

## Continuum discretized coupled channels calculations for reactions of ${}^6\text{Li}$ with ${}^{28}\text{Si}$ and ${}^{58}\text{Ni}$ : Effect of resonances of ${}^6\text{Li}$ on elastic scattering angular distributions

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Calculations of elastic scattering angular distributions for reactions of the weakly bound projectile  ${}^6\text{Li}$  with targets  ${}^{28}\text{Si}$  and  ${}^{58}\text{Ni}$  at energies above the barrier are performed with the Continuum Discretized Coupled-Channel method (CDCC). Ground, resonant and non-resonant continuum states of  ${}^6\text{Li}$  are included up to some maximum energy  $\epsilon_{\text{max}}$  for which convergence is achieved. In the three-body system, global interactions are used for the  $\alpha$ - $d$ ,  $\alpha$ -target and  $d$ -target sub-systems. The effect of continuum resonant states of  ${}^6\text{Li}$ , *i.e.*,  $l=2$ ,  $j^\pi = 3^+$ ,  $2^+$  and  $1^+$ , on elastic scattering angular distributions is investigated by removing these states from the continuum space. It is found that the calculated elastic scattering angular distributions are in good agreement with the measurements for most of the cases studied, where consideration of couplings to continuum states are essential. It is also found that the effect of resonances in the continuum space is, in some cases important to obtain a good agreement with the data.

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## 1. Introduction

Lately, reaction mechanisms involving weakly bound nuclei, both stable and radioactive, have been a subject of strong research [1,2,3]. One of the most interesting topics, is the effect of breakup on elastic scattering and fusion reaction mechanisms. It is now known that couplings of the elastic channel to continuum breakup states of the weakly bound projectile, as well as, continuum-continuum couplings are essential to fit the experimental data. This is so since, couplings to and between continuum states give rise to a strong repulsive polarization potential that decreases the reaction absorption. On the other side, besides the non-capture breakup process, other nuclear mechanisms are present for reactions with weakly bound nuclei. For instance, sequential and direct complete fusion, incomplete fusion and nucleon transfer. Coupled channel calculations show that couplings between the incident elastic channel to fusion ones, give place to an attractive dynamical polarization potential  $V_F(E,r)$  that lowers the nominal Coulomb barrier  $V_B$ . So, the net effect of  $V_F$  is to enhance fusion mainly for energies around the barrier. Similarly, couplings to breakup channels produce a repulsive dynamical polarization potential  $V_{BU}(E,r)$ , that is particularly strong at energies below the barrier. In this energy range,  $V_{BU}(E,r)$  overcomes  $V_F(E,r)$ , so that the net effect is twofold: a) An increase in the fusion barrier, that leads to a net fusion suppression, and b) Most of the reaction cross section is due to breakup. Several CDCC calculations have been proposed to calculate the effect of breakup couplings on elastic and fusion cross sections of several weakly bound systems. For instance, the  $2n$ -halo  ${}^6\text{He}$  on  ${}^{59}\text{Co}$  and  ${}^{208}\text{Pb}$  [4,5],  ${}^6\text{Li}$  with targets  ${}^{28}\text{Si}$ ,  ${}^{59}\text{Co}$ ,  ${}^{58}\text{Ni}$ ,  ${}^{144}\text{Sm}$  and  ${}^{208}\text{Pb}$  [4,6-13], also  ${}^7\text{Li}$  with  ${}^{28}\text{Si}$  [14,15] and  ${}^{144}\text{Sm}$  [16], the  $n$ -halo  ${}^{11}\text{Be}$  with  ${}^{208}\text{Pb}$  [17] and the  $p$ -halo  ${}^8\text{B}$  with  ${}^{58}\text{Ni}$  [10,18]. Most of the CDCC calculations in these works, use either, microscopic density dependent double-folding interactions with normalization factors  $N_R$ ,  $N_I$  or Woods-Saxon potentials obtained from experimental systematic analyses. Except for a few cases, where contradictory results are found, e.g.,  ${}^8\text{B}+{}^{58}\text{Ni}$  [10,18], most of the CDCC calculations show that couplings to continuum states of the weakly bound projectile have a strong effect and their consideration is very important to attain agreement with the experimental data. However, most of the studies for the weakly bound nucleus  ${}^6\text{Li}$  do not distinguish the effect that the resonance character of the continuum states  $3+$ ,  $2+$  and  $1+$  can have on elastic scattering. The purpose of the present work is to calculate this effect on reactions with the spherical targets  ${}^{28}\text{Si}$  and  ${}^{58}\text{Ni}$ , at energies above the Coulomb barrier. The CDCC [6,19] method is used with global  $d$ -target and  $\alpha$ -target interactions, which depend only on the target mass  $A$  and incident collision energy  $E_{lab}$ . The global Woods-Saxon potentials, given in Ref. [20] for the  $d$ -target interaction  $V_{dT}$ , are obtained from a large analysis of experimental elastic and inelastic cross sections for reactions of deuteron with a large variety of target masses ( $12 < A < 238$ ) and range of incident energies  $E_{lab} < 183$  MeV. As a matter of fact, this interaction is an improvement of other widely used systematic parametrizations for shorter range of masses and energies [21,22,23]. It is the intention of the calculations performed in this work, to test this improved parametrization of the  $d$ -target interaction, when used to describe continuum states of the deuteron after the breakup of  ${}^6\text{Li}$ . The density dependent double-folding Sao Paulo potential (SPP) [24] with adequate  $\alpha$ -particle mass densities is used to describe the  $\alpha$ -target interaction  $V_{\alpha T}$ . An extensive systematics

