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The nuclear matrix elements of double beta decay in the $\widetilde{\mathrm{SU}}(4) \times \widetilde{\mathrm{SU}}(6)$ model

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This work establishes a formalism for the study of the $2\nu\beta\beta$ decay $^{76}\mathrm{Ge} \rightarrow {}^{76}\mathrm{Se}$ by employing an algebraic model that involves the symmetry of pseudo-spin and pseudo-orbit. The shell model space of the two nuclei consists of the $g_{9/2}$ orbit (denoted as the g-subshell) and the orbits $p_{1/2} - p_{3/2} - f_{5/2}$, which are transformed into the orbits $\tilde{s}_{1/2} - \tilde{d}_{3/2} - \tilde{d}_{5/2}$ (denoted as the $\widetilde{ds}$-subshell). While to the g-subshell the seniority-zero restriction applies, in the $\widetilde{ds}$-subshell the $\widetilde{\mathrm{SU}}(4) \times \widetilde{\mathrm{SU}}(6)$ model dominates. The nuclear structure data of the two relevant nuclei strongly suggest the $\widetilde{\mathrm{SU}}(3)$ limit, which reflects a strong interaction between the proton and the neutron sectors.

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1. Introduction

Due to the importance in determining the neutrino mass, the study of the neutrinoless double beta decay $(0\nu\beta\beta)$ has gained very much attention in recent years. In the perspective of nuclear structure the focus is on the calculation of the nuclear matrix elements (NME) of the relevant nuclei. One way to tackle the problem is to study the NME of the $2\nu\beta\beta$ decay that takes place between the same pair of nuclei for the $0\nu\beta\beta$ decay. To this end various models are explored, i.e. the Interacting Shell Model [1], the Interacting Boson Model [2] as well as the Deformed Hatree-Fock Model [3]. Nuclear Shell Model provides a quantitative description of the $2\nu\beta\beta$ NME calculations, however the huge space involved causes a formidable technical difficulty. As an alternative, algebraic or symmetry adapted models were developed [4, 5].

For heavy nuclei, the $2\nu\beta^-\beta^-$ decay has been studied extensively by the SU(3) symmetry model [4]. For light nuclei there also exists a SU(4) model study [5] (where the strong spin-orbit coupling is omitted). However, for medium nuclei, e.g. for the $2\nu\beta\beta$ decay $^{76}\mathrm{Ge} \to {}^{76}\mathrm{Se}$, there has never been a study based on symmetry models. For the two nuclei the relevant shell model space consists of the single-particle orbits $p_{1/2}$, $p_{3/2}$, $f_{5/2}$ and $g_{9/2}$. Recently there are studies on the structure of $^{76}\mathrm{Ge}$ [6] by using the Effective Shell Model interaction JJB4 and JUN45. For producing the best fit to the low-excited energies and $B(E_2)$ values, the single-particle energies (in Mev) are:

$$\begin{array}{lll} JJ4B: & p_{3/2}=0.0 & f_{5/2}=0.3707, \\ & p_{1/2}=1.3871 & g_{9/2}=3.7622, \end{array} \qquad \begin{array}{lll} JUN45: & p_{3/2}=0.0 & f_{5/2}=1.1193, \\ & p_{1/2}=1.9892 & g_{9/2}=3.5663. \end{array} \tag{1.1}$$

One easily observes from these results that the energy level of the orbit $p_{3/2}$ is much closer to that of $f_{5/2}$ than that of $p_{1/2}$ (its spin-orbit partner), and the energy of $g_{9/2}$ is far away from those of the pf ones. These results are a clear indication of the existence of the pseudo-spin-pseudo-orbit symmetry, i.e. the pf orbits labeled by the the quantum numbers (η, l, j) can be relabeled by $(\widetilde{\eta}, \widetilde{l}, j)$. This symmetry is a consequence of the strong spin-orbit coupling that causes a breaking of the SU(4) symmetry [7] and facilitates a new pseudo symmetry. Therefore the single-particle orbits are now relabeled as $\widetilde{d}_{3/2}$, $\widetilde{d}_{5/2}$, $\widetilde{s}_{1/2}$ and $g_{9/2}$, respectively. For the g-subshell, following the previous studies, the seniority model dominates and the ground state is restricted to seniority-zero. In the $\widetilde{ds}$ subshell the $\widetilde{\mathrm{SU}}(4) \times \widetilde{\mathrm{SU}}(6)$ model [8] applies. Based on these symmetries we develop a formalism to calculate the NME of the $2\nu\beta\beta$ $^{76}\mathrm{Ge} \to {}^{76}\mathrm{Se}$.

