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Masses of light and heavy mesons in a $U(4)_r \times U(4)_l$ linear sigma model

Walaa I. Eshraim[*†]
Institute for Theoretical Physics, Goethe University, Max-von-Laue-Str. 1, 60438 Frankfurt am Main, Germany
E-mail: weshraim@th.physik.uni-frankfurt.de

We extend the three-flavor linear sigma model with (axial-)vector mesons to four flavors. We compute the masses of (pseudo)scalar and (axial-)vector mesons including open and hidden charmed mesons as well as weak decay constants. The results are in good agreement with experimental data.

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*Speaker.
†In collaboration with Francesco Giacosa and Dirk H. Rischke

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1. Introduction

For N_f massless quark flavors, quantum chromodynamics (QCD), the fundamental theory of the strong interaction, has a global chiral $U(N_f)_r \times U(N_f)_l = SU(N_f)_r \times SU(N_f)_l \times U(1)_V \times U(1)_A$ symmetry, where $V = r + l$, and $A = l - r$. The $U(1)_V$ symmetry corresponds to baryon number conservation. Effective models in the linear [1] and nonlinear [2] realization of the chiral symmetry are widely used to investigate the low-energy sector of the strong interaction, e.g. vacuum properties of hadrons [3, 4]. The nonlinear realization (the so-called nonlinear sigma model) contains only the lightest degrees of freedom, the pseudoscalar mesons. It forms the basis of chiral perturbation theory as shown in Ref. [5]. The linear representation of chiral symmetry (the so-called linear sigma model) contains both scalar and pseudoscalar degrees of freedom. (Axial-)vector mesons can also be included in the model [6, 7, 8, 9].

The extended Linear Sigma Model (eLSM) has been successfully used to study the vacuum phenomenology of the nonets of (pseudo)scalar, (axial-)vector, and tensor mesons, as shown in Refs. [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15] for non-strange hadrons ($N_f = 2$), and Refs. [15, 16, 17] for strange hadrons ($N_f = 3$). The eLSM emulates the global symmetries of the QCD Lagrangian; the global chiral symmetry (which is exact in the chiral limit), the discrete C, P, and T symmetries, and the classical dilatation (scale) symmetry. When working with colorless hadronic degrees of freedom, the local color symmetry of QCD is automatically preserved. In QCD (and thus also in the eLSM) the global chiral symmetry is explicitly broken by non-vanishing quark masses and quantum effects [18], and spontaneously by a non-vanishing expectation value of the quark condensate. The dilatation symmetry is broken explicitly by the logaritmic term of the dilaton potential, by the mass terms, and by the $U(1)_A$ anomaly.

In the present work, we outline the extension of the eLSM from the three-flavor case ($N_f = 3$) to the four-flavor case ($N_f = 4$) which includes charm degrees of freedom. In nature, this symmetry is strongly explicitly broken by the large charm quark mass. Nevertheless, it is still of principle interest to see how a linear sigma model fares in describing charmed hadron vacuum properties. As we shall see, this works surprisingly well.

Although the present work represents a straightforward implementation of the principles of the linear realization of chiral symmetry, this is the first time that all these degrees of freedom are considered within a single linear chiral framework which includes twelve new charmed mesons in addition to the nonstrange-strange sector. The new charmed mesons of lowest mass, the D, D_S, and the higher mass η_C, are quark-antiquark spin-singlet states with quantum number $J^{pc} = 0^{-+}$, i.e., pseudoscalar mesons. The scalar mesons D_0^*, D_{S0}^*, and χ_{C0} are spin-singlet states with $J^p = 0^{++}$. The vector mesons D^*, D_S^*, and J/ψ are quark-antiquark spin triplets with $J^{pc} = 1^{--}$. The axial-vector mesons D_1, D_{S1}, and χ_{C1} are quark-antiquark spin triplets with $J^{pc} = 1^{++}$. Most parameters of our linear sigma model are taken directly from Ref. [17] where the nonstrange-strange mesons were considered. There are three new parameters pertaining to the charm degree of freedom. In these proceedings, we calculate all meson masses in the model including open and hidden charmed mesons, and the decay constants of the pseudoscalar D and D_S mesons.