of nuclear densities has been performed by L.C. Chamon *et al.*, [25] to produce a parameter-free interaction. In our calculations, the  $\alpha$ -target interaction depends also only on the target mass  $A$  and the kinetic energy of the  $\alpha$ -particle which is a fraction of the initial incident energy of  ${}^6\text{Li}$ . As for the  $\alpha$ -d cluster structure of  ${}^6\text{Li}$ , use is made of the Woods-Saxon parametrization given in Ref. [26]. Ground, and unbound continuum scattering states of  ${}^6\text{Li}$  are generated with this interaction. In order to calculate the effect of the resonances of  ${}^6\text{Li}$ , we assume that there is a gap in the continuum states  $l=2, s=1, \pi=(-)^l$ , that is,  $j^\pi=3+, 2+$  and  $1+$ . This will be explained in section 2. The results and a discussion of the calculations are presented in section 3

## 2. CDCC formalism for a three-body system

In the CDCC formalism [6,19], continuum scattering states are replaced by a limited summation over bin-states. A bin-state  $N_i, l=1, \dots, l_{max}$  is a set of discrete states, with a given angular momentum  $l$ , with parity  $(-1)^l$ , total angular momentum  $j$ , excitation energy  $\varepsilon_{k,l}$  (respect to threshold energy), with discretization step  $\Delta\varepsilon_i$  up to a maximum energy  $\varepsilon_{max}$ . The  $N_i$  bin-states are constructed in sequential intervals of energy, *i.e.*,  $\varepsilon_{i-1} < \varepsilon_k < \varepsilon_i$  with  $i=1, N_i$ , where  $\varepsilon_0=0$  and  $\varepsilon_{N_i}=\varepsilon_{max}$ . Within the CDCC model, bin states  $\phi_{l,s,j}^{(i)}(r), i=1, N_i$  are square integrable and are constructed as a superposition of scattering wave functions within the interval  $[k_{i-1}, k_i]$  with excitation energy corresponding to the mean value energy,

$$\varepsilon_i = \frac{\varepsilon_k + \varepsilon_{k-1}}{2}. \quad (2.1)$$

That is,

$$\phi_{l,s,j}^{(i)}(r) = \frac{1}{\eta_i^{1/2}} \int_{k_{i-1}}^{k_i} w_i(k) \psi_{l,s,j}^{(scatt.)}(r, k) dk, \quad (2.2)$$

where  $w_i(k)$  is a weight function that satisfies [6,19],

$$\int_{k_{i-1}}^{k_i} |w_i(k)|^2 dk = \eta_i.$$

For the case of  ${}^6\text{Li}$ , the resonance energies  $\varepsilon_{i, res}$  and widths  $\Gamma_i$  correspond to the resonant states  $3+, 2+$  and  $1+$  with values  $\varepsilon_{3+}=0.716$  MeV and  $\Gamma_{3+}=0.024$  MeV,  $\varepsilon_{2+}=2.84$  MeV,  $\Gamma_{2+}=1.7$  MeV and  $\varepsilon_{1+}=4.18$  MeV,  $\Gamma_{1+}=1.5$  MeV, respectively. Within the CDCC formalism, a set of coupled channel equations is obtained for the radial part of the relative motion wave function,

$$\left[ T_{\mathcal{R}} + U_{\beta\beta}^{(J)}(R) - E - \varepsilon_{\beta} \right] u_{\beta}^{(J)}(R) = - \sum_{\beta'} U_{\beta\beta'}^{(J)}(R) u_{\beta'}^{(J)}(R), \quad (2.3)$$