The article is organized as follows: In Section 2 we introduce the $\widetilde{\mathrm{SU}}(4) \times \widetilde{\mathrm{SU}}(6)$ model, in Section 3 this model is applied to the calculation of the $2\nu\beta\beta\,(0_1^+ \to 0_1^+)$ decay of $^{76}\mathrm{Ge} \to {}^{76}\mathrm{Se}$ and the conclusion is in Section 4.

2. The $\widetilde{\mathrm{SU}}(4) \times \widetilde{\mathrm{SU}}(6)$ model for the $\widetilde{ds}$ subshell

A many-particle state is expressed as a direct product of the states of the $\widetilde{ds}$- and g-subshell,

$$|N, JM, T\mu\rangle = \sum_{\substack{J_1 T_1 \\ J_2 T_2}} \left(\left| N_{\widetilde{ds}}, J_1 T_1(\widetilde{ds}) \right\rangle \otimes |N_g, J_2 T_2(g)\rangle \right)^{(J,T)}_{(M,\mu)} . \tag{2.1}$$

where J,T are the total angular momentum and isospin and M,μ their projections, $N_{\widetilde{ds}}$ and N_g the number of nucleons in the $\widetilde{ds}$ and g subspace, respectively. N is the total number of nucleons and $N = N_g + N_{\widetilde{ds}}$. The Five-dimensional quasispin model is used for a classification of the states in the g-subshell [9, 10]:

$$\begin{array}{ccccc} \Big(\mathrm{SO}(5) \supset & \Big(\mathrm{SU_T}(2)\otimes \mathrm{U}(1)\Big)\Big) & \otimes & \Big(\mathrm{S_p}(10) \supset & \mathrm{SU_J}(2)\Big) \\ \downarrow & \downarrow & & \downarrow & \downarrow \\ \Big| \left(g_{9/2}\right)^{N_g} (v,t) & \beta^{'}\, H\, T_2, n & & \langle\sigma\rangle & J_2 \quad \Big\rangle \end{array}, \tag{2.2}$$

the configuration of protons and neutron in a single g-subshell is given by two group chains beginning with the direct product of $\mathrm{SO}(5)\otimes\mathrm{Sp}(10)$, where (v,t) (seniority and the reduced isospin) to label the irreps of SO(5) and $\langle\sigma\rangle$ that of Sp(10), respectively. The others quantum numbers are H with n the number of particles, $\beta^{'}$ additional quantum number, and T_2, J_2 the total isospin and the total angular momentum, respectively. The states of the $\widetilde{\mathrm{SU}}(3)$ limit of the $\widetilde{\mathrm{SU}}(4)\times\widetilde{\mathrm{SU}}(6)$ model in $\widetilde{ds}$ subshell are,

$$\begin{array}{ccccccc} \widetilde{\mathrm{SU}}(24) \supset & \Big(\widetilde{\mathrm{SU}}(4) \supset & \Big(\widetilde{\mathrm{SU}}(2)_{\widetilde{S}}\otimes\widetilde{\mathrm{SU}}(2)_T\Big)\Big) \otimes & \Big((\widetilde{\mathrm{SU}}(6) \supset & (\widetilde{\mathrm{SU}}(3) \supset & (\widetilde{\mathrm{SO}}(3) \supset & \mathrm{SU_J}(2)\Big) \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \Big|\widetilde{\{1^{N_{\widetilde{ds}}}\}} & \widetilde{(\alpha_1,\alpha_2,\alpha_3)} & \left(\widetilde{S}_1, T_1\right) & \widetilde{\{f\}} & \widetilde{(\lambda,\mu)}\widetilde{K}_R & \widetilde{L}_1 & J_1 \quad \Big\rangle \end{array}, \tag{2.3}$$

