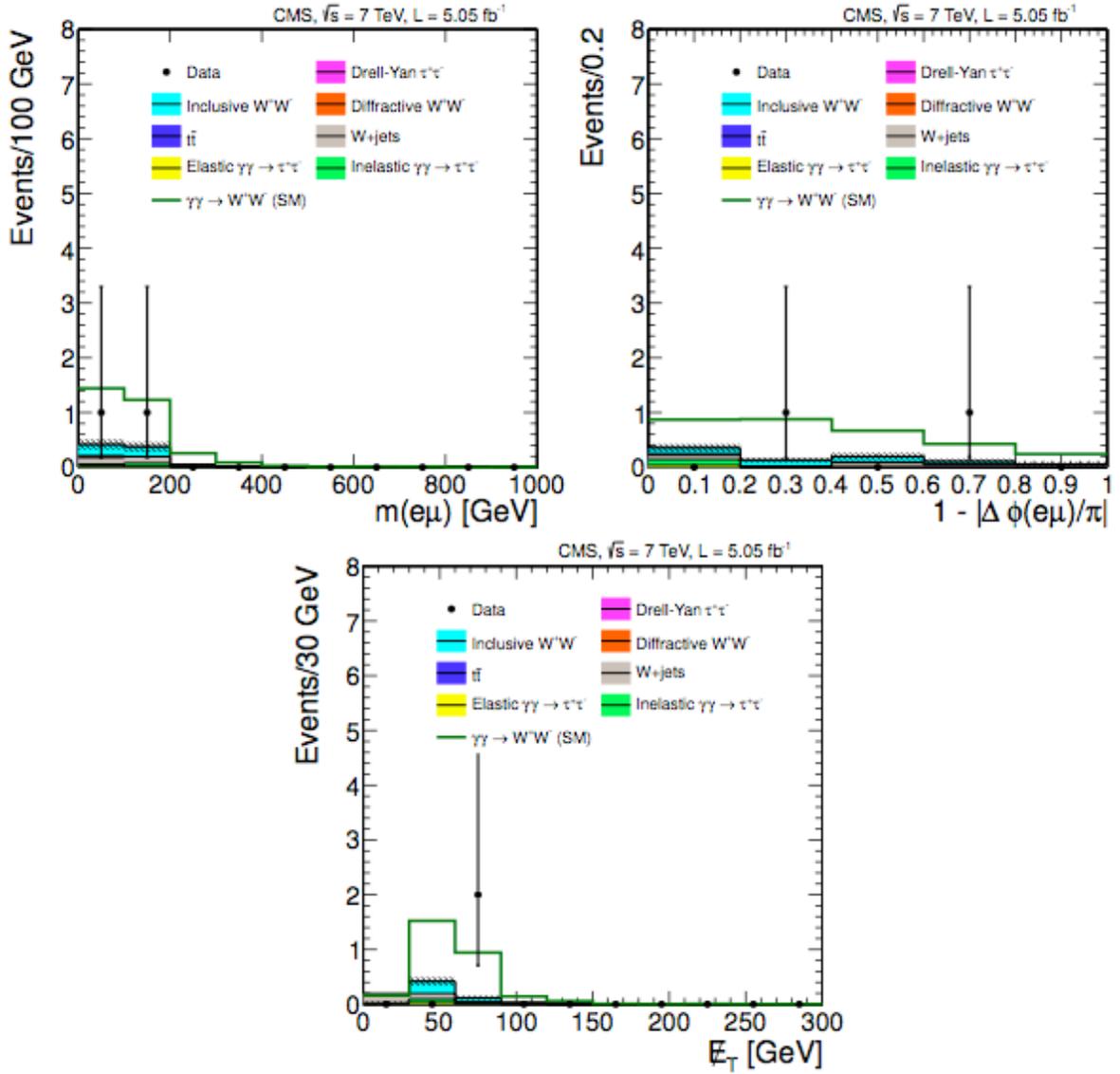


Figure 5: The $\mu^\pm e^\mp$ invariant mass (top left), acoplanarity (top right), and missing transverse energy (bottom) distributions, for events in the signal region with zero extra tracks on the $\mu^\pm e^\mp$ vertex and $p_T(\mu^\pm e^\mp) > 30 \text{ GeV}$. The backgrounds (solid histograms) are stacked with statistical uncertainties indicated by the shaded region, the signal (open histogram) is stacked on top of the backgrounds.

