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Quantum simulation of gauge potentials with cold atoms in optical lattices: a tunable platform for relativistic fermions and axions

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We offer here a brief introduction to the idea of quantum simulations with cold atomic gases, with focus on the recent efforts towards artificial gauge potentials and fields. This is mainly intended as a sort of "pedestrian guide" for people not yet working in the field, but curious to get a first contact with it; longer and deeper reviews are addressed for deeper details. As a special case, we focus here on reviewing some own previous contributions about a flexible toolbox based on bichromatic optical lattices and Raman assisted tunnelling. Such a scheme would allow good control on the mass and kinetic terms of a lattice Hamiltonian in different effective dimensions. If realized with fermionic atoms, the low-energy sector effectively describes their relativistic dynamics in a variety of situations, among which 3D axion electrodynamics. We conclude with a short discussion of possible future directions.

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1. Introduction

The study of quantum many-body systems (QMBS) in a strongly-correlated regime happens to be a crucial and challenging task both in condensed matter theory and in high-energy physics. Indeed, gauge theories are a special kind of QMBS, where states of the system are forced to obey local symmetry constraints. They stand at the core of our understanding of known fundamental interactions (electro-weak, strong, and, more broadly, gravitational) [1, 2]; on the other hand, they are also candidates to describe the low-energy physics of frustrated anti-ferromagnets [3] and high-temperature superconductors [4]. Despite the apparent simplicity of models involving interactions among only few of their constituents, we still very often miss generic computational tools to extract reliable quantitative information from them. The complexity of numerical simulations increases exponentially with the number of the QMBS constituents, making it impossible to deal exactly with more than few tens of particles [5]. Different approximate methods often provide different results, so that physical properties of these models remain in most cases controversial. Relevant examples in the high-energy domain include the hot and dense nuclear matter probed in heavy ion collisions or the predictions of hadronic masses from first principles, as discussed in this conference.

A possible way out dates back to a famous Feynman's conjecture [6] on the usefulness of a universal quantum simulator [7], capable of solving relevant Hamiltonian models. Although the feasibility of such an "all-purpose" quantum computer is still to be demonstrated, "purpose-built" "analog" quantum simulators (emulators), whose constituents (photons, ions, atoms, electrons) are used to encode the desired QMBS, exist and are already providing precious contributions to answer open problems [8, 9]. The rationale of a quantum simulator, i.e. a purpose-built quantum computer, is indeed to tailor the quantum Hamiltonian of interest in a highly controllable physical system (i.e., one on which the detailed microscopic knowledge is accompanied by a clean tunability of the parameters), and to retrieve all the desired information with a measurement of its properties, a task as crucial as the efficient preparation and simulation itself [9, 10]. A careful analysis of reliability and efficiency in the presence of unavoidable imperfections is of course also relevant [10]. Having such a device at disposal would also offer the possibility to bring abstract models into life, i.e., to realise theoretical models that have been up to now only "blackboard" exercises not observed in nature. One could then access unexplored regimes to pose new questions and possibly advance the understanding of an *a priori* distant phenomenology.

In the following, we first offer a brief overview of the use of ultracold atoms as quantum simulators [11 – 13], especially in the direction of gauge potentials and fields [14, 15] (Sec. 2); then we sketch the main ideas of a proposal of ours [16 – 18] for a flexible toolbox to embody different effective relativistic fermionic models (Sec. 3); finally we close with some remarks (Sec. 4).

2. Ultracold atoms as quantum simulators

2.1 Cold atoms and optical lattices

Among the many possible physical realizations of quantum simulators [8, 9], ultracold atomic gases have already demonstrated to be extraordinarily useful tools for the investigation of effects originally predicted in condensed-matter physics [11 – 13], and offer new exciting possibilities with respect to the simulation of nuclear and high-energy physics (e.g. for color superconductivity and

exotic fermionic superfluidity [19, 20]). Ultracold atoms could also be capable of emulating several aspects of hadronic matter in extreme conditions, relevant for compact stars [21] and for the quark epoch of the Universe [22]. Moreover, such devices have provided information on the long timescale behaviour of some non-equilibrium phenomena, for which classical numerical simulations are bound to loose reliability at much shorter times [23], and this could be one of the most useful ways to employ them also in the context of gauge phenomena.

