

Form factors and the effective PS-PV equivalence in eta-meson photoproduction

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In the eta-meson photoproduction from proton the equivalence between the pseudo-scalar (PS) and the pseudo-vector (PV) couplings is broken considerably. However, with properly chosen form factors an effective PS-PV equivalence can be reached for all the spin- $\frac{1}{2}$ resonances with mass less than 2 GeV, i.e. the resonances $S_{11}(1535)$, $S_{11}(1650)$ and $P_{11}(1710)$. Therefore for fitting data one only needs the PS coupling for the spin- $\frac{1}{2}$ resonances. However, for the Born channel it seems impossible to obtain such an effective PS-PV equivalence.

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1. Introduction

In recent decades there has been substantial progress in the study of η -meson photoproduction, that is motivated by the special role of η -meson in identifying new resonances which are weakly coupled to the πN channel. Experimentally the use of new accelerators and the state-of-art detectors have produced considerable amount of high-precision data [1]. On the theory side the Effective Lagrangian Approach (ELA) has shown its effectiveness and relative simplicity in interpreting data. However, in the application of ELA there exists an ambiguity concerning the couplings: while some authors use the pseudo-scalar (PS) coupling only [2], the others employ both the PS and the PV couplings [3], thus an in-depth study is needed for revealing the characteristics of the two couplings.

The relation between the PS and PV couplings has been studied by many authors from different angles. In this work we are studying the possibility of establishing an effective equivalence between the two couplings through applying form factors. We can use the the recent high-precision data [1] as a testing ground.

The article is arranged as follows: In Section 2 the spin- $\frac{1}{2}$ resonances are discussed, Section 3 deals with the Born channel and Section 4 is the conclusion and discussion.

2. The spin- $\frac{1}{2}$ resonances

First we deal with the three spin- $\frac{1}{2}$ resonances: i.e. $S_{11}(1535)$, $S_{11}(1650)$ and $P_{11}(1710)$, widths of which are 150, 165 and 100 MeV, respectively [4]. We start with the most important resonance $S_{11}(1535)$, whose Lagrangian for the PS and PV couplings are

$$\begin{aligned}\mathcal{L}_{\eta NR}^{\text{PS}} &= -ig_R \bar{N} R \eta + h.c.; & \mathcal{L}_{\eta NR}^{\text{PV}} &= -\frac{g'_R}{M_R - M} \bar{N} \gamma_\mu R \partial^\mu \eta + h.c. \\ \mathcal{L}_{\gamma NR} &= \frac{ek_R}{2(M_R + M)} \bar{R} \gamma_5 \sigma_{\mu\nu} N F^{\mu\nu} + h.c.,\end{aligned}$$

respectively. The amplitudes of eta-meson photoproduction are

$$\begin{aligned}i\mathcal{M}_{fi}^{\text{R,PS}} &\equiv eg_R C_R \bar{u}(p_f) \left[\frac{\not{p}_i + \not{k} + M_R}{s - M_R^{*2}} \gamma_5 \not{k} \not{\epsilon} + \gamma_5 \not{k} \not{\epsilon} \frac{\not{p}_f - \not{k} + M_R}{u - M_R^{*2}} \right] u(p_i), \\ i\mathcal{M}_{fi}^{\text{R,PV}} &\equiv \frac{eg'_R C_R}{M_R - M} \bar{u}(p_f) \left[\not{q} \frac{\not{p}_i + \not{k} + M_R}{s - M_R^{*2}} \gamma_5 \not{k} \not{\epsilon} - \gamma_5 \not{k} \not{\epsilon} \frac{\not{p}_f - \not{k} + M_R}{u - M_R^{*2}} \not{q} \right] u(p_i).\end{aligned}$$

The PS and PV amplitudes for helicity=2 are depicted in Fig. 1, where the left part presents the results without any form factor while the right part the results with form factors. The qualitative features of the amplitude of helicity=4 are the same as those of helicity=2, and the amplitudes of helicity=1 and 3 can be neglected, since they are small in magnitude [5] and there is no interference between different helicity channels. The real parts of the PV and PS amplitudes are denoted as PV-R and PS-R, and the imaginary parts as PV-I and PS-I, respectively. In Fig. 1, PV-R and PV-I are represented by solid lines whereas PS-R and PS-I by dashed lines, respectively. In the left part of Fig. 1 one observes the following features of the PS and PV amplitudes: First, the imaginary parts, PV-I and PS-I, are very close and they behave like Breit-Wigner curves with their peak at

