

Gauge fields beyond perturbation theory.

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A new formulation of nonabelian gauge theories, introducing new ghost fields and new symmetry is proposed. This formulation does not suffer from Gribov ambiguity and allows to quantize nonabelian gauge fields beyond perturbation theory.

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In this talk I am going to discuss a new approach to nonabelian gauge theories, which allows to quantize them unambiguously, provides a gauge invariant infrared regularization of these theories and may lead to a possibility of finding solitons in the topologically nontrivial sectors of nonabelian gauge theories.

Quantum Chromodynamics (QCD) is considered as theoretical basis of strong interaction physics. No experimental facts contradicting QCD were discovered. At the same time from the point of view of the theory QCD is far from being completed. A consistent theory of color confinement is absent. Even the quantization of nonabelian gauge fields beyond perturbation theory strictly speaking does not exist .

1. A general scheme of the method.

Progress in physics was usually related to the introduction of new symmetries.

Recent examples are given by gauge theories. QED may be formulated in terms of the stress tensor, depending only on the electric and magnetic fields, however much more transparent formulation is presented by the quantization in a manifestly covariant gauge. But using a covariant gauge we inevitably introduce unphysical excitations, corresponding to temporal and longitudinal photons. Simultaneously the theory acquires a new symmetry, gauge invariance. This invariance provides decoupling of unphysical excitations and guarantees unitarity of the scattering matrix in the space of transversal photons. Yang-Mills theory became really popular only after its formulation in the Lorentz covariant terms and explicit proof of its renormalizability. The gauge invariance of the Higgs model allows to give a manifestly renormalizable theory describing a massive gauge theory.

In this talk I wish to make a propaganda for a class of symmetries, which were introduced in my paper rather long ago [1] , but recently were applied successfully to the nonperturbative quantization of non-Abelian gauge theories, construction of the infrared regularization, applicable beyond perturbation theory.

This symmetry is based on the equivalence theorems. It is well known that the physical content of the theory does not change under canonical transformations. The same statement with some reservations related to the renormalization properties is true also for point transformations $\varphi = \varphi' + f(\varphi')$

One can also consider more general transformations, which contain explicitly the time derivatives of the fields. Let us transform the fields as follows

$$\varphi = \frac{\partial^n \varphi'}{\partial t^n} + f\left(\frac{\partial^{n-1} \varphi'}{\partial t^{n-1}}, \dots, \frac{\partial \varphi'}{\partial t}\right) = \tilde{f}(\varphi') \quad (1.1)$$

The spectrum is obviously changed under this transformation. New unphysical excitations appear. The question about the unitarity of the transformed theory arises.

Some ideas about possible violations of unitarity by this transformation are given by the path integral representation for the scattering matrix

$$S = \int \exp\{i \int L(\varphi) dx\} d\mu(\varphi); \quad \lim_{t \rightarrow \pm\infty} \varphi(x) = \varphi_{out,in}(x) \quad (1.2)$$

If the change (1.1) does not change the asymptotic conditions, then the only effect of such transformation is the appearance of a nontrivial jacobian

$$L(\varphi) \rightarrow \tilde{L}(\varphi') = L[\varphi(\varphi')] + \bar{c}^a \frac{\delta \varphi^a}{\delta \varphi'^b} c^b \quad (1.3)$$

For all new excitations one should take the vacuum boundary conditions. But it is by no means obvious that such boundary conditions may be imposed. To answer this question we note that the transformed lagrangian (1.3) is invariant with respect to a new symmetry

$$\begin{aligned} \delta \varphi'_a &= c_a \varepsilon \\ \delta c_a &= 0; \quad \delta \bar{c}_a = \frac{\delta L}{\delta \varphi_a}(\varphi') \varepsilon \end{aligned} \quad (1.4)$$

In these equations ε is a constant anticommuting parameter. On mass shell these transformations are nilpotent and generate a conserved charge Q , belonging to the Grassmann algebra. In this case there exists an invariant subspace of states annihilated by Q , which has a semidefinite norm. ([1]). For asymptotic space this condition reduces to

$$Q_0 |\phi\rangle_{as} = 0 \quad (1.5)$$

The scattering matrix is unitary in the subspace which contains only excitations of the original theory. However the theories described by the L and the \tilde{L} are different, and only expectation values of the gauge invariant operators coincide.

A very nontrivial generalization is obtained if one transforms the \tilde{L} further shifting the fields φ in the topologically trivial sector by constants. It is not an allowed change of variables in the path integral as it changes the asymptotic of the fields. The unitarity of the "shifted" theory is not guaranteed and a special proof (if possible) is needed.