The success of these systems relies on their ultra-low coupling to the environment, which enables the measurement of ground state properties, as well as of the pure quantum dynamics, of specifically-tailored quantum Hamiltonian models. Very powerful tools exist for this "quantum engineering" task with cold atoms, primarily the possibility of trapping them in periodic potentials created with standing waves of light, the so-called "optical lattices" [11 – 13], thus shaping the kinetic part of the model. Such artificial crystals, almost free from defects and vibrations, can be loaded with particles obeying either Bose or Fermi statistics, and even mixtures of them. Moreover, a good control over collision properties of the system constituents can be achieved by tuning external parameters (e.g., the magnetic field) across so-called Feshbach resonances [24]; this allows to scan many orders of magnitude of the interaction term in both repulsive and attractive side, including the extremes of unitarity [25, 26] and non-interacting regimes [27, 28]. Powerful tools are also at disposal for the characterization and study of the quantum phases realized in these setups, ranging from momentum distributions to single-site imaging in real and spin space,thus fulfilling the requirements for a good quantum simulator.

Let us sketch here the construction of optical lattice models, in order to get acquainted with the language used later in Sec. 2.2 and 3. An atom illuminated by a laser of intensity I, detuned by Δ from one of its (optical) adsorption lines, behaves as an effective two-level system with an off-diagonal coupling proportional to the dipole matrix element Γ between the electronic clouds of the two levels (Fig. 1a). This originates a so-called AC Stark shift of the levels $\propto (\Gamma/\Delta)I$, that effectively acts as a potential for the center of mass dynamics of the atoms when they are "cold" enough [29][1]. Strong confinement in some direction may reduce the low-energy spectrum to that of a system in a lower dimensionality. Maxima (or minima) of a periodic light pattern then identify the lattice sites $\mathbf{r}_j$, around which one could define a basis of localized single-particle wavefunctions $w(\mathbf{r}-\mathbf{r}_j)$, named after Wannier. By taking an expansion of the field operator $\psi(\mathbf{r})$ onto such a basis, i.e. $\psi(\mathbf{r}) = \sum_j w(\mathbf{r}-\mathbf{r}_j)\, c_j$, the Hamiltonian of the system gets transformed into an Hubbard-like model, where c_j now describes the annihilation of a particle on a given site [30][2]:

$\varepsilon\, c_j^\dagger c_j$ is an onsite energy term due to some external confinement;

$J\, c_j^\dagger c_l + h.c.$ is the kinetic term (J<0) due to the hopping (tunnelling) between adiacent sites;

$U c_j^\dagger c_j^\dagger c_j c_j$ is a on-site interaction arising from the quartic term describing two-body collisions.

These coefficients depend also on the optical depth of the potential, since it changes the Wannier wavefunctions themselves and therefore the relevant overlap integrals. Experimentalists have therefore various knobs at disposal to tailor the lattice topology and its effective dimensionality, the amplitude and the internal state (spin) dependence of tunnel coupling between the sites.

[1] typical potentials are of order 100 KHz, i.e. μK or 0.1 neV

[2] sub-indexing due to the internal state (spin) of the atoms, and the proper accounting for the bosonic/fermionic statistics is here omitted for the sake of simplicity (see, e.g., [11 – 13] for more details)

2.2 Artificial gauge potentials with cold atoms

When coming to the simulation of gauge phenomena, one has to distinguish two main approaches: the first one is to engineer a static configuration of the gauge field, i.e. a gauge potential or classical field, like a static magnetic field affecting the orbits of charged particles moving in it; the second one consists into donating indeed dynamics to the gauge fields, thus giving rise to bosonic force carriers interacting with fermionic matter particles, as needed for the simulation of fully-quantum gauge theories. Let us start reviewing the first one, which is currently the most experimentally developed and lies at the basis of the proposal of ours sketched in Sec. 3.