where $\widetilde{\{1^{N_{\widetilde{ds}}}\}}$ is the total antisymmetric irrep of the $\widetilde{\mathrm{SU}}(24)$ group, $\widetilde{(\alpha_1,\alpha_2,\alpha_3)}$ the irrep of $\widetilde{\mathrm{SU}}(4)$, $\left(\widetilde{S}_1,T_1\right)$ the multiplet pseudo-spin, isospin related with the irrep of $\widetilde{\mathrm{SU}}(4)$, $\widetilde{\{f\}}$ the irrep of $\widetilde{\mathrm{SU}}(6)$, $\widetilde{(\lambda,\mu)}$ the $\widetilde{\mathrm{SU}}(3)$ representation, and $\widetilde{K}_R$ the rotational quantum number, respectively. The relations between $\widetilde{K}_R$ and $\widetilde{L}$ are given by [11, 12] as follows

$$\begin{aligned} \widetilde{K} &= min(\lambda,\mu), min(\lambda,\mu)-2,\cdots,1 \quad \text{or} \quad 0 \\ \widetilde{L} &= (\lambda+\mu),(\lambda+\mu)-2,\cdots,1,0 \quad \text{if} \quad \widetilde{\mathrm{K}}=0 \\ &= \widetilde{K},\widetilde{K}+1,\widetilde{K}+2,\cdots,(\lambda+\mu)-\widetilde{K}+1 \quad \text{if} \quad \widetilde{\mathrm{K}}\neq 0, \end{aligned} \tag{2.4}$$

in which $\vec{\widetilde{L}}_1$ is the pseudo-angular momentum, $\vec{\widetilde{L}}_1 = \sum_i \vec{\widetilde{l}}_i$, and $\vec{J}_1$ is the total angular momentum associated with the $\widetilde{ds}$ subspace, and $\vec{J}_1 = \vec{\widetilde{L}}_1 + \vec{\widetilde{S}}_1$.

We note that this model has been used in the study of the nuclear structure of medium nuclei [8] and the reasonably good results provide a support to this model.

3. Application to $2\nu\beta\beta\,(0_1^+ \to 0_1^+)$ decay of ^{76}Ge $\to$ ^{76}Se

3.1 The closure approximation

The $2\nu\beta^-\beta^-$ decay is a process in which a nucleus (Z,A) decays to a neighboring nucleus $(Z+2,A)$ and emits two electrons and two antineutrinos. The inverse half-life can be calculated by using the closure approximation in the form [13]

$$\left[T_{1/2}^{2\nu}\left(0_1^+ \to 0_1^+\right)\right]^{-1} \approx G_{2\nu}^{(0)}\left|m_e c^2 M_{2\nu}\right|^2, \tag{3.1}$$

where $G_{2\nu}^{(0)}$ is a phase space factor (PSF). The nuclear matrix element is defined as $M_{2\nu} = g_A M^{2\nu}$ [13] in which

$$M^{2\nu} = \frac{\langle 0_{1,f}^+, T_f \,|\, \vec{\mathscr{T}}(GT)^- \cdot \vec{\mathscr{T}}(GT)^- \,|\, 0_{1,i}^+, T_i \rangle}{\langle \bar{E} \rangle}, \tag{3.2}$$

where the closure energy $\langle \bar{E} \rangle$ stands for

$$\langle \bar{E} \rangle = \frac{1}{2}\left(Q_{\beta\beta} + 2m_e c^2\right) + \langle E_{1^+,N} \rangle - E_i, \tag{3.3}$$

in which $\langle E_{1^+,N} \rangle$ is an adequately chosen energy of the intermediate odd-odd nuclei. For the ^{76}Ge $\to$ ^{76}Se decay the values of $G_{2\nu}^{(0)}$ and $\langle \bar{E} \rangle$ are taken as $40.17 \times 10^{-21} yr^{-1}$ and 9.411 MeV, respectively [13, 14].