These proceedings are organized as follows: in Sec. 2 we present the $U(4)_R \times U(4)_L$ linear sigma model with (axial-)vector mesons and its implications. In Sec. 3 we fix the parameters and present the results, and in Sec. 4 we provide our conclusions and an outlook. Our units are

$\hbar = c = 1$, the metric tensor is $g^{\mu\nu} = \mathrm{diag}(+,-,-,-)$.

2. The $U(4)_r \times U(4)_l$ linear sigma model and its implications

In this section we extend the eLSM to the case $N_f = 4$ which includes open and hidden charmed mesons. We consider isospin multiplets as a single degree of freedom, which gives 28 resonances for $N_f = 4$:
(i) scalar mesons: σ_N, σ_S, a_0, K_0^*, D_0^*, D_{S0}^*, χ_{C0}.
(ii) pseudoscalar mesons: η_N, η_S, π, K, D, D_S, η_C.
(iii) vector mesons: ω_N, ω_S, ρ, K^*, D^*, D_S^*, J/ψ.
(iv) axial-vector mesons: f_{1N}, f_{1S}, a_1, K_1, D_1, D_{S1}, χ_{C1}.

When we assign a state from our model to a physical resonance we assume that the resonance is a $q\bar{q}$ state. There are 16 light (i.e., with mass $\lesssim 2$ GeV) resonances as discussed in Refs. [15, 17], and twelve new heavy resonances. We include all of these mesons in our model by introducing 4×4 matrices as follows:
(i) The multiplet of the scalar, S_i, and the pseudoscalar, P_i, quark-antiquark states:

$$\Phi = \sum_{i=0}^{15} (S_i + iP_i)T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{*+} + iK^+ & D_0^{*0} + iD^0 \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_0^{*0} + iK^0 & D_0^{*-} + iD^- \\ K_0^{*-} + iK^- & \overline{K}_0^{*0} + i\overline{K}^0 & \sigma_S + i\eta_S & D_{S0}^{*-} + iD_S^- \\ \overline{D}_0^{*0} + i\overline{D}^0 & D_0^{*+} + iD^+ & D_{S0}^{*+} + iD_S^+ & \chi_{C0} + i\eta_C \end{pmatrix}, \tag{2.1}$$

where T_i $(i = 0, \ldots, 15)$ denote the generators of $U(4)$.
(ii) The left-handed and right-handed matrices containing the vector, V_i^μ, and axial-vector, A_i^μ, degrees of freedom:

$$L^\mu = \sum_{i=0}^{15} (V_i^\mu + iA_i^\mu)T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} + \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ + a_1^+ & K^{*+} + K_1^+ & D^{*0} + D_1^0 \\ \rho^- + a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} + \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{*0} + K_1^0 & D^{*-} + D_1^- \\ K^{*-} + K_1^- & \overline{K}^{*0} + \overline{K}_1^0 & \omega_S + f_{1S} & D_S^{*-} + D_{S1}^- \\ \overline{D}^{*0} + \overline{D}_1^0 & D^{*+} + D_1^+ & D_S^{*+} + D_{S1}^+ & J/\psi + \chi_{C1} \end{pmatrix}^\mu, \tag{2.2}$$

$$R^\mu = \sum_{i=0}^{15} (V_i^\mu - iA_i^\mu)T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} - \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ - a_1^+ & K^{*+} - K_1^+ & D^{*0} - D_1^0 \\ \rho^- - a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} - \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{*0} - K_1^0 & D^{*-} - D_1^- \\ K^{*-} - K_1^- & \overline{K}^{*0} - \overline{K}_1^0 & \omega_S - f_{1S} & D_S^{*-} - D_{S1}^- \\ \overline{D}^{*0} - \overline{D}_1^0 & D^{*+} - D_1^+ & D_S^{*+} - D_{S1}^+ & J/\psi - \chi_{C1} \end{pmatrix}^\mu. \tag{2.3}$$