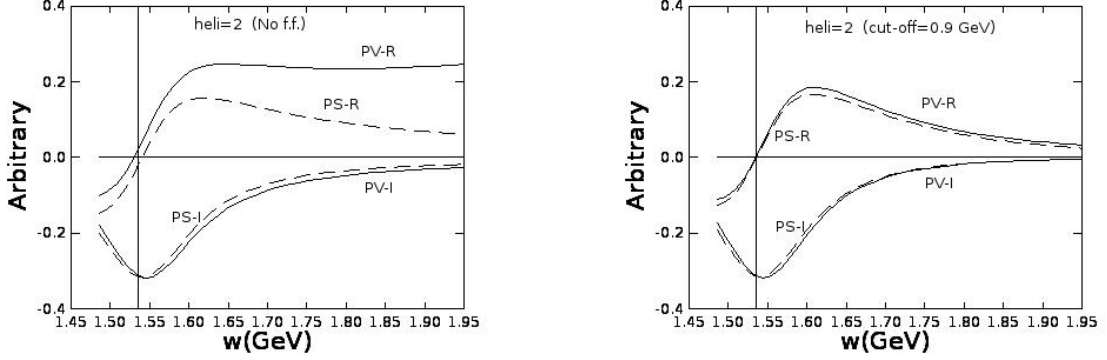


Figure 1: The amplitudes of $S_{11}(1535)$ resonance

the resonance point. The real parts PV-R and PS-R are almost zero at the resonance point and their absolute value increases when going away from the resonance point. Thus the difference between PV-R and PS-R is similar to the behaviors of a widely-adopted monopole form factor, which has the following form:

$$f(\rho) = \frac{\Lambda^4}{\Lambda^4 + (\rho - M_R^2)^2} \quad (\rho = s, u), \quad (2.1)$$

where Λ is the cut-off. Since form factors are indispensable in fitting experimental data for suppressing the amplitudes at higher energy, we may have a chance to use form factors to reduce the difference between the PS and PV amplitudes. Therefore we apply a form factor as is given in Eq. (2.1) with $\Lambda=0.9$ GeV, and indeed both the imaginary and the real parts of PS and PV are becoming very close, as is shown in the right part of Fig. 1.

To further test this effective equivalence we calculate the cross section by using the pure PV and pure PS amplitudes, respectively, and the results are given in Fig. 2, where the left part gives results without form factors while in the right part those with the application of a form factor with $\Lambda=0.9$ GeV. In both parts dashed line represents the cross section by using only the PS amplitude whereas solid line by only that of PV. It is worth mentioning that for the pure PV case with a small shift of resonance energy (from 1535 MeV to 1527 MeV) plus a small modification of Λ (from 0.90 GeV to 0.85 GeV), the small discrepancy between the PS and PV cross section shown in the right part of the figure can be further reduced. The shift of resonance energy is caused by the interplay between the real part and the imaginary part. Due to a big width of the resonance such a tiny shift of resonance energy is justifiable. Therefore in the case of sole the $S_{11}(1535)$ resonance we do reach an effective PS-PV equivalence by using form factors.

Through a similar study for the other two resonances, i.e. $S_{11}(1650)$ and $P_{11}(1710)$, the same effective equivalence can also be reached.

To understand this effective PS-PV equivalence we study the equivalence-breaking terms which are as follows

$$\Delta^R(s+u) = [i\mathcal{M}^{\text{PV}} - i\mathcal{M}^{\text{PS}}](s+u) = \frac{eC_R}{M_R - M} [Z(s) + Z(u)]D_1(k), \quad (2.2)$$

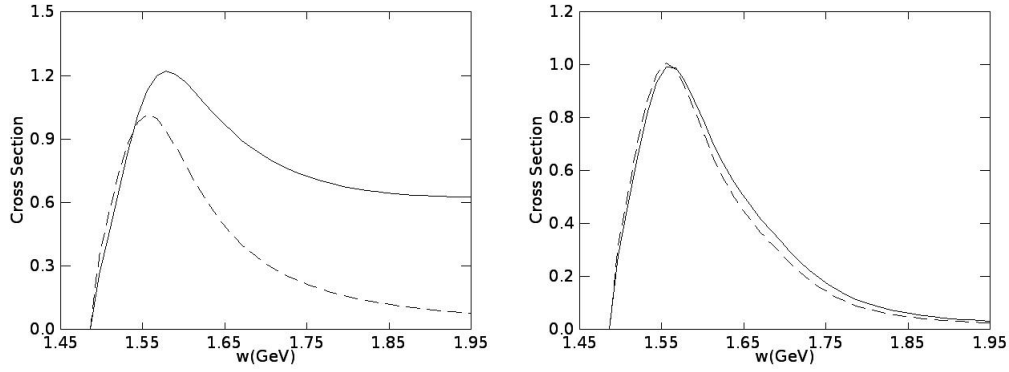


Figure 2: The effective PS-PV equivalence for $S_{11}(1535)$ resonance

where

$$D_1(k) = \bar{u}(p_f) \gamma_5 \not{k} \not{u}(p_i),$$

and

$$Z(\rho) = \frac{\rho - M_R^2 - iM_R\Gamma}{(\rho - M_R^2)^2 + (M_R\Gamma)^2} (\rho - M_R^2) \quad (\rho = s, u).$$