3.2 The $2\nu\beta\beta$ operator in the $\widetilde{\mathrm{SU}}(4) \times \widetilde{\mathrm{SU}}(6)$ scheme

For the $2\nu\beta\beta$ operator we first express it in the normal shell model space and then transform it into the pseudo space. The Gamow-Teller operator for a single-β decay writes as

$$\vec{\mathscr{T}}(GT)^{(1,1)}_{\mathscr{M},-1} = \sum_{j,j',l,\eta} \sigma\left((\eta,l,\tfrac{1}{2})j,j'\right) \mathscr{A}^{(1,1)}_{\mathscr{M},-1}\left((\eta,l)j,\tfrac{1}{2};(\eta,l)j',\tfrac{1}{2}\right), \tag{3.4}$$

where

$$\sigma\left((\eta,l,\tfrac{1}{2})j,j'\right) = \sqrt{2(2j+1)(2j'+1)}\,(-1)^{\eta+l+j+\frac{3}{2}} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ j' & j & l \end{Bmatrix} \delta_{l,l'}\delta_{\eta,\eta'}, \tag{3.5}$$

and

$$\mathscr{A}^{(1,1)}_{\mathscr{M},-1}\left((\eta,l)j,\tfrac{1}{2};(\eta,l)j',\tfrac{1}{2}\right) = \left(a^\dagger_{((\eta,0)l,\frac{1}{2})j;\frac{1}{2}}\,\bar{a}_{((0,\eta),l,\frac{1}{2})j';\frac{1}{2}}\right)^{(1,1)}_{(\mathscr{M},-1)}, \tag{3.6}$$

which destroys a neutron and create a protons in the same orbit. Note, to the initial and final nuclear states we impose the isospin symmetry, i.e. $T = T_z = (N-Z)/2$. Note that the symbol $\bar{a}$ is the covariant form of the annihilation operator a, and the use of a bar instead of a tilde on top of it is to avoid confusion with the pseudo sign.

The the double Gamow-Teller $2\nu\beta\beta$ operator can be transformed into the following pair-form:

$$\begin{aligned} &\vec{\mathscr{T}}(GT)^- \cdot \vec{\mathscr{T}}(GT)^- = 3\textstyle\sum_{\mathscr{J}} \sum_{\substack{j,j' \\ l,\eta}} \sum_{\substack{j'',j''' \\ l'',\eta''}} \sqrt{(2\mathscr{J}+1)}\,(-1)^{j'+j''+\mathscr{J}} \\ &\begin{Bmatrix} j & j' & 1 \\ j''' & j'' & \mathscr{J} \end{Bmatrix} \sigma\left((\eta,l,\tfrac{1}{2})\,j,j'\right)\sigma\left(\left(\eta'',l'',\tfrac{1}{2}\right)j'',j'''\right) \\ &\left(G^\dagger_{\mathscr{J};1}\left((\eta,l,\tfrac{1}{2})j,(\eta'',l'',\tfrac{1}{2})j''\right)\bar{G}_{\mathscr{J};1}\left((\eta,l,\tfrac{1}{2})\,j',\left(\eta'',l'',\tfrac{1}{2}\right)j'''\right)\right)^{0;2}_{0;-2}, \end{aligned} \tag{3.7}$$

where

$$
\begin{aligned}
G^{\dagger}_{\mathscr{J};1}\left((\eta,l,\tfrac{1}{2})j,(\eta'',l'',\tfrac{1}{2})j''\right) &= \left(a^{\dagger}_{(\eta,l,\frac{1}{2})j;\frac{1}{2}}\, a^{\dagger}_{(\eta'',l'',\frac{1}{2})j'';\frac{1}{2}}\right)^{(\mathscr{J};1)} \\
\bar{G}_{\mathscr{J};1}\left((\eta,l,\tfrac{1}{2})j',\left(\eta'',l'',\tfrac{1}{2}\right)j'''\right) &= \left(\bar{a}_{(\eta,l,\frac{1}{2})j';\frac{1}{2}}\, \bar{a}_{(\eta'',l'',\frac{1}{2})j''';\frac{1}{2}}\right)^{(\mathscr{J};1)}.
\end{aligned} \tag{3.8}
$$