A classical magnetic field influences the quantum mechanical motion of a charge e via the insertion of its (gauge) vector potential $\mathbf{A}$ into the definition of a covariant derivative $\boldsymbol{\nabla} \to \boldsymbol{\nabla} + i(e/c)\mathbf{A}$: this leads then to the accumulation of an Aharanov-Bohm phase $\gamma = (e/\hbar c)\oint \mathbf{A}\cdot \mathrm{d}\mathbf{l} = 2\pi\Phi/(hc/e)$ around a closed, non-contractible, loop pierced by a magnetic flux Φ. A first idea to engineer the same effect with neutral atoms relies on the equivalence between the Coriolis force in a rotating system and the Lorentz force in a uniform magnetic field: experimentalists have realized such $\mathbf{A} = m\boldsymbol{\Omega}_{\mathrm{rot}} \times \mathbf{r}$ since the early times of atomic Bose-Einstein condensates and have achieved quite spectacular results, though not managing to access the field intensities needed for Quantum Hall regimes [31, 32]. A more general idea consists in letting the ground manifold $|\psi_\alpha(\mathbf{r})\rangle$ depend on the spatial coordinates of the center of mass $\mathbf{r}$, thus realising an effective geometric vector potential $\mathbf{A}_{\alpha,\beta}(\mathbf{r}) = i\hbar\langle\psi_\alpha(\mathbf{r})|\boldsymbol{\nabla}|\psi_\beta(\mathbf{r})\rangle$. If the atom moves now adiabatically around a closed loop in real space, the exponential of the circulation integral of such a potential results into a non-trivial unitary operation on the vectors of the manifold, in analogy to the Abelian phase factor $e^{i\gamma}$ described before[3]. An experimental technique based on these ideas has been applied so far not only to single-component Bose gases [33, 34] but also to BEC with two components, and a spin-orbit coupling has been realized [35]. The reader interested in a wider overview of the current theoretical and experimental status is addressed to a couple of excellent recent reviews [14, 15].

In order to emulate either condensed matter or high-energy phenomena, it is useful to extend these ideas to the powerful platform of optical lattices. The kinetic term of matter fields (i.e., the hopping matrix in the correspondent Hubbard model) needs then to be designed to embody the parallel transporter of the gauge group one aims to engineer. Such a term can be understood as a discretisation across a lattice link of the unitary operator introduced above, in a way that the circulation around a plaquette is preserved. In the simple case of the electromagnetic $U(1)$, the hopping coefficient becomes complex and link dependent, i.e. $J \to J_{jl} = J\,e^{i\varphi_{jl}}$, with $\varphi_{jl} = (e/\hbar c)\int_j^l \mathbf{A}\cdot\mathrm{d}\mathbf{r}$ also known as Peierls phase. This can be achieved by time-dependent modulation or rotation of the lattice itself [36–38],or also by a "laser-assisted tunnelling" between the sites, as first outlined in [39, 40] and realized recently for staggered [41, 42] and uniform [43, 44] magnetic fluxes. In this latter scheme, the spontaneous tunnelling is suppressed by raising the optical lattice power and some additional laser beam (often a Raman pair of them) is arranged to induce a transition of the center of mass coordinate of the atom between the initial and final lattice positions. While doing that, a phase proportional to the scalar product of the light wave vector and the displacement is left imprinted on the atomic wavefunction. Expanding on the same idea, it would be possible to

[3] the accumulated phase is often referred to as a Berry phase, and the described mechanism is analog to the Jahn-Teller effect in molecular dynamics

couple also the internal degrees of freedom (i.e. the mentioned hyperfine levels) and therefore to create non-abelian gauge potentials as well, by interpreting the atomic levels as color indices of the gauge group [45]. Even more exotic scenarios are possible if the hyperfine levels are seen as position indices of some extra-dimension [46]. Non-abelian gauge potentials also open new possibilities connected to emergent relativistic massless Dirac dispersions (including exotic effects like Klein tunnelling and Zitterbewegung) and to the formation of topological insulators ([18] and references therein) or to ground-states with non-abelian excitations, i.e. anyons, possibly useful in the direction of quantum computation [47, 48].

In this framework of creating "synthetic" gauge potentials, interactions between the underlying constituents of the simulator are often tuned to zero in order to get clean dispersion relations (as done below in Sec. 3) or are exploited as independent knobs to achieve exotic many-body regimes (e.g., quantum Hall states), but they never enter as key ingredients to engineer the gauge itself. A tentative to translate the interatomic potentials into relativistic invariant interactions between the emergent Dirac fermions has been put forward in Ref. [49]. We also already mentioned that strongly-interacting fermionic atoms can mimic many aspects of confining gauge theories and several aspects of hadronic matter in extreme conditions. Unfortunately all these approaches still lack the dynamical character that gauge fields have in high-energy theory.