where  $\beta=l, L, J$  are the quantum numbers of the ground and discrete states,  $\varepsilon_{\beta}$  is the eigenvalue of internal Hamiltonian of the nuclear system.  $U_{\beta\beta}^{(J)}$  represents the diagonal matrix elements of the interactions and  $U_{\beta\beta'}^{(J)}$  non-diagonal elements. In particular, the diagonal matrix element  $U_{00}^{(J)}(R)$  corresponds to the optical potential in the elastic channel. Inelastic excitations of the target are not explicitly considered in our calculations, while all continuum couplings are included. The deuteron-target potential  $V_{dT}(r)$  is that taken from Ref. [20]. This potential is the result of a large systematic study of existing experimental data of elastic scattering angular distributions

and non-elastic cross sections of incident deuteron on a wide range of target nuclei and energies. It is given in terms of Woods-Saxon shapes as,

$$V(r) = -V_0 f_r(r) - iW_V f_V(r) + 4ia_s W_s \frac{df_S}{dr} + \lambda_\pi^2 \frac{V_{so} + iW_{so}}{r} \frac{df_{so}}{dr} \sigma \cdot \mathbf{l} + V_{Coul}(r), \quad (2.4)$$

where  $f_j(r) = 1 / [1 + \exp(r - R_j) / a_j]$ ,  $R_j = r_j A^{1/3}$  with  $j=r, V, S, SO$ . All potentials strengths, diffuseness and radii are given, by the systematics, in terms of the target mass and deuteron energy  $E_d$ . The non-local, double-folding density dependent interaction SPP is used for the  $\alpha$ -target sub-system.

$$V_{\alpha T}(r, E) = [N_R(E) + iN_I(E)] V_F(r) e^{-v(E)^2 / c^2}, \quad (2.5)$$

$r$  being the separation between the  $\alpha$ -particle and target and  $v(E)$  their relative velocity. The imaginary part is assumed with the same radial dependence as the real part.  $N_R(E)$  and  $N_I(E)$  are normalization coefficients that take into account polarization effects arising from direct reaction couplings and  $V_F(r)$  is the folding potential given by,

$$V_F(r) = \int \rho_{A1}(\mathbf{r}_1) \rho_{A2}(\mathbf{r}_2) V_0 \delta[\mathbf{r} - (\mathbf{r}_1 - \mathbf{r}_2)] d\mathbf{r}_1 d\mathbf{r}_2, \quad (2.6)$$

with Fermi density distributions,  $\rho = \rho_{0i} / [1 + \exp(r - R_i) / a_i]$ . with  $i=A_1, A_2$ . Global values have been determined for the zero-range potential strength  $V_0 = -456$  MeV-fm<sup>3</sup>, the mass density diffuseness  $a_i = 0.56$  fm and the radius parameter  $R_i = 1.31 A^{1/3} - 0.84$  fm,  $i=A_1, A_2$ . Thus the SPP is a parameter-free interaction which depends only on the masses  $A_1, A_2$  of the interacting nuclei and their collision energy  $E_{lab}$ . It should be noticed that the non-local feature of Eq.(2.6) is not important at energies near the Coulomb barrier.

### 3. CDCC calculations of elastic scattering using global interactions

In this section CDCC calculations of elastic scattering angular distributions are presented for the projectile  ${}^6\text{Li}$  with targets  ${}^{28}\text{Si}$ ,  ${}^{58}\text{Ni}$  for incident energies just above the corresponding Coulomb barriers. In our calculations, the code FRESKO is used [27], where regarding the  $V_{\alpha T}$  interaction, care is taken for the radial parameters  $R_j$ . This is so, since in the code FRESKO, the general relation  $R_j = r_j (A_1^{1/3} + A_2^{1/3})$  is considered while a shorter  $R_j = r_j A^{1/3}$  is used in Ref. [20], being  $A_1 = 2$  and  $A$  the target mass. To calculate the ground state of  ${}^6\text{Li}$ ,  $\psi(l=0, E_{thre} = -1.47$  MeV), resonant states  $l=2, j^\pi=3+, 2+, 1+$  and non-resonant breakup discrete states, we have used the parametrization for the interaction  $V_{\alpha d}(r)$ , given in Ref. [26]. The discretization is made as follows; the maximum angular momentum for the relative motion of the  $\alpha$ - $d$  fragments is  $l_{max}=3$ , larger values do not have any effect on the calculations. So, bin states are constructed for  $l=0, j^\pi=1+$  and  $l=1, j^\pi=0^-, 1^-, 2^-$ , with step  $\Delta\varepsilon = 0.5$  MeV up to  $\varepsilon_{max} = 6.8$  MeV. Finer and variable steps are used for resonant states  $l=2, j^\pi=3+, 2+, 1+$ , so as to obtain centroid excitation energies and widths close to the corresponding measured values. For bin states with  $l=3, j^\pi=4^-, 3^-, 2^-$ , a larger step  $\Delta\varepsilon = 1.0$  MeV is used. Convergence tests at  $\varepsilon_{max} = 7.0, 7.5$  and  $8.0$  MeV were done with no effect on elastic angular distributions. Similarly, larger steps  $\Delta\varepsilon = 0.75$  and  $1.0$  MeV were used with no appreciable effect on elastic scattering. Partial waves for