A numerical calculation shows that $D_1(k)$ is a function moderately increasing with the center-of-mass energy ($w = \sqrt{s}$), thus the most important factor is $Z(s) + Z(u)$. In Fig. 3 the contribution

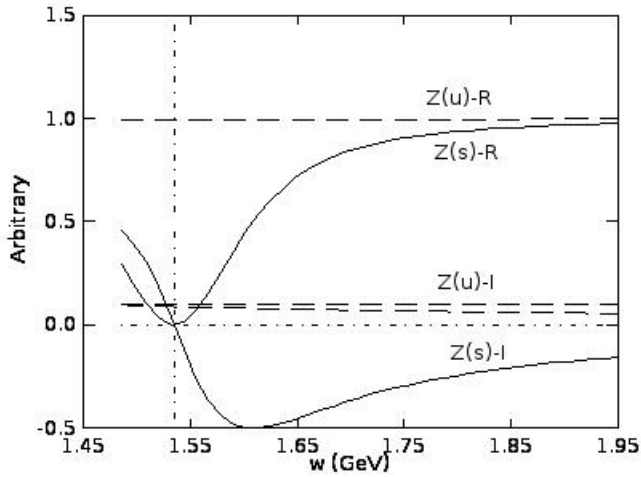


Figure 3: The equivalence-breaking terms

from s -channel ($Z(s)$) are represented by solid lines while that from u -channel ($Z(u)$) by dashed lines, with the suffices -R and -I labeling the real part and the imaginary part, respectively. The two dashed-dotted lines are for guiding the eye: the vertical is to mark the resonance energy while

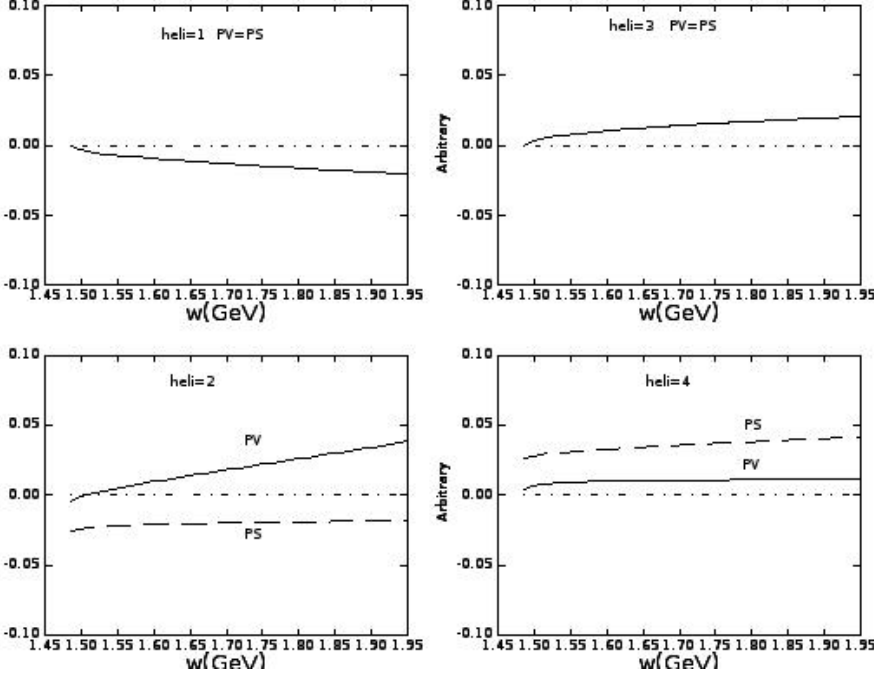


Figure 4: The amplitudes of Born channel

the horizontal represents the zero value. For $Z(u)$ -I the two lines correspond to the angle of $\theta = 5$ and 175 degrees, respectively. Since the $Z(u)$ -I is very small in magnitude and $Z(s)$ -I very small compared to the imaginary part of the s -channel amplitude, the equivalence-breaking is mainly caused by the $Z(s)$ -R and $Z(u)$ -R terms. It is easy to show that a form factor as that in Eq. (2.1) will greatly suppress the contribution from $Z(s)$ -R, in particular at higher energy. As for the form factor for u -channel, there has been no well established rule, thus we follow Ref. [2] to use the same cut-off (0.9 GeV) for both the s - and u -channel. Due to the small absolute value of u (also a small angular dependence), this form factor $f(u)$ almost kills $Z(u)$ -R. In this way all the contributing factors to the equivalence-breaking terms are greatly reduced, thus an effective PS-PV equivalence is obtained.