2.3 Towards dynamical gauge fields with cold atoms

The first requirement for a physical setup aiming to emulate a fully quantum (lattice) gauge theory is quite clearly to treat bosons and fermions on an equal footing: this task is indeed well accomplished by several experiments in ultracold atoms since many years [11 – 13]. The counter requirement for high-energy problems is that they have to be formulated in a clear Hamiltonian formalism, possibly with discrete degrees of freedom, in order to resemble as much as possible the situation of cold atomic particles into an optical lattice. A first strategy is offered by Ref. [50], where the low energy sector of interpenetrating 1D lattices loaded with bosons and fermions is shown to display emergent Dirac fermions coupled to a dynamical scalar field, thus realising the Gross-Neveu model. Another possibility is to use quantum link models [51 – 53], where the elements of the continuous gauge group are replaced by discrete quantum spins living on the lattice links. This offers an alternative non-perturbative approach that deals with a quantum problem rather than with a classical statistical mechanics one, like in the traditional Wilson formulation of lattice gauge theories [54]. Moreover, these models are also relevant in condensed matter contexts, like spin liquids and frustrated systems [55, 56]. A quite intense recent theoretical activity has focused on working out fairly demanding optical-atomic tabletop experimental setups to emulate U(1) quantum gauge theories, namely compact QED in different dimensionality [57 – 67]. Active experiments are still to be scheduled in this direction, though the ideas are in principle all feasible.

The first generation of quantum emulators of gauge theories will, of course, not aim at solving right away ultimate problems in QCD and not even at performing high-precision measurements on other models. A profitable use of them will instead be to investigate some classically hardly accessible regime, like real-time evolution and equilibration properties after quenches or phase diagrams at high baryonic matter density [5, 51]. After a validation by comparison with a brute force computation of small scale systems, the devices can be employed to highlight the presence (or absence) and the character of specific phases or phenomena otherwise almost undecidable.

The literature on the topic is vastly flourishing right now and many other working directions may elude the simple partial lists presented here and in the previous section. For a nice vocabulary translation between the two communities we address the reader to the other Colloquia in this Workshop [68] and to a recent review by U. Wiese [51].

3. A tunable platform for relativistic fermions and axions

We revise here a proposal of an experimental scheme, pursuable with an increasing degree of intricacy, aiming to realise a fermionic D-dimensional Hubbard Hamiltonian [17, 18]:

$$H_{\rm sys} = \sum_{\mathbf{r}} \sum_{\tau\tau'} \left[\sum_{\boldsymbol{\nu}} t_\nu \, c^\dagger_{\mathbf{r}+\boldsymbol{\nu}\tau'} [U_{\boldsymbol{\nu}}]_{\tau'\tau} c_{\mathbf{r}\tau} + \Omega \, c^\dagger_{\mathbf{r}\tau'} [\Lambda]_{\tau'\tau} c_{\mathbf{r}\tau} + \text{H.c.} \right] \tag{3.1}$$

with tunneling strengths t_ν and hopping operators $U_{\boldsymbol{\nu}}$ dependent on the nearest-neighbour direction $\boldsymbol{\nu}$, as well as on-site energy operators Λ with amplitude Ω. The dependence of both $U_{\boldsymbol{\nu}}$ and Λ on the N_D internal degrees of freedom of the atoms, labeled by τ, is exploited to shape the dispersion relation around the Fermi surface and to obtain an effective relativistic field theory, containing different kind of fermions, from "naive" Dirac ones to "axion-like" particles. In Sec. 3.1 we expose the sequence of steps from a theoretical point of view, while in Sec. 3.2 we substantiate these ideas with details on the envisaged laser assisted schemes for cold atoms in optical lattices. Here we deal only with the non-interacting quadratic part of the Hubbard Hamiltonian, since the atomic interactions can then be set to zero via the cited Feshbach resonances [24].

3.1 General strategy towards relativistic lattice field theories

Let us start by denoting with α_ν, β the Dirac matrices[4], fulfilling a Clifford algebra, i.e. $\{\alpha_\nu, \alpha_\mu\} = 2\delta^{\nu\mu}$ and $\{\alpha_\nu, \beta\} = 0$, thus requiring $N_D = 2$ for $D = 1, 2$ and $N_D = 4$ for $D = 3$. By implementing translationally invariant $U_{+\nu} = i\,\alpha_\nu$ and $\Lambda = \beta$, the Hamiltonian (3.1) becomes

