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Determining Neutrino Mass and Mixing Observables and the Nature of Massive Neutrinos Using Atoms

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We discuss how the absolute neutrino mass scale, the neutrino mass hierarchy and the nature (Dirac or Majorana) of massive neutrinos can be determined, in principle, from the measurement of the continuous spectrum of the photon produced in the process of collective de-excitation of atoms in a metastable level into emission mode of a single photon plus a neutrino pair ("radiative emission of neutrino pair (RENP)"). We use the example of a transition between specific levels of the Yb atom. The possibility of determining the nature of massive neutrinos and, if neutrinos are Majorana fermions, of obtaining information about the Majorana phases in the neutrino mixing matrix, is analised in the cases of normal hierarchical, inverted hierarchical and quasi-degenerate types of neutrino mass spectrum. It is found, in particular, that the sensitivity to the nature of massive neutrinos depends critically on the atomic level energy difference relevant in the RENP.

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1. Introduction

Determining the absolute scale of neutrino masses, the type of neutrino mass spectrum, which can be either with normal or inverted ordering [1] (NO or IO), the nature (Dirac or Majorana) of massive neutrinos, and getting information about the Dirac and Majorana CP violation phases in the neutrino mixing matrix, are the most pressing and challenging problems of the future research in the field of neutrino physics (see, e.g., [2]). It is remarkable that all these neutrino mass and mixing observables and massive neutrino properties can be determined, in principle, as we will discuss in the present article, in a single atomic physics experiment by using the process of cooperative de-excitation of atoms in a metastable level into emission mode of single photon plus a neutrino pair [3, 4]. This process is usually called radiative emission of neutrino pair (RNEP).

The analysis which we will present in this article and which is based on the publication [1], will be performed within the reference 3-neutrino mixing scheme. As is well known, in this scheme the left-handed flavour neutrino fields $\nu_{l\mathrm{L}}(x)$, which enter into the expressions for the charged lepton and neutral neutrino currents in the weak interaction Lagrangian, are linear combinations of the fields of three neutrinos ν_j, having masses $m_j \neq 0$:

$$\nu_{l\mathrm{L}}(x) = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}}(x), \;\; l = e, \mu, \tau\,, \tag{1.1}$$

where $\nu_{j\mathrm{L}}(x)$ is the LH component of the field of ν_j possessing a mass m_j and $U \equiv U_{\mathrm{PMNS}}$ is a unitary matrix - the Pontecorvo, Maki, Nakagawa, Sakata (PMNS) neutrino mixing matrix. All compelling neutrino oscillation data is compatible with 3-neutrino mixing. It is firmly established on the basis of the current data that the three light neutrinos ν_1, ν_2 and ν_3, have masses $m_{1,2,3} \lesssim 1$ eV, and that $m_1 \neq m_2 \neq m_3$.

It is also well known that the PMNS neutrino mixing matrix U, which is a 3×3 unitary matrix, can be parametrised in terms of three mixing angles θ_{12}, θ_{23} θ_{13}, and depending on whether the massive neutrinos are Dirac or Majorana particles, by one Dirac (δ), or one Dirac (δ) and two Majorana [5] (α and β), CP violation (CPV) phases. In the widely used "standard parametrisation" of U (see, e.g., [2]), the elements of the first row of the PMNS matrix, U_{ei}, $i = 1, 2, 3$, which play important role in our further discussion, are given by

$$U_{e1} = c_{12}\, c_{13}\,,\; U_{e2} = s_{12}\, c_{13}\, e^{i\alpha}\,,\; U_{e3} = s_{13}\, e^{i(\beta - \delta)}\,, \tag{1.2}$$

where we have used the standard notation $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ with $0 \leq \theta_{ij} \leq \pi/2$, $0 \leq \delta \leq 2\pi$ and, in the case of interest for our analysis [2], $0 \leq \alpha, \beta \leq \pi$, (see, however, [6]). If CP invariance holds, we have $\delta = 0, \pi$, and [7] $\alpha, \beta = 0, \pi/2, \pi$.