3. The Born channel

For the Born channel the PS and PV couplings are

$$\begin{aligned} \mathcal{L}_{\eta NN}^{\text{PS}} &= -ig_N \bar{N} \gamma_5 N \eta; & \mathcal{L}_{\eta NN}^{\text{PV}} &= \frac{g'_N}{2M} \bar{N} \gamma_\mu \gamma_5 N \partial^\mu \eta \\ \mathcal{L}_{\gamma NN} &= -e \bar{N} \gamma_\mu N A^\mu + \frac{ek_P}{4M} \bar{N} \sigma_{\mu\nu} N F^{\mu\nu}, \end{aligned}$$

respectively. Fig. 4 shows the amplitudes of those couplings. In the figure the PS and PV amplitudes (all real) are depicted for the helicity 1 to 4 in the four parts with the order of from left to right and from top to bottom. The PV amplitudes is presented with solid line, the PS with dashed line and a dashed-dotted line presents zero value for guiding the eye. For the helicity 1 and 3 the

amplitudes of PS and PV are equal, while for helicity 2 and 4 they are different. It seems obvious that the effective PS-PV equivalence of the three spin- $\frac{1}{2}$ resonances do not exist for the Born channel. However, in fitting data with the application of form factor the contribution of Born channel will be reduced for higher energy, thus this channel has importance only for the energy region near threshold.

In a multi-channel process the interference among channels always play an important role. To test the effective PS-PV equivalence, we make a calculation with all the three spin- $\frac{1}{2}$ resonances plus the Born channel, and compare the theoretical results with the data from Ref. [1]. In Fig. 5, we show the comparison. In the left part of the figure the two theoretical predictions are given: the solid and dashed lines represent the cross section with pure PV and pure PS amplitudes, respectively. The form factors are the same as given in the Fig 1. The left part shows that the effective PS-PV equivalence does exist in the case of multi-channels. The right part compares the theoretical predictions with the experimental data of Ref. [1]. We note that no sophisticated and quantitative fitting procedure was intended, rather our sole purpose is to show that the difference between the cross section calculated with PS only and PV only is much less than the error bar of the data, which means that the PS and PV amplitudes, after an application of form factors, are really equivalent as far as data interpretation is concerned. We note that in the two parts of the Fig. 5 the Born channel contains both the PS and the PV amplitudes. It is worth mentioning that in the above discussion the

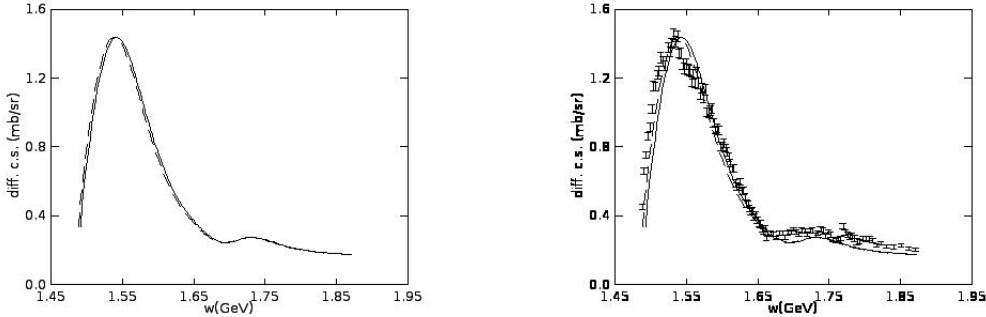


Figure 5: Cross sections with three spin- $\frac{1}{2}$ resonances plus Born channel

angle is taken as $\theta=90$, however, for other angles the same effective equivalence can be reached, but the value of cut-off may be slightly different.

4. conclusion and discussion

It has been shown in this work that for the spin- $\frac{1}{2}$ resonances (with mass less than 2 GeV) an effective PS-PV equivalence can be established with properly chosen form factors. Since in fitting data the form factors are indispensable, therefore for simplicity we need only the PS couplings for those resonances. However, for Born channel there seems no such an effective equivalence existing, thus in fitting data both the PS and PV couplings are necessary, especially for the data near

threshold. For a comprehensive quantitative fitting the resonances with spin- $\frac{3}{2}$, i.e. the $D_{13}(1520)$ and $P_{13}(1720)$, need to be included and it will be the subject of a subsequent study,

As is indicated in Section 3 all the calculations are done for $\theta=90$. We have tried other angles and the same conclusion on the PS-PV equivalence can be obtained except that the cut-off in the form factor are slightly different from what we presented in the above. Therefore if one would intend a comprehensive interpretation on both the energy dependence and the angular distribution of the experimental data, more investigation is needed.

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