The assignment of fields in the $N_f = 3$ sector to physical resonances is the following [17, 19, 20, 21]: the fields $\vec{\pi}$ and K represent the pions and kaons, respectively. The fields ω_N, ω_S, $\vec{\rho}$, f_{1N}, f_{1S}, $\vec{a}_1$, K^*, K_0^*, and K_1 are assigned to the $\omega(782)$, $\phi(1020)$, $\rho(770)$, $f_1(1285)$, $f_1(1420)$, $a_1(1260)$, $K^*(892)$, $K_0^*(1430)$, and $K_1(1270)$, or $K_1(1400)$ mesons, respectively. The field $\vec{a}_0$ is the physical isotriplet state $a_0(1450)$ (the details of this assignment are given in Ref. [17]). The bare non-strange field $\eta_N \equiv |\bar{u}u + \bar{d}d\rangle/\sqrt{2}$ and strange field $\eta_S \equiv |\bar{s}s\rangle$ mix to yield the physical η and η' fields, with the pseudoscalar mixing angle $\varphi \simeq -44.6^\circ$ [17, 21]. The non-strange and strange isoscalar σ_N and σ_S fields mix to give the physical isoscalar resonances $f_0(1370)$ and $f_0(1710)$, respectively [6, 21].

In the present work, we assign the additional charmed fields D^{*0}, D^*, D_0^{*0}, D_0^*, D_{S0}^*, χ_{c1}, χ_{c0}, and J/ψ to the physical resonances $D^*(2007)0$, $D^*(2010)^\pm$, $D_0^*(2400)^0$, $D_0^*(2400)^\pm$, $D_{S0}^*(2317)$, $\chi_{c1}(1P)$, $\chi_{c0}(1P)$, and $J/\psi(1S)$, respectively. The isospin doublet D_1^0 is $D_1(2420)$. The isospin singlet D_{S1} can be assigned to two different physical resonances, $D_{S1}(2460)$ and $D_{S1}(2536)$. Reference [22] found $D_{S1}(2460)$ to be a molecule, so we assign D_{S1} to $D_{S1}(2536)$.

The eLSM contains also the scalar glueball, G, and the pseudoscalar glueball, $\widetilde{G}$. The scalar glueball is included in the dilaton Lagrangian [17, 23, 24]

$$\mathscr{L}_{dil} = \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4}\frac{m_G^2}{\Lambda^2}\left(G^4 \ln\frac{G^2}{\Lambda^2} - \frac{G^4}{4}\right), \tag{2.4}$$

where Λ is a constant and is the minimum of the dilaton potential. The dilaton potential breaks the dilatation symmetry explicitly. The mass of the glueball is about 1.6 GeV which obtained from lattice-QCD calculations [25].

The Lagrangian of the $N_f = 4$ model with global chiral invariance has an analogous form as the corresponding eLSM Lagrangian for $N_f = 3$ [15, 16, 17]. For a better fit to the masses, we add a new mass term $-2\,\mathrm{Tr}[\varepsilon\Phi^\dagger\Phi]$. The Lagrangian then reads:

$$\begin{aligned}\mathscr{L} = {} & \mathscr{L}_{dil} + \mathrm{Tr}[(D^\mu\Phi)^\dagger(D^\mu\Phi)] - m_0^2\left(\frac{G}{G_0}\right)^2\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2 \\ & + \mathrm{Tr}[H(\Phi+\Phi^\dagger)] - 2\,\mathrm{Tr}[\varepsilon\Phi^\dagger\Phi] + c(\det\Phi - \det\Phi^\dagger)^2 + ic_{\tilde{G}\Phi}\tilde{G}(\det\Phi - \det\Phi^\dagger) \\ & - \frac{1}{4}\mathrm{Tr}[(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \mathrm{Tr}\left\{\left[\left(\frac{G}{G_0}\right)^2\frac{m_1^2}{2} + \Delta\right][(L^\mu)^2 + (R^\mu)^2]\right\} \\ & - 2ig_2\{\mathrm{Tr}(L_{\mu\nu}[L^\mu, L^\nu]) + \mathrm{Tr}(R_{\mu\nu}[R^\mu, R^\nu])\} + \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}[(L^\mu)^2 + (R^\mu)^2] \\ & + h_2\mathrm{Tr}[(\Phi R^\mu)^2 + (L^\mu\Phi)^2] + 2h_3\mathrm{Tr}(\Phi R_\mu\Phi^\dagger L^\mu),\end{aligned} \tag{2.5}$$