According to the initial and the final orbits of the two decaying neutrons the $2\nu\beta\beta$ operator can be decomposed into four parts, i.e. the $(g\to g)$, $(g\to fp)$, $(fp\to g)$ and $(fp\to pf)$ parts, respectively. By considering the conservation laws, we can obtain the the expression for the decomposition of the $2\nu\beta\beta$ operator as follows

$$
\begin{aligned}
\vec{\mathscr{T}}(GT)^{-}\cdot\vec{\mathscr{T}}(GT)^{-} = {} & \beta_1 G^{\dagger}_{\mathscr{J};1}(a^{\dagger},a^{\dagger})_{(g\to g)}\bar{G}_{\mathscr{J};1}(\bar{a},\bar{a})_{(g\to g)} \\
& +\beta_2 G^{\dagger}_{\mathscr{J};1}(a^{\dagger},a^{\dagger})_{(fp\to fp)}\bar{G}_{\mathscr{J};1}(\bar{a},\bar{a})_{(fp\to fp)} \\
& +\beta_3 G^{\dagger}_{\mathscr{J};1}(a^{\dagger},a^{\dagger})_{(fp\to g)}\bar{G}_{\mathscr{J};1}(\bar{a},\bar{a})_{(fp\to g)} \\
& +\beta_4 G^{\dagger}_{\mathscr{J};1}(a^{\dagger},a^{\dagger})_{(g\to fp)}\bar{G}_{\mathscr{J};1}(\bar{a},\bar{a})_{(g\to fp)}
\end{aligned} \tag{3.9}
$$

Due to the seniority-zero restriction on the g-subshell, the only parts of the operator that give non-zero contribution are those with the subscripts $(fp\to fp)$ and $(g\to g)$. The $(fp\to fp)$ part of the operator can be transformed from the normal shell space into the pseudo space as follows

$$
\begin{aligned}
&\beta_2 G^{\dagger}_{\mathscr{J};1}(a^{\dagger},a^{\dagger})_{(fp\to fp)}\bar{G}_{\mathscr{J};1}(\bar{a},\bar{a})_{(fp\to fp)} = \textstyle\sum_{\mathscr{J}}\sum_{j_a,j_b}\sum_{j_c,j_d}\sum_{l_a,l_c} 3\sqrt{(2\mathscr{J}+1)}\,(-1)^{j_b+j_c+\mathscr{J}} \\
&\quad \sigma\left((3,l_a,\tfrac{1}{2})\,j_a,j_b\right)\sigma\left((3,l_c,\tfrac{1}{2})\,j_c,j_d\right)\begin{Bmatrix} j_a & j_b & 1 \\ j_d & j_c & \mathscr{J}\end{Bmatrix} \\
&\left(G^{\dagger}_{\mathscr{J};1}\left((2,\widetilde{l}_a,\tfrac{1}{2})j_a,(2,\widetilde{l}_c,\tfrac{1}{2})j_c\right)\otimes\bar{G}_{\mathscr{J};1}\left(\left(2,\widetilde{l}_a,\tfrac{1}{2}\right)j_b,\left(2,\widetilde{l}_c,\tfrac{1}{2}\right)j_d\right)\right)^{0;2}_{0;-2},
\end{aligned} \tag{3.10}
$$

where for the normal orbits j_a, j_b, j_c, j_d take the value of $\frac{1}{2}$, $\frac{3}{2}$, or $\frac{5}{2}$, l_a, l_c take the value 1 or 3, $\eta=3$; whereas for the pseudo orbits $\widetilde{l}_a,\widetilde{l}_c$ can have the values 0 or 2 and $\widetilde{\eta}=2$. The operators $G^{\dagger}_{\mathscr{J};1}\left((2,\widetilde{l}_a,\tfrac{1}{2})j_a,(2,\widetilde{l}_c,\tfrac{1}{2})j_c\right)$ and $\bar{G}_{\mathscr{J};1}\left(\left(2,\widetilde{l}_a,\tfrac{1}{2}\right)j_b,\left(2,\widetilde{l}_c,\tfrac{1}{2}\right)j_d\right)$ can be recoupled to the $\widetilde{\mathrm{SU}}(4)\times\widetilde{\mathrm{SU}}(3)$ tensors, e.g.