Using this method one can construct a renormalizable formulation of nonabelian gauge theories free of ambiguity.

In fact it is not necessary to introduce higher derivatives. Necessary ingredients are new ghost excitations, and new symmetry of the Lagrangian.

These ideas were successfully implemented in the papers ([2], [3], [4], [5]) A problem of unambiguous quantization of nonabelian gauge theories beyond perturbation theory originates from the classical theory: Even in classical theory the equation

$$D_\mu F_{\mu\nu} = 0 \quad (1.6)$$

does not determine the Cauchy problem. To deal with gauge theory one has to impose the gauge condition, selecting a unique representative in a gauge equivalent class.

Differential gauge conditions: $L(A_\mu, \varphi) = 0 \rightarrow$ which contains a differential operator as we shall see lead to appearance of Gribov ambiguity. One can try to avoid this problem by applying so called algebraic gauge conditions: $\tilde{L}(A_\mu, \varphi) = 0$. The most known condition of this kind is so called Hamiltonian gauge $A_0 = 0$. However these gauges also lead to problems. From practical point of view the most important problem is the absence of a manifest Lorentz invariance.

Let us consider the problem of ambiguity for the case of Coulomb gauge. To answer the question about ambiguity in the choice of a representative in the class of gauge invariant configurations

in the case of the Coulomb gauge, we must consider a possibility of existence of several solutions of the equation $\partial_i A_i = 0$.

$$\begin{aligned} \partial_i A_i &= 0 \\ A'_i &= (A^\Omega)_i \\ \Delta \alpha^a + ig \varepsilon^{abc} \partial_i (A_i^b \alpha^c) &= 0 \end{aligned} \quad (1.7)$$

The last equation has nontrivial solutions rapidly decreasing at spatial infinity, therefore the Coulomb gauge does not select a unique representative among gauge equivalent configurations. This fact was firstly noticed by V.N.Gribov [6] and later generalized by I.Singer [7] to arbitrary gauge. I wish to emphasize that in perturbation theory the only solution of the (eq.1.7) is $\alpha = 0$. So in perturbation theory the problem of ambiguity is absent. There are two possibilities to solve the problem of ambiguity :

1. Use of this phenomenon to try to explain confinement e.t.c. (Series of works by D.Zwanziger [8] and others.)
2. To avoid the Gribov problem by using new (equivalent) formulation of the Yang-Mills theory using more ghost fields. In the following I consider in more details the second option.

2. Formulation of the Yang-Mills theory free of Gribov ambiguity

Let us consider the classical ($SU(2)$)Lagrangian

$$\begin{aligned} L = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a - m^{-2} (D^2 \tilde{\phi})^* (D^2 \tilde{\phi}) + (D_\mu e)^* (D_\mu b) + (D_\mu b)^* (D_\mu e) \\ + \alpha^2 (D_\mu \tilde{\phi})^* (D_\mu \tilde{\phi}) - \alpha^2 m^2 (b^* e + e^* b) \end{aligned} \quad (2.1)$$

where ϕ is a two component complex doublet, and

$$\tilde{\phi} = \phi - \hat{\mu}; \quad \hat{\mu} = (0, \mu \sqrt{2} g^{-1}) \quad (2.2)$$

μ is an arbitrary constant. D_μ denotes the usual covariant derivative. To save the place we consider here the group $SU(2)$.

In the following we shall use the parametrization of ϕ in terms of Hermitean components

$$\phi = \left(\frac{i\phi^1 + \phi^2}{\sqrt{2}} \left(1 + \frac{g}{2\mu} \phi^0\right), \frac{\phi^0 - i\phi^3 \left(1 + \frac{g}{2\mu} \phi^0\right)}{\sqrt{2}} \right) \quad (2.3)$$

The complex anticommuting scalar fields b, e will be parameterized as follows

$$\begin{aligned} b &= \left(\frac{ib^1 + b^2}{\sqrt{2}}, \frac{b^0 - ib^3}{\sqrt{2}} \right) \left(1 + \frac{g}{2\mu} \phi^0\right) \\ e &= \left(\frac{ie^1 + e^2}{\sqrt{2}}, \frac{e^3}{\sqrt{2}} \right) \end{aligned} \quad (2.4)$$

where the components e^α are Hermitean, and b^α are antihermitean. This particular parametrization of the classical fields is used as we want to get rid off the ambiguity in choosing the gauge for quantization completely.