$$H = \sum_{\mathbf{k}\in \mathrm{BZ}} \tilde{c}^\dagger_{\mathbf{k}} \left(\sum_\nu 2 t_\nu \sin k_\nu \alpha_\nu + 2\Omega\beta \right) \tilde{c}_{\mathbf{k}} \,, \tag{3.2}$$

where $\tilde{c}_{\mathbf{k}}$ are the field operators in momentum space (BZ being the Brillouin zone). A set of $\mathcal{N}_D = 2^D$ isolated extremal points $\mathbf{K_d} = \boldsymbol{\pi}\mathbf{d} = (d_x\pi, d_y\pi, d_z\pi)$ (where $d_\nu \in \{0,1\}$) can be identified: their energy $\varepsilon(\mathbf{K_d}) = \pm\Omega$ becomes degenerate and exhibits Dirac cones for $\Omega = 0$. Around them we can define translated field operators $\Psi(\mathbf{p_d}) = \tilde{c}_{\mathbf{K_d}+\mathbf{p_d}}$ and thus reformulate Eq.(3.2) as

$$H_{\rm eff} = \sum_{\mathbf{d}} \sum_{\mathbf{p_d}} \Psi^\dagger(\mathbf{p_d}) \left[c\, \boldsymbol{\alpha}^{\mathbf{d}} \cdot \mathbf{p_d} + mc^2\beta \right] \Psi(\mathbf{p_d}), \qquad (\boldsymbol{\alpha}^{\mathbf{d}})_\nu = (-1)^{d_\nu} \alpha_\nu. \tag{3.3}$$

The quantum statistics forces then the low energy excitations of the cold atomic gas at half filling to embody a relativistic field theory in terms of Dirac fermions, where the tiny Fermi velocity plays the role of the speed of light, i.e. $c = 2t_x = 2t_y = 2t_z \simeq 5\cdot 10^{-6} \div 5\cdot 10^{-5}$ m/s, and the onsite energy scale sets the effective mass $mc^2 = 2\Omega$. This path is usually dubbed as "naive" since it leads to an even number of relativistic-fermion species, each located around a different Dirac point $\mathbf{K_d}$.

[4] with respect to the notation of γ matrices, it holds $\alpha_\nu = \gamma_0\gamma_\nu$ and $\beta = \gamma_0$

In order to remove these spurious fermion doublers, a first solution is to synthesize the so-called Wilson fermions by adding extra nearest-neighbour hoppings of the form $\tilde{U}_{\boldsymbol{\nu}} = -\beta$. This indeed induces a term $-2\tilde{t}_\nu \cos k_\nu \beta$ in Eq.(3.2) and results into:

$$H_{\rm eff} = \sum_{\mathbf{d}} \sum_{\mathbf{p_d}} \Psi^\dagger(\mathbf{p_d}) \left[c\boldsymbol{\alpha}^{\mathbf{d}} \cdot \mathbf{p_d} + m_{\mathbf{K_d}} c^2 \beta \right] \Psi(\mathbf{p_d}), \qquad m_{\mathbf{K_d}} = m - \sum_\nu (-1)^{d_\nu} m_\nu. \tag{3.4}$$

By tuning the direction dependent mass shift $m_\nu c^2 = 2\tilde{t}_\nu$ properly, i.e. $\sum_\nu m_\nu = m$, a single massless Dirac fermion survives at $K_{\mathbf{d}} = \mathbf{0}$ and is therefore decoupled from all its doublers.

An alternative route is provided by "domain-wall" fermions, namely massless chiral Dirac fermions bound to a lower-dimensional surface where the mass gets inverted, as introduced by Kaplan [69]. The same happens on the boundaries of a finitely thick slab, inside which the Wilson mass is tuned to be negative: as soon as the localisation length is much smaller than the slab thickness, the two copies can be considered independent and thus circumventing the doubling issue. Such strategy can be implemented within an ultracold atomic setup, e.g., by making the on-site energy scale Ω space dependent, while keeping the hopping rates t_ν, $\tilde{t}_\nu$ translationally invariant.