In the anomalous quartic gauge coupling search region $p_T(\mu^\pm e^\mp) > 100 \text{ GeV}$, zero events are observed in data, which is consistent with the SM expectation of 0.14, dominated by $p p \rightarrow p^{(*)} W^+W^- p^{(*)}$. This results in an upper limit of 1.8 fb on the partial cross section times branching fraction at 95% CL with the selections $p_T(\mu, e) > 20 \text{ GeV}$, $|\eta(\mu, e)| < 2.4$, and $p_T(\mu^\pm e^\mp) > 100 \text{ GeV}$. The expected number of events observed as a function of the anomalous quartic

gauge coupling parameters is interpolated from simulated samples and used to construct 95% CL intervals according to the Feldman-Cousins prescription.

With a dipole form factor of $\Lambda_{\text{cutoff}} = 500$ GeV, the limits obtained on each anomalous quartic gauge coupling parameter with the other fixed to zero are:

$$\begin{aligned} -0.00015 < a_0^W/\Lambda^2 < 0.00015 \text{ GeV}^{-2} & (a_C^W/\Lambda^2 = 0, \Lambda_{\text{cutoff}} = 500 \text{ GeV}), \\ -0.0005 < a_C^W/\Lambda^2 < 0.0005 \text{ GeV}^{-2} & (a_0^W/\Lambda^2 = 0, \Lambda_{\text{cutoff}} = 500 \text{ GeV}). \end{aligned}$$

These limits are approximately 20 times more stringent than the best limits obtained at the Tevatron [5] with a dipole form factor of $\Lambda_{\text{cutoff}} = 500$ GeV, and approximately two orders of magnitude more stringent than the best limits obtained at LEP [7, 8, 12, 13]. We perform a similar procedure to derive two dimensional limits on the a_0^W/Λ^2 and a_C^W/Λ^2 parameters, see figure 6.