In our analysis we will use as input the best fit values of the neutrino oscillation parameters, Δm^2_{21}, $\sin^2\theta_{12}$, $|\Delta m^2_{31(32)}|$, $\sin^2\theta_{13}$ and $\sin^2\theta_{13}$, which have been determined with a relatively high precision in the global analysis of the neutrino oscillation data performed in [8]:

$$\Delta m^2_{21} = 7.54 \times 10^{-5}\ \mathrm{eV}^2\,,\; |\Delta m^2_{31(32)}| = 2.47\ (2.46) \times 10^{-3}\ \mathrm{eV}^2\,, \tag{1.3}$$

$$\sin^2\theta_{12} = 0.307, \qquad \sin^2\theta_{13} = 0.0241\ (0.0244)\,, \tag{1.4}$$

[1]We use the convention adopted in [2].

[2]Note that the two Majorana phases α_{21} and α_{31} defined in [2] are twice the phases α and β: $\alpha_{21} = 2\alpha$, $\alpha_{31} = 2\beta$.

where the values (the values in brackets) correspond to neutrino mass spectrum with normal ordering (inverted ordering) (see, e.g., [2]). We will neglect the small differences between the NO and IO values of $|\Delta m^2_{31(32)}|$ and $\sin^2\theta_{13}$ and will use $|\Delta m^2_{31(32)}| = 2.47\times 10^{-3}$ eV^2, $\sin^2\theta_{13} = 0.024$ in our numerical analysis.

For a single atom the process of radiative emission of neutrino pair (RNEP) of interest is $|e\rangle \to |g\rangle + \gamma + (\nu_i + \nu_j)$, $i,j = 1,2,3$, where ν_i's are neutrino mass eigenstates. If ν_i are Dirac fermions, $(\nu_i + \nu_j)$ should be understood as a pair of neutrino anti-neutrino with masses m_i and m_j, respectively. If neutrino are Majorana particles, we have $\overline{\nu}_i \equiv \nu_i$ and $(\nu_i + \nu_j)$ are the Majorana neutrinos with masses m_i and m_j. The proposed experimental method is to measure, under irradiation of two counter-propagating trigger lasers, the continuous photon (γ) energy spectrum below each of the six thresholds ω_{ij} corresponding to the production of the six different pairs of neutrinos, $\nu_1\nu_1$, $\nu_1\nu_2$,..., $\nu_3\nu_3$: $\omega < \omega_{ij}$, ω being the photon energy, and [3, 4]

$$\omega_{ij} = \omega_{ji} = \frac{\varepsilon_{eg}}{2} - \frac{(m_i + m_j)^2}{2\varepsilon_{eg}}\,, \quad i,j = 1,2,3, \;\; m_{1,2,3} \geq 0\,, \tag{1.5}$$

where ε_{eg} is the energy difference between the two relevant atomic levels. The disadvantage of the proposed experiment is the smallness of RENP rate, which is proportional to G_F^2, $G_F = 10^{-23}\text{eV}^{-2}$. This can possibly be overcome by "macro-coherence" amplification of the rate [9, 10], the amplification factor being $\propto n^2V$, where n is the number density of excited atoms and V is the volume irradiated by the trigger laser. For n at the order of N_A/cm^3, where N_A is Avogadro's number, and $V \sim 1cm^3$, the rate is observable. The macro-coherence of interest is developed by irradiation of two trigger lasers of frequencies ω_1, ω_1, satisfying $\omega_1 + \omega_2 = \varepsilon_{eg}$. It is a complicated dynamical process. The asymptotic state of fields and target atoms in the latest stage of trigger irradiation is described by a static solution of the master evolution equation. In many cases there is a remnant state consisting of field condensates (of the soliton type) accompanied with a large coherent medium polarisation. This asymptotic target state is stable against two photon emission (except for minor "leakage" from the edges of the target), while RENP occurs from any point in the target [9, 10]. A Group at Okayama University, Okayama, Japan, is working on the experimental realisation of the macro-coherent RENP [11].