Finally, by controlling additional on-site terms $\tilde{\Omega}(i\beta\gamma_5)$, we could even envisage a so-called *axion term*, i.e. a complex Wilson mass of argument θ. When combined with (synthetic) electromagnetic gauge fields, this would introduce a term $\tilde{\alpha}(\theta/2\pi)\mathbf{E}\cdot\mathbf{B}$ in the Hamiltonian density and a set of modified Maxwell equations, $\boldsymbol{\nabla}\cdot\mathbf{E} = \rho - \tilde{\alpha}/2\pi\,\boldsymbol{\nabla}\theta\cdot\mathbf{B}$ and $\boldsymbol{\nabla}\times\mathbf{B} = \partial_t\mathbf{E} + \mathbf{j} + \tilde{\alpha}/2\pi\,(\boldsymbol{\nabla}\theta\times\mathbf{E} + \partial_t\theta\,\mathbf{B})$[5]. These would lead to anomalous magneto-electric effects at the interface between standard ($\theta = 0$) and topological ($\theta = \pi$) bulk regions, as in the integer quantum Hall effect and other topological insulators [70–75]. In Ref. [17] we suggested to look by in-situ imaging for fractional accumulation of atomic density to certify the presence of domain wall fermions in the above introduced thick slab arrangement - i.e., to realize a sort of 3D fractional magnetic capacitor.

3.2 Some details on our Superlattice+Raman proposal

We consider a spin-independent optical potential[6] $V(\mathbf{x}) = -V_0 \sum_{j\in D} \left[\cos^2(qx_j) + \xi\cos^2(2qx_j)\right]$, formed by a principal laser beam with wavevector $q = 2\pi/\lambda_L$ and a secondary one at double frequency with relative intensity ξ; the energy scale V_0 (of order 100 KHz) is set by the laser power and frequency, as sketched in Sec. 2.1. The resulting cubic array of main minima, $\mathbf{r} \in (\lambda_L/2)\,\mathbb{Z}^D$, is taken as the simulation lattice[7], while Wannier states localised in secondary minima on the middle of each lattice link are employed as ancillary tools to realize the laser-assisted hoppings[8]. We focus on fermionic alkali isotopes[9] and identify a proper subset of N_D hyperfine sub-levels $|F, m_F\rangle$ with the field components τ of Eq.(3.1), while a second hyperfine manifold F' at energy $\simeq \Delta_{\rm HF}$ serves as a further ancilla. Such levels can be made individually addressable by exploiting the linear Zeeman splitting they undergo in presence of a weak magnetic field[10]: $E_{F,m_F} = +g_F\,\mu_B\,B\,m_F$ and $E_{F',m_{F'}} = \Delta_{\rm HF} - g_F\,\mu_B\,B\,m_{F'}$, with $g_F\,\mu_B\,B$ easily of order $10 \div 50$ MHz (see Fig. 1b-c).

[5] $\tilde{\alpha}$ is an effective fine structure constant

[6] such a choice allows in principle for longer lifetimes, due to larger detuning and lower photon adsorption [29]

[7] the lattice spacing is of order 0.3-0.5 μm as dictated by optical frequencies of order 300-500Thz

[8] additional higher-order minima, not lying across lattice links, will not play any relevant role

[9] alkalis have only one external electron and thus an hyperfine structure similar to hydrogen

[10] the Landé factors are opposite for the two hyperfine manifolds, i.e. $g_{F'} = -g_F$