$$
\begin{aligned}
G^{\dagger}_{\mathscr{J};1}\left((2,\widetilde{l}_a,\tfrac{1}{2})j_a,(2,\widetilde{l}_c,\tfrac{1}{2})j_c\right) &= \left(a^{\dagger}_{(2,\widetilde{l}_a,\frac{1}{2})j_a;\frac{1}{2}}\, a^{\dagger}_{(2,\widetilde{l}_c,\frac{1}{2})j_c;\frac{1}{2}}\right)^{(\mathscr{J};1)} \\
&= \sum_{\widetilde{L}_{ac},\widetilde{S}_{ac}\widetilde{K}_{ac}}\ \sum_{\substack{\widetilde{(\lambda\mu)}_{ac}\\ \widetilde{(\alpha_1\alpha_2\alpha_3)}_{ac}}} \overline{\begin{Bmatrix}\widetilde{l}_a & \frac{1}{2} & j_a \\ \widetilde{l}_c & \frac{1}{2} & j_c \\ \widetilde{L}_{ac} & \widetilde{S}_{ac} & \mathscr{J}\end{Bmatrix}} \left\langle \begin{matrix}(20) & (20) \\ \widetilde{l}_a & \widetilde{l}_c\end{matrix}\ \middle|\ \begin{matrix}\widetilde{K}_{ac}\,\widetilde{(\lambda\mu)}_{ac} \\ \widetilde{L}_{ac}\end{matrix}\right\rangle \\
&\qquad \left(a^{\dagger}_{\widetilde{\lambda}_a}\, a^{\dagger}_{\widetilde{\lambda}_c}\right)^{\widetilde{(\lambda\mu)}_{ac},\widetilde{(\alpha_1\alpha_2\alpha_3)}_{ac};\mathscr{J},1}_{\widetilde{K}_{ac},\widetilde{L}_{ac},\widetilde{S}_{ac}},
\end{aligned} \tag{3.11}
$$

with $\widetilde{\lambda}_a=(20),\widetilde{l}_a;(100)(\frac{1}{2}\frac{1}{2})$, $\widetilde{\lambda}_c=(20),\widetilde{l}_c;(100)(\frac{1}{2}\frac{1}{2})$, and

$$
\overline{\begin{Bmatrix}\widetilde{l} & \frac{1}{2} & k \\ \widetilde{l}' & \frac{1}{2} & t \\ \widetilde{L} & \widetilde{S} & \mathscr{J}\end{Bmatrix}} = \sqrt{(2k+1)\,(2t+1)\left(2\widetilde{L}+1\right)\left(2\widetilde{S}+1\right)}\begin{Bmatrix}\widetilde{l} & \frac{1}{2} & k \\ \widetilde{l}' & \frac{1}{2} & t \\ \widetilde{L} & \widetilde{S} & \mathscr{J}\end{Bmatrix}. \tag{3.12}
$$

The $(g\to g)$ part of the the operator can be decomposed into the seniority (or quasispin) tensors:

$$\beta_1 G^\dagger_{\mathscr{J};1}(a^\dagger,a^\dagger)_{(g\to g)}\bar{G}_{\mathscr{J};1}(\bar{a},\bar{a})_{(g\to g)} = \frac{220}{27}\sum_{\mathscr{J}}\sum_{(W_1,W_2)} C\left((W_1,W_2)\right)\sqrt{2\mathscr{J}+1}(-1)^{\mathscr{J}-\mathscr{M}} \begin{Bmatrix} 9/2 & 9/2 & 1 \\ 9/2 & 9/2 & \mathscr{J} \end{Bmatrix} \mathscr{T}\left(\mathscr{J}^2;(1,1)1^2\right)^{(W_1,W_2);0}_{02-2;0}. \tag{3.13}$$

Therefore the calculation of NME can be done in the $\widetilde{\mathrm{SU}}(4)\times\widetilde{\mathrm{SU}}(6)$ scheme and in the seniority scheme separately. While the $SU(6)\supset SU(3)$ cfp presents a complicity, the calculation in the seniority model is relatively easy.