The process of RENP has a rich variety of neutrino phenomenology. The RENP related photon spectral shape is sensitive to various observables: the absolute neutrino mass scale, the type of neutrino mass spectrum, the nature of massive of neutrinos and the Majorana CPV phases in the case of massive Majorana neutrinos. All these observables can be determined in one experiment, each observable with a different degree of difficulty, once the RENP process is experimentally established. For atomic energy available in the process of the order of a fraction of eV, the observables of interest can be ranked in the order of increasing difficulty of their determination as follows:
(1) The absolute neutrino mass scale, which can be fixed by, e.g., measuring the smallest photon energy threshold $\min(\omega_{ij})$ near which the RENP rate is maximal: $\min(\omega_{ij})$ corresponds to the production of a pair of the heaviest neutrinos ($\max(m_j) \gtrsim 50$ meV).
(2) The neutrino mass hierarchy, i.e., distinguishing between the normal hierarchical (NH), inverted hierarchical (IH) and quasi-degenerate (QD) spectra, or a spectrum with partial hierarchy.
(3) The nature (Dirac or Majorana) of massive neutrinos.

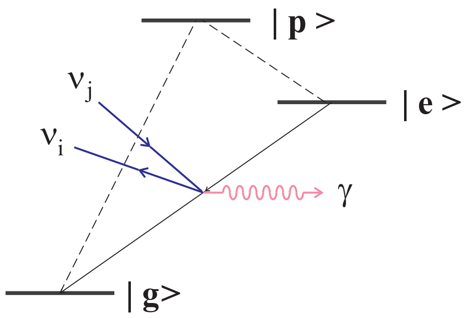

Figure 1: $\Lambda-$type atomic level for RENP $|e\rangle \to |g\rangle + \gamma + \nu_i\nu_j$ with ν_i a neutrino mass eigenstate. Dipole forbidden transition $|e\rangle \to |g\rangle + \gamma + \gamma$ may also occur via weak E1$\times$ M1 couplings to $|p\rangle$.

(4) The measurement on the Majorana CPV phases if the massive neutrinos are Majorana particles. The last two items are particularly challenging.

In what follows we show how each of the neutrino observables listed above can be determined, in principle, in a RENP experiment (for a more detailed discussion see [1, 11]).

2. Photon Energy Spectrum in RENP

The process of interest occurs in the 3rd order (counting the four Fermi weak interaction as the 2nd order) of electroweak theory as a combined weak and QED process, as depicted in Fig. 1. Its effective amplitude has the form of

$$\langle g|\vec{d}|p\rangle \cdot \vec{E}\,\frac{G_F\sum_{ij}a_{ij}\nu_j^\dagger\vec{\sigma}\nu_i}{\varepsilon_{pg}-\omega}\cdot\langle p|\vec{S}_e|e\rangle\,, \tag{2.1}$$

$$a_{ij} = U_{ei}^*U_{ej} - \frac{1}{2}\delta_{ij}\,, \tag{2.2}$$

where U_{ei}, $i = 1,2,3$, are the elements of the first row of the neutrino mixing matrix U_{PMNS}, given in eq. (1.2). The atomic part of the probability amplitude involves three states $|e\rangle, |g\rangle, |p\rangle$, where the two states $|e\rangle, |p\rangle$, responsible for the neutrino pair emission, are connected by a magnetic dipole type operator, the electron spin $\vec{S}_e$. The $|g\rangle - |p\rangle$ transition involves a stronger electric dipole operator $\vec{d}$. From the point of selecting candidate atoms, E1$\times$M1 type transition must be chosen between the initial and the final states ($|e\rangle$ and $|g\rangle$). The field $\vec{E}$ in eq. (2.1) is the one stored in the target by the counter-propagating fields. The formula has some similarity to the case of stimulated emission.