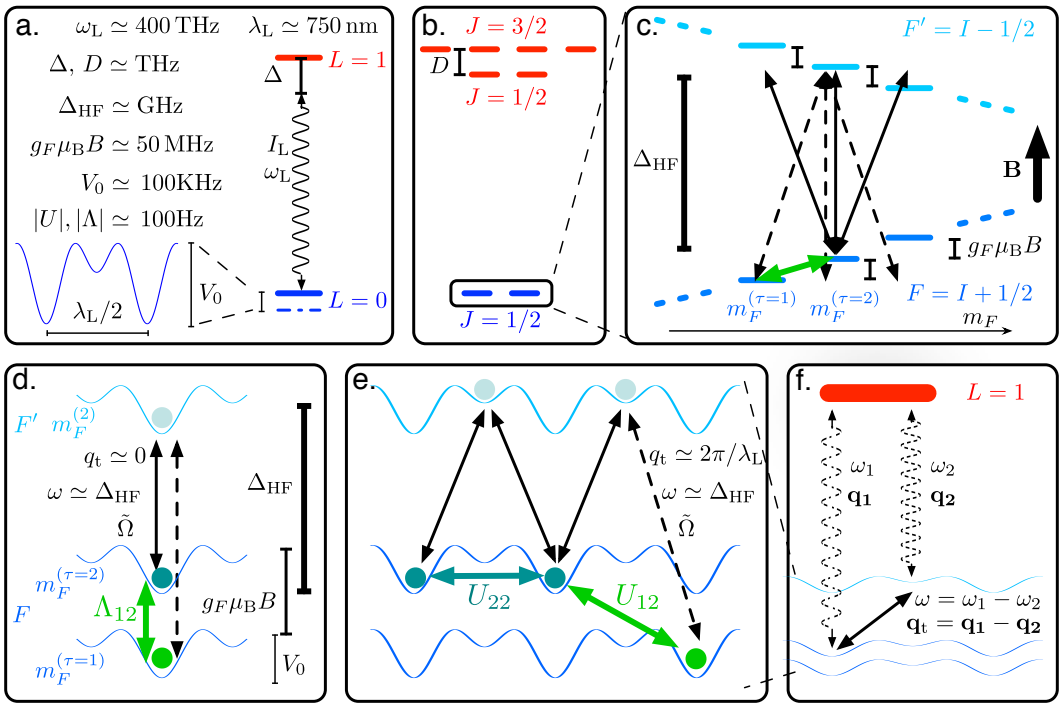

Figure 1: **a.** The basic principle of optical trapping, by considering only the electronic structure of the atom; some typical numbers are quoted about the exploited energy hierarchy. **b.** The fine structure of an alkali atom, $J = L+S$, with $S = 1/2$ of the single electron. **c.** The hyperfine structure, $F = I+J$, with I the nuclear spin (e.g., $I = 4$ for the fermionic ^{40}K); the energy selection rule used in (d.; e.) is also depicted (B not parallel to the laser/lattice axes, thus all three polarization involved, but only one coupling much more effective than others). **d.** The generation of on-site matrix elements $\Lambda_{\tau,\tau'}$ via microwave Raman couplings through the ancillary level in F'. **e.** Same for the hopping matrix $[U_{\boldsymbol{\nu}}]_{\tau,\tau'}$, this time needing a momentum transfer $\mathbf{q}_t \parallel \boldsymbol{\nu}$ and an ancilla in the intermediate minima between main lattice sites. **f.** The further optical Raman scheme to generate the momentum and energy transfer needed in (e.). More details, including further typical numbers and numerical checks of the adiabatic eliminations, can be found in Ref. [18].

We then suggest to freeze the natural tunnelling by raising V_0 and to design a collection of Raman couplings between the simulation sites/spins through the above introduced ancillary states. These excited levels are then integrated out in a process called adiabatic elimination in order to obtain the desired matrix element of the on-site (Λ) and nearest-neighbors ($U_{\boldsymbol{\nu}}$) operators of Eq. (3.1). Some distinction is needed due to the form of the electromagnetic coupling $\tilde{\Omega}_{\tau_2\mathbf{r}_2;\tau_1\mathbf{r}_1}(t) = S_{\mathbf{r}_2,\mathbf{r}_1}\,\Omega_{\tau_2,\tau_1}\,e^{-i\omega t}$, where:

Ω_{τ_2,τ_1} is a function of the dipole matrix element and the polarization of the light;

$S_{\mathbf{r}_2,\mathbf{r}_1}$ contains the dependence on the center-of-mass degrees of freedom;

$\mathbf{q}_t$ and ω are the transferred momentum and energy, respectively.

The factor $S_{\mathbf{r}_2,\mathbf{r}_1}$ is proportional to the overlap integral of the Wannier functions with the momentum transferred by the photon, i.e. $\langle w_{n_2,\mathbf{r}_2}|e^{-i\mathbf{q}_t\cdot\mathbf{x}}|w_{n_1,\mathbf{r}_1}\rangle$, and is not zero only if $2\pi/|\mathbf{q}_t|$ is of the order of the distance $|\mathbf{r}_2-\mathbf{r}_1|$. Due to the microwave nature of the hyperfine splitting ($\Delta_{HF} \simeq$ GHz), a direct coupling of the levels would transfer a negligible momentum, sufficient to design only on-site matrices Λ (Fig. 1d). In order to control hopping terms $U_{\boldsymbol{\nu}}$, we suggest to employ a further two-photon Raman process involving an electronically excited manifold (100's THz far away) allowing for "optical kicks" comparable to the lattice spacing (Fig. 1e-f). The obtained synthetic hopping will be then ruled essentially only by laser pairs aligned with the link direction. Furthermore, the

use of coherent laser light for the Raman transitions entails the additional advantage of being able to imprint complex-phases, $\varphi = \mathbf{q}_t \cdot (\mathbf{r}_1 + \mathbf{r}_2)/2$, even if this would require to slightly violate the just mentioned direction rule [39 – 44].