In this parametrization the Lagrangian (2.1) is invariant with respect to "shifted" gauge transformations

$$\begin{aligned}
A_\mu^a &\rightarrow A_\mu^a + \partial_\mu \eta^a - g \varepsilon^{abc} A_\mu^b \eta^c \\
\phi^a &\rightarrow \phi^a + \frac{g^2}{4\mu} \phi^a \phi^b \eta^b + \mu \eta^a \\
\phi^0 &\rightarrow \phi^0 - \frac{g}{2} \phi^a \eta^a (1 + \frac{g}{2} \phi^0) \\
b^a &\rightarrow b^a + \frac{g}{2} \varepsilon^{abc} b^b \eta^c + \frac{g}{2} b^0 \eta^a + \frac{g^2}{4\mu} b^a \phi^b \eta^b \\
e^a &\rightarrow e^a + \frac{g}{2} \varepsilon^{abc} e^b \eta^c + \frac{g}{2} e^0 \eta^a \\
b^0 &\rightarrow b^0 - \frac{g}{2} b^a \eta^a + \frac{g^2}{4\mu} (\phi^a \eta^a) \\
e^0 &\rightarrow e^0 - \frac{g}{2} e^a \eta^a.
\end{aligned} \tag{2.5}$$

The field ϕ^a is shifted by an arbitrary function, therefore one can put $\phi^a = 0$. Contrary to the common wisdom this gauge is algebraic, but Lorentz invariant. It may be used beyond perturbation theory as well.

This Lagrangian is also invariant with respect to the supersymmetry transformations

$$\begin{aligned}
\phi &\rightarrow \phi - b \varepsilon \\
e &\rightarrow e - \frac{D^2(\phi - \hat{\mu})}{m^2} \varepsilon \\
b &\rightarrow b
\end{aligned} \tag{2.6}$$

where ε is a constant Hermitean anticommuting parameter. This symmetry plays a crucial role in the proof of decoupling of unphysical excitations. It holds for any α , but for $\alpha = 0$ these transformations are also nilpotent.

Note that for further discussion we need only the existence of the conserved charge Q and nilpotency of the asymptotic charge Q_0 , as the physical spectrum is determined by the asymptotic dynamics. In the case under consideration the nilpotency of the asymptotic charge requires $\alpha = 0$, and the massive theory with $\alpha \neq 0$ is gauge invariant but not unitary. It may seem strange as usually the gauge invariance is a sufficient condition of unitarity, because one can pass freely from a renormalizable gauge to the unitary one, where the spectrum includes only physical excitations. In the present case there is no "unitary" gauge. Even in the gauge $\phi^a = 0$, there are unphysical excitations.

For gauge transformations (2.5) the gauge $\phi^a = 0$ is admissible both in perturbation theory and beyond it. Indeed, if $\phi^a = 0$, then under the gauge transformations (2.5) the variables ϕ^a become

$$\delta \phi^a = \mu \eta^a \tag{2.7}$$

and the condition $\phi^a = 0$ implies that $\eta^a = 0$. It is also obvious that for $\alpha \neq 0$ the Lagrangian (2.1) describes a massive vector field and does not produce infrared singularities.

In terms of shifted variables the Lagrangian (2.1) looks as follows

$$\begin{aligned}
L = & -\frac{1}{4}F_{\mu\nu}^a F_{\mu\nu}^a - m^{-2}(D^2\phi)^*(D^2\phi) + m^{-2}(D^2\phi)^*(D^2\hat{\mu}) \\
& + m^{-2}(D^2\hat{\mu})^*(D^2\phi) - m^{-2}(D^2\hat{\mu})^*(D^2\hat{\mu}) + (D_\mu e)^*(D_\mu b) \\
& + (D_\mu b)^*(D_\mu e) + \alpha^2(D_\mu\phi)^*(D_\mu\phi) - \alpha^2(D_\mu\phi)^*(D_\mu\hat{\mu}) \\
& - \alpha^2(D_\mu\hat{\mu})^*(D_\mu\phi) + \alpha^2(D_\mu\hat{\mu})^*(D_\mu\hat{\mu}) - \alpha^2 m^2(b^*e + e^*b)
\end{aligned} \tag{2.8}$$

The shift of the variables ϕ produces the term

$$\alpha^2(D_\mu\hat{\mu})^*(D_\mu\hat{\mu}) = \frac{\alpha^2\mu^2}{2}A_\mu^2 \tag{2.9}$$

which gives a mass to the vector field. The term

$$m^{-2}(D^2\hat{\mu})^*(D^2\hat{\mu}) = \frac{\mu^2}{2m^2}[(\partial_\mu A_\mu)^2 + \frac{g^2}{2}(A^2)^2] \tag{2.10}$$

makes the theory renormalizable for any α . To avoid complications due to the presence of the Yang-Mills dipole ghosts at $\alpha = 0$ we put $\mu^2 = m^2$.