where $\mathscr{L}_{dil}$ is the dilaton term (2.5), $D^\mu\Phi \equiv \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu)$ is the covariant derivative; $L^{\mu\nu} \equiv \partial^\mu L^\nu - \partial^\nu L^\mu$, and $R^{\mu\nu} \equiv \partial^\mu R^\nu - \partial^\nu R^\mu$ are the left-handed and right-handed field strength tensors. H, Δ, and ε are constant external fields defined as

$$H = H_0 T_0 + H_8 T_8 + H_{15} T_{15} = \frac{1}{2}\begin{pmatrix} h_{0N} & 0 & 0 & 0 \\ 0 & h_{0N} & 0 & 0 \\ 0 & 0 & \sqrt{2}h_{0S} & 0 \\ 0 & 0 & 0 & \sqrt{2}h_{0C}\end{pmatrix}, \tag{2.6}$$

where $h_{0N} = const.$, $h_{0S} = const.$, and $h_{0C} = const.$,

$$\Delta = \Delta_0 T_0 + \Delta_8 T_8 + \Delta_{15} T_{15} = \begin{pmatrix} \delta_{0N} & 0 & 0 & 0 \\ 0 & \delta_{0N} & 0 & 0 \\ 0 & 0 & \delta_{0S} & 0 \\ 0 & 0 & 0 & \delta_{0C}\end{pmatrix}, \tag{2.7}$$

and

$$\varepsilon = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon_C \end{pmatrix}, \tag{2.8}$$

where $\delta_N \sim m_N^2, \delta_S \sim m_S^2, \delta_C \sim m_C^2$, and $\varepsilon_C \sim m_C^2$. In our framework the isospin symmetry is exact for up and down quarks, so in Eqs. (2.6 - 2.7) the first two diagonal elements are identical. Then, only the scalar-isoscalar fields σ_N, σ_S, G, and χ_{C0} have the quantum numbers of the vacuum and can have nonzero expectation values. Moreover with no loss of generality, one can set $\delta_{0N} = 0$.

In order to implement spontaneous symmetry breaking, we shift σ_N and σ_S by their respective vacuum expectation values ϕ_N, ϕ_S, and ϕ_C as

$$\sigma_N \rightarrow \sigma_N + \phi_N \text{ and } \sigma_S \rightarrow \sigma_S + \phi_S\ , \tag{2.9}$$

as obtained in Refs. [6, 7, 15], and similarly for χ_{C0},

$$\chi_{C0} \rightarrow \chi_{C0} + \phi_C\ . \tag{2.10}$$

The spontaneous symmetry breaking generates in $\eta_N - f_{1N}$, $\vec{\pi} - \vec{a}_1$ [7], $\eta_S - f_{1S}$, $K_S - K^*$, and $K - K_1$ mixing terms [11]:

$$\begin{aligned} &-g_1\phi_N(f_{1N}^{\mu}\partial_\mu\eta_N + \vec{a}_1^{\,\mu}\cdot\partial_\mu\vec{\pi}) - \sqrt{2}g_1\phi_S f_{1S}^{\mu}\partial_\mu\eta_S + ig_1(\sqrt{2}\phi_S - \phi_N)(\overline{K}^{*\mu 0}\,\partial_\mu K_S^0 \\ &\quad + K^{*\mu -}\,\partial_\mu K_S^+)/2 + ig_1(\phi_N - \sqrt{2}\phi_s)(K^{*\mu 0}\,\partial_\mu\overline{K}_S^0 + K^{*\mu +}\,\partial_\mu K_S^-)/2 - g_1(\phi_N + \sqrt{2}\phi_S) \\ &\quad (K_1^{\mu 0}\,\partial_\mu\overline{K}^0 + K_1^{\mu +}\,\partial_\mu K^-)/2 - g_1(\phi_N + \sqrt{2}\phi_S)(\overline{K}_1^{\mu 0}\,\partial_\mu K^0 + K_1^{\mu -}\,\partial_\mu K^+)/2, \end{aligned} \tag{2.11}$$

respectively, as well as in $\eta_C - \chi_{C1}$, $D_S - D_{S1}$, $D_{S0}^* - D_{S1}^*$, $D_0^* - D^*$, and $D - D_1$ mixing terms:

$$\begin{aligned} &-g_1\phi_C\chi_{C1}^{\mu}\,\partial_\mu\eta_C - \sqrt{2}g_1\phi_S(D_{S1}^{\mu -}\,\partial_\mu D_S^+ + D_{S1}^{\mu +}\,\partial_\mu D_S^-)/2 + \sqrt{2}ig_1\phi_S(D_{S1}^{*\mu -}\,\partial_\mu D_{S0}^{*+} \\ &\quad - D_{S1}^{*\mu +}\,\partial_\mu D_{S0}^{*-})/2 + ig_1\phi_N(D^{*\mu -}\,\partial_\mu D_0^{*+} - D^{*\mu +}\,\partial_\mu D_0^{*-})/2 + ig_1\phi_N(D^{*\mu 0}\,\partial_\mu\overline{D}_0^{*0} \\ &\quad - \overline{D}^{*\mu 0}\,\partial_\mu D_0^{*0})/2 - g_1\phi_N(D_1^{0\mu}\,\partial_\mu\overline{D}^0 + \overline{D}_1^{\mu 0}\,\partial_\mu D^0 + D_1^{\mu +}\,\partial_\mu D^- + D_1^{\mu -}\,\partial_\mu D^+)/2, \end{aligned} \tag{2.12}$$

respectively. Note that our Lagrangian is real despite the imaginary $K_S - K^*$, $D_{S0}^* - D_{S1}^*$, and $D_0^* - D^*$ coupling because the $K_S - K^*$, $D_{S0}^* - D_{S1}^*$, and $D_0^* - D^*$ mixing terms are equal to their hermitian conjugates.

In order to obtain canonically normalized fields, we introduce wave-function renormalization constants labelled $Z_{\eta_{N,S}}$ for $\eta_{N,S}$, Z_π for $\vec{\pi}$, Z_{K_S} for K_S, and Z_K for K, Z_{η_C} for η_C, Z_{D_S} for D_S, $Z_{D_{S0}}$ for D_{S0}, $Z_{D_0^*}$ for D_0^*, $Z_{D_0^{*0}}$, and Z_D for D. We obtain the following formulas:

$$Z_\pi \equiv Z_{\eta_N} = \frac{m_{a_1}}{\sqrt{m_{a_1}^2 - g_1^2\phi_N^2}}, \quad Z_{\eta_S} = \frac{m_{f_{1S}}}{\sqrt{m_{f_{1S}}^2 - 2g_1^2\phi_S^2}}, \tag{2.13}$$

$$Z_K = \frac{2m_{K_1}}{\sqrt{4m_{K_1}^2 - g_1^2(\phi_N + \sqrt{2}\phi_S)^2}}, \quad Z_{K_S} = \frac{2m_{K_*}}{\sqrt{4m_{K_*}^2 - g_1^2(\phi_N - \sqrt{2}\phi_S)^2}}, \tag{2.14}$$

as in Ref. [16], and additionally

$$Z_{\eta_C} = \frac{m_{\chi_{C1}}}{\sqrt{m_{\chi_{C1}}^2 - 2g_1^2\phi_C^2}}, \quad Z_{D_S} = \frac{\sqrt{2}m_{D_{S1}}}{\sqrt{2m_{D_{S1}}^2 - g_1^2(\phi_S + \phi_C)^2}}, \tag{2.15}$$

$$Z_{D_{S0}} = \frac{\sqrt{2}m_{D_S^*}}{\sqrt{2m_{D_S^*}^2 - g_1^2(\phi_S - \phi_C)^2}}, \quad Z_{D_0^*} = \frac{2\,m_{D^*}}{\sqrt{4m_{D^*}^2 - g_1^2(\phi_N - \sqrt{2}\phi_C)^2}}, \tag{2.16}$$

$$Z_{D_0^{*0}} = \frac{2\,m_{D^{*0}}}{\sqrt{4m_{D^{*0}}^2 - g_1^2(\phi_N - \sqrt{2}\phi_C)^2}}, \quad Z_D = \frac{2\,m_{D_1}}{\sqrt{4m_{D_1}^2 - g_1^2(\phi_N + \sqrt{2}\phi_C)^2}}, \tag{2.17}$$

where $\phi_N = Z_\pi f_\pi$ [6], $\phi_S = \frac{2Z_K f_K - \phi_N}{\sqrt{2}}$ for the non-strange and strange condensates, with $f_\pi = 92.4$ MeV and $f_K = 155/\sqrt{2}$ MeV being the pion and kaon decay constants, respectively, as shown in Ref. [17]. For the charm condensates, we have $\phi_C = \frac{1}{\sqrt{2}}(2Z_D f_D - \phi_N)$ or $\phi_C = \sqrt{2}Z_{D_S} f_{D_S} - \phi_S$, where f_D, and f_{D_S} are the decay constants of the pseudoscalar D and D_S mesons, respectively.