the relative motion of  ${}^6\text{Li}$  and target are considered up to  $L_{max} = 600$  and nuclear and Coulomb potential multipoles are included up to  $L_Q = 4$  for which convergence is achieved. As regards the normalization coefficients of the  $\alpha$ -target interaction  $V_{\alpha T}$  of Eq. (2.5), we set these as,  $N_R = 1$  and  $N_I = 1$ . This is so, since for the range of collision energies studied in this work, the  $\alpha$ -target relative kinetic energy is, in most cases, above the corresponding Coulomb barrier, thus polarization effects are negligible. The effect of resonances of  ${}^6\text{Li}$  ( $l=2, j^\pi = 3+, 2+, 1+$ ) on elastic scattering differential cross sections is studied by removing the narrow range of continuum bins around themselves. Figs.(1) and (2) show the results for the elastic scattering angular distributions for the systems  ${}^6\text{Li}+{}^{28}\text{Si}$  and  ${}^6\text{Li}+{}^{58}\text{Ni}$  for laboratory energies just above the barrier. Solid-lines show the full calculations with all couplings. As seen, in most cases, a close agreement with the data is achieved. Dashed-lines represent the results for the effect on elastic scattering when the resonances are extracted from the continuum space. For the  ${}^6\text{Li}+{}^{28}\text{Si}$  system, no appreciable effect is observed, except at  $E_{lab}=20$  MeV at large angles. As for  ${}^6\text{Li}+{}^{58}\text{Ni}$ , the effect of resonances is small as to be, in most cases within experimental error bars. However, at  $E_{lab}=14$  MeV consideration of the resonance continuum is necessary to fit the data. The dotted-lines in the figures represent the calculations for the elastic channel without continuum couplings. As seen, the sole consideration of the optical potential  $U^{(j)}_{00}(R)$  of Eq. (2.3) is not enough to fit the data and couplings to continuum states are essential.

In summary, CDCC calculations of the weakly bound projectile  ${}^6\text{Li}$  with targets  ${}^{28}\text{Si}$  and  ${}^{58}\text{Ni}$  have been performed with global fragment-target interactions dependent on the target mass and incident energy. In most cases, the results for the elastic scattering angular distributions at energies just above the barrier, closely agree with the data, when couplings to continuum states of  ${}^6\text{Li}$  are considered. Optical model calculations, without continuum couplings predict elastic scattering angular distributions well below the experimental values. This could mean that continuum couplings to breakup channels of  ${}^6\text{Li}$ , produce a net repulsive polarization potential that increases the Coulomb barrier. This fact, consequently leads to less absorption from the elastic channel. On the same footing, the effect of resonance continuum states of  ${}^6\text{Li}$  on elastic scattering has been studied. The effect appears to be negligible in most cases, since the calculations without consideration of these states are within the experimental error bars.

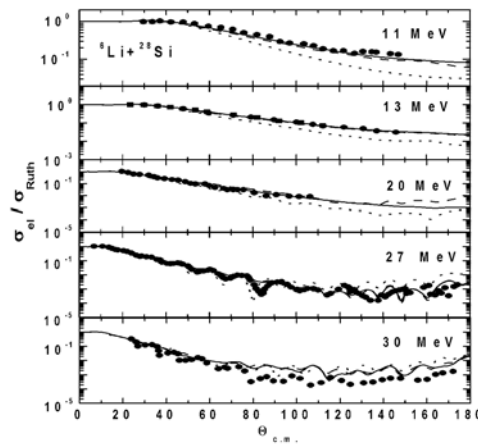


Fig.1. Elastic scattering angular distributions for  ${}^6\text{Li}+{}^{28}\text{Si}$ . See text for details. Data taken from Refs.[28-31].

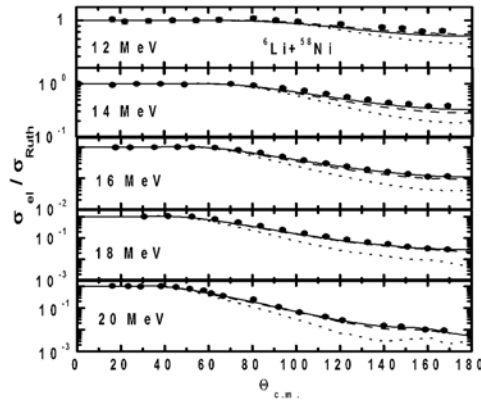


Fig.2. Elastic scattering angular distributions for  ${}^6\text{Li}+{}^{58}\text{Ni}$ . See text for details. Data taken from Refs.[32,33].

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