As mentioned above, the quantity that is experimentally measured is the single photon spectrum. In the case with no external magnetic field, after neglecting the atomic recoil, the spectral rate, which is the rate of number of events per unit time at each photon energy ω, can be written as:

$$\Gamma_{\gamma 2\nu}(\omega) = \Gamma_0 I(\omega)\eta_\omega(t)\,, \qquad \Gamma_0 \sim \frac{n^2 V G_F^2\varepsilon_{eg}n}{2\varepsilon_{pg}^3}\,, \tag{2.3}$$

$$I(\omega) = \frac{1}{(\varepsilon_{pg}-\omega)^2}\sum_{ij}|a_{ij}|^2\Delta_{ij}(\omega)\left(I_{ij}(\omega) - \delta_M m_i m_j B_{ij}^M\right)\,, \tag{2.4}$$

$$B_{ij}^M = \frac{\Re(a_{ij}^2)}{|a_{ij}|^2} = \left(1 - 2\,\frac{(\mathrm{Im}(a_{ij}))^2}{|a_{ij}|^2}\right), \quad a_{ij} = U_{ei}^* U_{ej} - \frac{1}{2}\delta_{ij}\,, \tag{2.5}$$

$$\Delta_{ij}(\omega) = \frac{1}{\varepsilon_{eg}(\varepsilon_{eg} - 2\omega)} \left\{ \left(\varepsilon_{eg}(\varepsilon_{eg} - 2\omega) - (m_i + m_j)^2\right) \left(\varepsilon_{eg}(\varepsilon_{eg} - 2\omega) - (m_i - m_j)^2\right) \right\}^{1/2}, \tag{2.6}$$

$$I_{ij}(\omega) = \left(\frac{1}{3}\varepsilon_{eg}(\varepsilon_{eg} - 2\omega) + \frac{1}{6}\omega^2 - \frac{1}{18}\omega^2 \Delta_{ij}^2(\omega) - \frac{1}{6}(m_i^2 + m_j^2) - \frac{1}{6}\frac{(\varepsilon_{eg} - \omega)^2}{\varepsilon_{eg}^2(\varepsilon_{eg} - 2\omega)^2}(m_i^2 - m_j^2)^2 \right). \tag{2.7}$$

Here $\eta_\omega(t)$ is the dynamical dimensionless factor, which will be taken as unity. The case $\delta_M = 1$ applies to Majorana neutrinos, $\delta_M = 0$ corresponds to Dirac neutrinos. The term $\propto m_i m_j (1 - 2(\mathrm{Im}(a_{ij}))^2/|a_{ij}|^2)$ is similar to, and has the same physical origin as, the term $\propto M_i M_j$ in the production cross section of two different Majorana neutralinos χ_i and χ_j with masses M_i and M_j in the process of $e^- + e^+ \to \chi_i + \chi_j$ [12]. The term $\propto M_i M_j$ of interest determines, in particular, the threshold behavior of the indicated cross section.

In the limit of massless neutrinos the spectral rate becomes

$$I(\omega; m_i = 0) = \frac{\omega^2 - 6\varepsilon_{eg}\omega + 3\varepsilon_{eg}^2}{12(\varepsilon_{pg} - \omega)^2}\,, \tag{2.8}$$

where the prefactor of $\sum_{ij} |a_{ij}|^2 = 3/4$ is calculated using the unitarity of the neutrino mixing matrix.

In what follows we will carry numerical analysis for the case of Yb atom with energy levels:

$$\mathrm{Yb}; \quad |e\rangle = (6s6p)\,{}^3P_0\,, \quad |g\rangle = (6s^2)\,{}^1S_0\,, \quad |p\rangle = (6s6p)\,{}^3P_1\,, \tag{2.9}$$

which are relevant for the emission of RENP. The atomic energy differences are [13]:

$$\varepsilon_{eg} = 2.14349 \text{ eV}\,, \quad \varepsilon_{pg} = 2.23072 \text{ eV}\,. \tag{2.10}$$

In the case of Yb atom considered, the overall rate factor Γ_0 is given by

$$\Gamma_0 \sim 0.37 \text{ mHz}\left(\frac{n}{10^{21}\text{ cm}^{-3}}\right)^3 \frac{V}{10^2\text{ cm}^3}\,, \tag{2.11}$$

where the number is valid for the Yb first excited state of $J = 0$ [3].

In the present article we concentrate on the elementary particle physics potential of the experiment proposed; the technical aspects of the proposed experiment are discussed in, e.g., [11, 14].