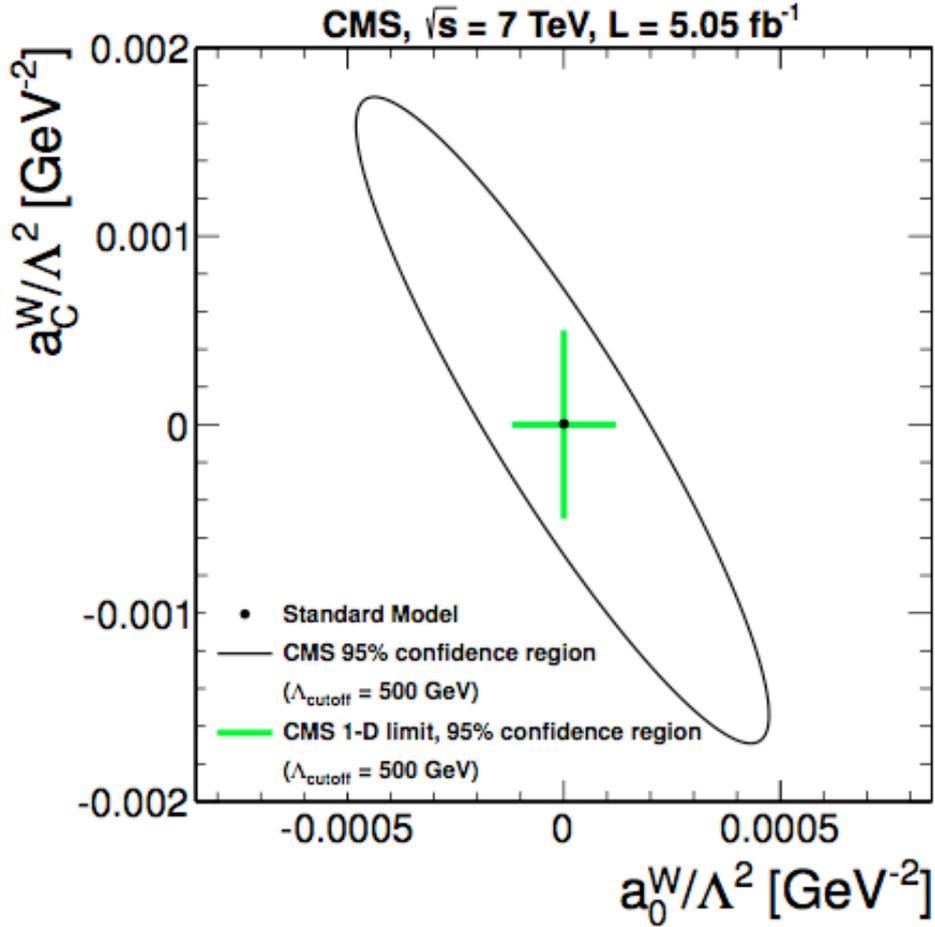


Figure 6: Excluded values of the anomalous coupling parameters a_0^W/Λ^2 and a_C^W/Λ^2 with $\Lambda_{\text{cutoff}} = 500$ GeV. The area outside the solid contour is excluded by this measurement at 95% CL, obtained for $p_T(\mu, e) > 20$ GeV, $|\eta(\mu, e)| < 2.4$, $p_T(\mu^\pm e^\mp) > 100$ GeV. The predicted cross sections are rescaled to include the contribution from proton dissociation.

6. Summary

In a region sensitive to SM $\gamma\gamma \rightarrow W+W-$ production with $p_T(\mu^\pm e^\mp) > 30$ GeV, two events are observed, with a background expectation of 0.84 ± 0.15 . The signal expectation is 2.2 ± 0.4 events, with the uncertainty on the theory reflecting the uncertainty on the proton dissociation contribution.

In the region with $p_T(\mu^\pm e^\mp) > 100$ GeV, where the SM contribution is expected to be small, no events are observed. We use this subsample to set limits on the anomalous quartic gauge coupling parameters, which results in values of the order of 1.5×10^{-4} GeV $^{-2}$ for $a_{W_0}^W/\Lambda^2$ and 5×10^{-4} GeV $^{-2}$ for $a_{C^W}^W/\Lambda^2$, assuming a dipole form factor with the energy cutoff scale at $\Lambda_{\text{cutoff}} = 500$ GeV. These limits are approximately 20 times more stringent than the best limits obtained at the Tevatron, and approximately two orders of magnitude more stringent than the best limits obtained at LEP. With no form factors, the limits on the anomalous quartic gauge coupling parameters would be of order 10^{-5} GeV $^{-2}$ and below, driven by high-energy $\gamma\gamma$ interactions beyond the unitarity bound.