Let us stress that our scheme does not need the enforcement of polarization selection rules, which would be problematic in $D = 3$. Besides the direction rule of overlap integrals, we suggest to strongly profit of the favourable atomic energy hierarchy to create effective energy selection rules [18]. For example, a magnetic field not aligned to any lattice axis would allow each photon to produce all spin changes $\delta m_F = 0, \pm 1$ independently of the beam polarization: the above described Zeeman splitting will then be sufficient to pinpoint which of these jumps is relevant (Fig. 1c). Such energy-based rules, however, do not prevent spurious on-site spin-flipping: these can however be compensated with the help of staggered lattices and microwave fields [40].

At this point we have introduced the main ingredients to individually tailor the matrix elements of the quadratic Hubbard operators in amplitude and phase, and direction dependence, as required to quantum simulate lattice field theories with the strategy of Sec. 3.1. By plugging realistic numbers, we can foresee assisted tunnelling rates of order 100Hz, comparable to ongoing frontier experiments. Temperature and entropy requirements to detect true ground state properties are subtle and are currently attacked with huge efforts, the aim being the detection of anti-ferromagnetic ordering of Hubbard models and its connection to high-T_c superconductivity [4, 76, 77]. Once a strategy will be found, we are therefore confident that these requirements will be automatically at reach also for a quantum simulator of lattice field theories as the one reviewed here. In Refs. [16 – 18] we checked numerically all the adiabatic eliminations against an exact time-evolution of the full atomic-optical Hamiltonian on a link. We showed that a validity within errors (i.e., essentially spurious couplings) of order few % can be achieved, even without a system dedicated optimization that would be relevant only at the moment of actual experimental design. On the other hand, it would be intrinsically interesting to see how robust is the phenomenology predicted by the ideal lattice field theory, in presence of such unavoidable imperfections of a quantum simulator [10].

4. Conclusion

In this Colloquium we briefly reviewed the idea of quantum simulations of artificial gauge potentials and fields by the use of ultracold atomic gases and quantum optics tools. The rapid and impressive experimental progresses are making possible to foresee that in a not-too-far future cold-atom experiments will be actively employed in the search for an answer to challenging theoretical problems. This may even be true for open-questions related to static and dynamical gauge models, as we have recently witnessed the realization of synthetic Abelian potentials in optical lattices [41 – 44], as well as the first non-Abelian ones in free-space [33, 35, 34]. We focused here on the design of a flexible toolbox to reproduce lattice field theories and classical gauge fields, leaving the task of quantum gauge theories to other Colloquia of this Workshop [68]. Quite interestingly, with the help of the same setup, we could follow a dimensional reduction approach to realize most of the instances of the celebrated periodic table of topological insulators [18].

Among the directions where an atomic-optical simulator can reveal its utility, we mentioned the possibility of studying time-dependent non-equilibrium physics. In this respect, it is natural to think at the periodic driving of the potentials in time and therefore at exploring the rich phe-

nomenology of quenches and equilibration. With respect to our specific proposal, we might even envision to combine the tailoring of the synthetic axion field with the controlled preparation and release of matter-wavepackets [78] in order to synthesize some analogue of the "light behind the wall" experiments [79]. This would indeed embody the spirit of what we dubbed as "bringing abstract models into life" in the introduction of the quantum simulator rationale.

A further interesting perspective is to analyse the situation once interactions between the simulator constituents are no longer tuned to zero as assumed in the present manuscript. On one hand they can be mapped to the relativistic language and exploited to realize truly dynamical gauge theories. The advantage will lie on dealing with finite densities of matter constituents without suffering sign problems felt by classical computations [51]. On the other hand, an intense search is also ongoing towards exotic fractionalization effects, in analogy to the fractional quantum Hall, with the hope of outcomes on quantum computation [48].

Even if the exciting results we reviewed are calling attention of the entire scientific community, we may be just starting to scratch an incredible world. If quantum simulators will hold their promise, the sinergy between theoretical and experimental studies which has always boosted the progress of physics will experience a novel era of joint efforts.

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