3.3 Consideration of the configuration space

For the nuclei ^{76}Ge and ^{76}Se, we take ^{56}Ni as an inert core, and the valence nucleons occupy the single particle orbits in the $\widetilde{ds}$ and g subshells, respectively. A remaining problem is how to distribute the valence nucleons between these two subshells. A recent measurement [1] for the ^{76}Ge and ^{76}Se gives the results of the g-orbit occupancy as $(6.48\pm0.30(\nu),\ 0.23\pm0.25(\pi))$ and $(5.80\pm0.30(\nu),\ 0.84\pm0.25(\pi))$, respectively, which can be used as a constraint for the configurations $(N_\pi,N_\nu)_g$ and $[M_\pi,M_\nu]_{\widetilde{ds}}$. where the numbers N_ρ and M_ρ ($\rho=\pi$ or ν) are the ρ-particle number in the g- and $\widetilde{ds}$-subshell, respectively. In the g-subshell the configuration can be reasonably well restricted to $(n_1,n_2)_g$ with $n_1=0,2$ and $n_2=4,6,8$, respectively for both nuclei. The corresponding configurations in the $\widetilde{ds}$-subshell are $[4-n_1,\ 16-n_2]$ for ^{76}Ge and $[6-n_1,\ 14-n_2]$ for ^{76}Se, respectively. We assume the only contribution of many-body states for the g are $J_2=0$ (seniority-zero restriction) and $\widetilde{S}=0$ for the $\widetilde{ds}$ subshells, then the initial states (for ^{76}Ge) can be written in as a product of states in the two subspaces:

$$\left|^{76}\mathrm{G_e};J^+=0^+,T=6\right\rangle = \sum_{n_1,n_2}\sum_{\widetilde{(\lambda,\mu)},\widetilde{\{f\}}}\left|(n_1,\ n_2)\ \left(g_{9/2}\right)^{N_g}(0,0)H\,T_2;\langle 0\rangle J_2=0\right\rangle_g \left|[4-n_1,\ 16-n_2]\widetilde{\{1^{N_{\widetilde{ds}}}\}}\,\widetilde{(0\,T_1\,0)}\,\widetilde{S}=0\,T_1\,\widetilde{\{f\}},\widetilde{(\lambda,\mu)}\widetilde{K}=\widetilde{L}=0;J_1=0\right\rangle_{\widetilde{ds}.}, \tag{3.14}$$

all states in the N_g space with $J_2=0$ have $(v,t)=(0,0)$. $\widetilde{(0\,T_1\,0)}=\widetilde{\{f^*\}}$ (Dynkin-Young diagram notation) is the $\widetilde{\mathrm{SU}}(4)$ most antisymmetric representation of $20-(n_1+n_2)$ nucleons with T_1, $\widetilde{\{f\}}$ is the $\widetilde{\mathrm{SU}}(6)$ representation associated with the conjugate of $\widetilde{\{f^*\}}$, $\widetilde{(\lambda,\mu)}$ is the $\widetilde{\mathrm{SU}}(3)$ representaton contained in the $\widetilde{\mathrm{SU}}(6)$ with the highest value of Casimir operator. The final state (^{76}Se) is of the same form but the relevant quantum numbers need to be replaced. According to the discussion in the subsection (3.2) on the $2\nu\beta\beta$ operators with non-zero contribution, there exist only two types of transition between different configuration spaces, i.e. the transition of $(n_1,n_2)\to(n_1+2,\ n_2-2)$ and that of $[4-n_1,\ 16-n_2]\to[6-n_1,\ 14-n_2]$. This feature greatly simplifies the calculation of the NME of $2\nu\beta\beta$ decay, a numerical calculation of which is currently underway.

4. Conclusion and discussion

Recent shell model studies on the structure of ^{76}Ge strongly indicates the approximate degeneracy in the single particle orbits of $p_{3/2}$ and $f_{5/2}$, that supports a symmetry model based on the pseudo-spin and pseudo-orbits in the subshell of $p_{1/2} - p_{3/2} - f_{5/2}$ (the $\widetilde{ds}$-subshell). Therefore for the relevant nuclei in this region we use the model $\widetilde{\mathrm{SU}}(4) \times \widetilde{\mathrm{SU}}(6)$ for the $\widetilde{ds}$ subshell, whereas for the g-subshell we adopt the well-established zero-seniority restriction. The model is applied to the calculation of the NME of the $2\nu\beta\beta\,(0_1^+ \to 0_1^+)$ decay ^{76}Ge $\to$ ^{76}Se. A prominent feature of this model is that it accounts for the large l-s coupling as well as the strong interaction between the protons and neutrons, therefore it is appropriate for the medium nuclei. This formalism may also facilitate the study of nuclear structure for the medium nuclei in this region, for which a considerable amount of data have been accumulated and theoretical explanation based on symmetry models is relatively scarce.

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