References

- [1] D. d'Enterria, M. Klasen and K. Piotrzkowski, Photon-LHC-2008. Proceedings of the International Workshop on High-energy Photon Collisions at the LHC, [Nucl. Phys. Proc. Suppl 179-180 \(2008\)](#).
- [2] J. de Favereau de Jeneret et al., High energy photon interactions at the LHC, [arXiv:0908.2020](#).
- [3] T. Pierzchała and K. Piotrzkowski, Sensitivity to anomalous quartic gauge couplings in photon-photon interactions at the LHC, [Nucl. Phys. Proc. Suppl. 179-180 \(2008\) 257](#) [[arXiv:0807.1121](#)].
- [4] E. Chapon, C. Royon and O. Kepka, Anomalous quartic $WW\gamma\gamma, ZZ\gamma\gamma$ and trilinear $WW\gamma$ couplings in two-photon processes at high luminosity at the LHC, [Phys. Rev. D 81 \(2010\) 074003](#) [[arXiv:0912.5161](#)].
- [5] D0 collaboration, V.M. Abazov et al., Search for anomalous quartic $WW\gamma\gamma$ couplings in $\sqrt{s} = 1.96$ TeV, [arXiv:1305.1258](#).
- [6] G. Belanger, F. Boudjema, Y. Kurihara, D. Perret-Gallix and A. Semenov, Bosonic quartic couplings at LEP-2, [Eur. Phys. J. C 13 \(2000\) 283](#) [[hep-ph/9908254](#)].
- [7] ALEPH collaboration, A. Heister et al., Constraints on anomalous QGC's in e^+e^- interactions from 183 GeV to 209 GeV, [Phys. Lett. B 602 \(2004\) 31](#).
- [8] OPAL collaboration, G. Abbiendi et al., Constraints on anomalous quartic gauge boson couplings from $\nu\bar{\nu}\gamma\gamma$ and $q\bar{q}\gamma\gamma$ events at LEP-2, [Phys. Rev. D 70 \(2004\) 032005](#) [[hep-ex/0402021](#)].

- [9] OPAL collaboration, G. Abbiendi et al., A Study of $W^+W^- \gamma$ events at LEP, *Phys. Lett. B* 580 (2004) 17 [[hep-ex/0309013](#)].
- [10] OPAL collaboration, G. Abbiendi et al., Measurement of the $W^+W^- \gamma$ cross-section and first direct limits on anomalous electroweak quartic gauge couplings, *Phys. Lett. B* 471 (1999) 293 [[hep-ex/9910069](#)].
- [11] L3 collaboration, P. Achard et al., The $e^+e^- \rightarrow Z\gamma\gamma \rightarrow q\bar{q}\gamma\gamma$ reaction at LEP and constraints on anomalous quartic gauge boson couplings, *Phys. Lett. B* 540 (2002) 43 [[hep-ex/0206050](#)].
- [12] L3 collaboration, P. Achard et al., Study of the $W^+W^- \gamma$ process and limits on anomalous quartic gauge boson couplings at LEP, *Phys. Lett. B* 527 (2002) 29 [[hep-ex/0111029](#)].
- [13] DELPHI collaboration, J. Abdallah et al., Measurement of the $e^+e^- \rightarrow W^+W^- \gamma$ cross-section and limits on anomalous quartic gauge couplings with DELPHI, *Eur. Phys. J. C* 31 (2003) 139 [[hep-ex/0311004](#)].
- [14] CMS collaboration, Exclusive photon-photon production of muon pairs in proton-proton collisions at $\sqrt{s} = 7$ TeV, *JHEP* 01 (2012) 052 [[arXiv:1111.5536](#)].
- [15] CMS collaboration, Search for exclusive or semi-exclusive photon pair production and observation of exclusive and semi-exclusive electron pair production in pp collisions at $\sqrt{s} = 7$ TeV, *JHEP* 11 (2012) 080 [[arXiv:1209.1666](#)].
- [16] CMS collaboration, Study of exclusive two-photon production of W^+W^- pairs in pp collisions at $\sqrt{s} = 7$ TeV, and constraints on anomalous quartic couplings, *JHEP* 07 (2013) 116 [[arXiv:1305.5596](#)].
- [17] G. Belanger and F. Boudjema, Probing quartic couplings of weak bosons through three vectors production at a 500 GeV NLC, *Phys. Lett. B* 288 (1992) 201.
- [18] S.L. Glashow, Partial symmetries of weak interactions, *Nucl. Phys.* 22 (1961) 579.
- [19] S. Weinberg, A model of leptons, *Phys. Rev. Lett.* 19 (1967) 1264.
- [20] A. Salam, Weak and electromagnetic interactions, in *Elementary particle physics: relativistic groups and analyticity*, N. Svartholm ed., Almqvist & Wiksell, Sweden (1968).