Invariance of the Lagrangian (2.8) with respect to the gauge transformation (2.5) and the supersymmetry transformations (2.6) makes the effective Lagrangian invariant with respect to the simultaneous BRST transformations corresponding to (2.5) and the supersymmetry transformations (2.6). The effective Lagrangian may be written in the form

$$L_{ef} = L + s_1 \bar{c}^a \phi^a = L(x) + \lambda^a \phi^a - \bar{c}^a (\mu c^a - b^a) \tag{2.11}$$

One can integrate over \bar{c}, c in the path integral determining expectation value of any operator corresponding to observable. It leads to the change $c^a = b^a \mu^{-1}$. After such integration the effective Lagrangian becomes invariant with respect to the transformations which are the sum of the BRST transformations and the supersymmetry transformations (2.6) with $c^a = b^a \mu^{-1}$. These transformations look as follows

$$\begin{aligned}
\delta A_\mu^a &= D_\mu b^a \mu^{-1} \varepsilon \\
\delta \phi^a &= 0 \\
\delta \phi^0 &= -b^0 \left(1 + \frac{g}{2\mu} \phi^0\right) \varepsilon \\
\delta e^a &= \left(\frac{g}{2\mu} \varepsilon^{abc} e^b b^c + \frac{g e^0 b^a}{2\mu} + i \frac{D^2(\tilde{\phi})^a}{\mu^2}\right) \varepsilon \\
\delta e^0 &= \left(-\frac{g e^a b^a}{2\mu} - \frac{D^2(\tilde{\phi})^0}{\mu^2}\right) \varepsilon \\
\delta b^a &= \frac{g}{2\mu} \varepsilon^{abc} b^b b^c \\
\delta b^0 &= 0
\end{aligned} \tag{2.12}$$

For the asymptotic theory these transformations acquire the form

$$\delta A_\mu^a = \partial_\mu b^a \mu^{-1} \varepsilon$$

$$\begin{aligned}
\delta\phi^a &= 0 \\
\delta\phi^0 &= -b^0\varepsilon \\
\delta e^a &= \partial_\mu A_\mu^a \mu^{-1} \\
\delta e^0 &= -\partial^2\phi^0 \mu^{-2} \\
\delta b^a &= 0 \\
\delta b^0 &= 0.
\end{aligned} \tag{2.13}$$

According to the Neuther theorem the invariance with respect to the supertransformations mentioned above generates a conserved charge Q , and the physical asymptotic states may be chosen to satisfy the equation

$$\hat{Q}_0|\psi\rangle_{as} = 0 \tag{2.14}$$

$$\begin{aligned}
Q_0 = \int d^3x [(\partial_0 A_i^a - \partial_i A_0^a)\mu^{-1} \partial_i b^a - \mu^{-1} \partial_\nu A_\nu^a \partial_0 b^a + \mu^{-2} \partial^2 (\partial_0 \phi^0) b^0 - \\
\mu^{-2} \partial_0 b^0 \partial^2 (\phi^0 - \mu \alpha^2 b^a A_0^a)]
\end{aligned} \tag{2.15}$$

Due to the conservation of the Neuther charge this condition is invariant with respect to dynamics. It was proven in the paper [9] that this symmetry guarantees the decoupling of all unphysical excitations at $\alpha = 0$ and the transitions between the states, annihilated by the charge Q include only three dimensionally transversal components of the Yang-Mills field. Therefore we succeeded to formulate the Yang-Mills theory in such a way that in a topologically trivial sector Gribov ambiguity is absent and the infrared regularization valid beyond perturbation theory is easily constructed. This approach opens also interesting possibilities to consider topologically nontrivial sectors and study the confinement problem.

Discussion

A renormalizable manifestly Lorentz invariant formulation of the non-Abelian gauge theories which allows a canonical quantization without Gribov ambiguity (including Higgs model) is possible.

In perturbation theory the scattering matrix and the gauge invariant correlators coincide with the standard ones.

On the basis of this approach infrared regularization of Yang-Mills theory beyond perturbation theory is constructed [9]

This approach seems to be appropriate for a study of existence of soliton excitations in Yang-Mills theory. This problem is under consideration.

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