3. Parameters and results

Our model (2.5) has 16 free parameters in the case of $N_f = 4$, where twelve of them have been determined in Ref. [17] for the case $N_f = 3$. Their values are:

Parameter	Value	Parameter	Value
m_1^2	0.4135×10^6 MeV^2	m_0^2	-0.9183$\times 10^6$ MeV^2
δ_N	0	δ_S	0.1511×10^6 MeV^2
g_1	5.8433	h_1	0
h_2	9.8796	h_3	3.8667
ϕ_N	164.6 MeV	ϕ_S	126.2 MeV
λ_1	0	λ_2	68.2972

Table 1 : Values of parameters.

The parameter c in the axial anomaly term is related to the corresponding parameter [17, 23] in the case $N_f = 3$ as follows:

$$c = \frac{2\,c_{N_f=3}}{\phi_c^2}\,, \tag{3.1}$$

where $c_{N_f=3} = 450.5420 \times 10^{-6}$ MeV^{-2} from the fit in Ref. [17].

The three new parameters δ_C, ϕ_C, and ε_C related to the mass of the charm quark have been determined by a fit including the experimental charmed meson masses from the PDG [26] and our equations for the charmed meson masses. Then we get $\phi_C = 198.103$ MeV, $\delta_C = 3.742 \times 10^6$ MeV^2, and $\varepsilon_C = 1.432 \times 10^6$ MeV^2. We obtain $c = 2.296 \times 10^{-8}$ in the axial anomaly term $c\,(\det\Phi - \det\Phi^\dagger)^2$. The decay constant of the isodoublet D is $f_D = 266.52$ MeV, and of the isosinglet D_S we get $f_{D_S} = 273.8$ MeV, while the experimental values from the PDG [26] are $f_D = 206.7$ MeV, and $f_{D_S} = 260.5$ MeV, respectively.

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The difference of the square charmed vector and axial-vector masses are:

$$m^2_{D_{S1}} - m^2_{D^*_S} = 2\,(g_1^2 - h_3)\phi_S\,\phi_C\;, \tag{3.2}$$

$$m^2_{D_1} - m^2_{D^*} = \sqrt{2}\,(g_1^2 - h_3)\phi_N\,\phi_C\;, \tag{3.3}$$

and

$$m^2_{\chi_{C1}} - m^2_{J/\psi} = 2\,(g_1^2 - h_3)\phi_C^2\;. \tag{3.4}$$

These mass differences are interesting because they do not depend on the charm mass. The results for the light mesons are reported in table 2. By construction, one finds the same values as in Refs. [16, 17].

observable	our value [MeV]	experimental value [MeV]
$m_{f_{1N}}$	1186	1281.8 ± 0.6
m_{a_1}	1185	1230 ± 40
$m_{f_{1S}}$	1372	1426.4 ± 0.9
m_{K^*}	885	891.66 ± 0.26
m_{K_1}	1281	1272 ± 7
m_{σ_1}	1362	(1200-1500)-i(150-250)
m_{a_0}	1363	1474 ± 19
m_{σ_2}	1531	1720 ± 60
m_{ω_N}	783	782.65 ± 0.12
m_{ω_S}	975	1019.46 ± 0.020
m_ρ	783	775.5 ± 38.8
m_η	509	547.853 ± 0.024
m_π	141	139.57018 ± 0.00035
$m_{\eta'}$	962	957.78 ± 0.06
m_{K_S}	1449	1425 ± 50
m_K	485	493.677 ± 0.016

Table 2 : Light meson masses.

The results for the (hidden and open) charmed mesons are reported in table 3. They have been obtained through a fit to experimental data.

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Observable	Our Value [MeV]	Experimental Value [MeV]
$m_{D_{s1}}$	2501	2535.12 ± 0.13
$m_{D_s^*}$	2188	2112.3 ± 0.5
m_{D^*}	2155	2010.28 ± 0.13
$m_{D^{*0}}$	2155	2006.98 ± 0.15
m_{D_1}	2448	2421.3 ± 0.6
$m_{\chi_{c1}}$	3282	3510.66 ± 0.07
$m_{\chi_{c0}}$	3160	3414.75 ± 0.31
$m_{J/\psi}$	2911	3096.916 ± 0.011
m_{D_0}	1882	1864.86 ± 0.13
m_{η_c}	2491	2981 ± 1.1
$m_{D_0^*}$	2416	$2403 \pm 14 \pm 35$
m_D	1882	1869.62 ± 0.15
$m_{D_{s0}^*}$	2470	2317.8 ± 0.6
m_{D_s}	1900	1968.49 ± 0.32
$m_{D_0^{*0}}$	2416	2318 ± 29

Table 3 : Open and hidden charmed meson masses.

4. Conclusions and outlook

We have extended a linear sigma model with (axial-)vector degrees of freedom, the so-called eLSM, to the case of four flavors, $N_f = 4$. The model implements the symmetries of QCD: the discrete C, P, and T symmetries and the global chiral $U(N_f)_r \times U(N_f)_l$ symmetry. The latter is broken spontaneously through the chiral condensate, explicitly through non-vanishing quark masses, and at the quantum level through the $U(1)_A$ anomaly. Furthermore, it implements the dilatation symmetry and its explicit breaking due to the trace anomaly. In the extension to $N_f = 4$ we have included twelve new charmed mesons in the model, which are the scalar mesons $(D_0^*, D_{s0}^*, \chi_{c0})$, the pseudoscalar mesons (D, D_s, η_c), the vector mesons $(D^*, D_s^*, J/\psi)$, and the axial-vector mesons (D_1, D_{s1}, χ_{c1}). To our knowledge, this the first time that a model was constructed, which contains (pseudo)scalars and (axial-)vectors with charm quarks (open and hidden charmed mesons), and the first time that the model has been used to describe meson states with high masses.

We have found that we need a new mass term to successfully fit the masses of charmed mesons. Implementing spontaneous symmetry breaking in the model yields not only the known $\eta_N - f_{1N}$, $\vec{\pi}$-$\vec{a}_1$, $\eta_S - f_{1S}$, $K_S - K^*$, and $K - K_1$ mixings [16] in Eq. (2.11) but also the $\eta_C - \chi_{C1}$, $D_S - D_{S1}$, $D_{S0}^* - D_{S1}^*$, $D_0^* - D^*$, and $D - D_1$ mixings in Eq. (2.12). Removing the non-diagonal terms in the Lagrangian and subsequently bringing the kinetic terms of $\eta_{N,S,C}$, $\vec{\pi}$, K_S, K, D_S, D_{S0}, D_0^*, D_0^{*0}, and D to the canonical form leads us to define the charmed renormalization coefficients Z_{η_C}, Z_{D_S}, $Z_{D_{S0}}$, $Z_{D_0^*}$, $Z_{D_0^{*0}}$, and Z_D. The squared mass differences $m_{D_{S1}}^2 - m_{D_S^*}^2$, $m_{D_1}^2 - m_{D^*}^2 - m^2$, and $m_{\chi_{C1}}^2 - m_{J/\psi}^2$ are, as seen in Eqs. (3.2 - 3.4), independent from the charm masses, they just depend on the condensates.

The eLSM containing open and hidden charmed mesons has 16 free parameters. We have fixed 13 of these parameters from the strange and non-strange sector. The three new unknown parameters have been fixed in a fit to the expermental values (details will be presented in Ref.

[27]). Moreover, the decay constants of D and D_S have been calculated. All meson masses in the Lagrangian (2.5) have been computed, with the same results for light mesons as in the strange and non-strange sector [15, 16, 17], and with (open and hidden) charmed meson masses being in good agreement with experimental data [26].

A study in progress is the calculation of the decay widths of the charmed mesons presented in this work [27]. In the future, we are planning to study the vacuum phenomenology of the light and heavy tetraquarks and the inclusion of the scalar and pseudoscalar glueballs in $N_f = 4$.

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