3. Sensitivity of the Spectral Rate to the Properties of Neutrinos

3.1 General Features of the Spectral Rate

It follows from eqs. (2.3) and (2.4) that the rate of emission of a given pair of neutrinos $(\nu_i + \nu_j)$ is suppressed, in particular, by the factor $|a_{ij}|^2$ which depends only on the mixing angles θ_{12} and θ_{13}. The numerical values of $|a_{ij}|^2$, corresponding to the best fit values of $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$ quoted in eq. (1.3), are given in the Table 1. It follows from Table 1 that the least suppressed

[3] If one chooses the other intermediate path, ${}^1P_1$, the rate Γ_0 is estimated to be of order, 1×10^{-3} mHz, a value much smaller than that of the ${}^3P_1$ path.

Table 1: The quantity $|a_{ij}|^2 = |U^*_{ei}U_{ej} - \frac{1}{2}\delta_{ij}|^2$

$\lvert a_{11}\rvert^2 = \lvert c^2_{12}c^2_{13} - \frac{1}{2}\rvert^2$	$\lvert a_{12}\rvert^2 = c^2_{12}s^2_{12}c^4_{13}$	$\lvert a_{13}\rvert^2 = c^2_{12}s^2_{13}c^2_{13}$
0.0311	0.2027	0.0162
$\lvert a_{22}\rvert^2 = \lvert s^2_{12}c^2_{13} - \frac{1}{2}\rvert^2$	$\lvert a_{23}\rvert^2 = s^2_{12}s^2_{13}c^2_{13}$	$\lvert a_{33}\rvert^2 = \lvert s^2_{13} - \frac{1}{2}\rvert^2$
0.0405	0.0072	0.2266

by the factor $|a_{ij}|^2$ is the emission of the pairs $(\nu_3 + \nu_3)$ and $(\nu_1 + \nu_2)$, while the most suppressed is the emission of $(\nu_2 + \nu_3)$. The smallest value of $|a_{ij}|^2$ is $|a_{23}|^2 \sim 10^{-3}$. Thus, in order to identify the emission of all thresholds, the RENP spectral rate should be measured with a relative precision not worse than approximately 5×10^{-3}.

As it follows from eqs. (2.4) and (2.5), the rate of emission of a pair of Majorana neutrinos with masses m_i and m_j differs from the rate of emission of a pair of Dirac neutrinos with the same masses by the interference term $\propto m_i m_j B^M_{ij}$. For $i = j$ we have $B^M_{ij} = 1$, the interference term is negative and tends to suppress the neutrino emission rate. In the case of $i \neq j$, the factor B^M_{ij}, and thus the rate of emission of a pair of different Majorana neutrinos, depends on specific combinations of the Majorana and Dirac CPV phases of the neutrino mixing matrix: from eqs. (2.5) and (1.2) we get

$$B^M_{12} = \cos 2\alpha\,,\;\; B^M_{13} = \cos 2(\beta - \delta)\,,\;\; B^M_{23} = \cos 2(\alpha - \beta + \delta)\,. \tag{3.1}$$

Note, however, that the rates of emission of $(\nu_1 + \nu_3)$ and of $(\nu_2 + \nu_3)$ are suppressed by $|a_{13}|^2 = 0.016$ and $|a_{23}|^2 = 0.007$, respectively. Thus, studying the rate of emission of $(\nu_1 + \nu_2)$ seems the most favorable approach to get information about the Majorana phase α, provided the corresponding interference term $\propto m_1 m_2 B^M_{12}$ is not suppressed by the smallness of the factor $m_1 m_2$. The mass m_1 can be very small or even zero in the case of NH neutrino mass spectrum, while for the IH spectrum we have $m_1 m_2 \gtrsim |\Delta m^2_{32}| \cong 2.5 \times 10^{-3}$ eV^2 . In contrast, the rate of emission of a pair of Dirac neutrinos does not depend on the CPV phases of the PMNS matrix.

In the case of $m_1 < m_2 < m_3$ (NO spectrum), the ordering of the threshold energies at $\omega_{ij} = \omega_{ji}$ is the following: $\omega_{11} > \omega_{12} > \omega_{22} > \omega_{13} > \omega_{23} > \omega_{33}$. It follows from eq.(1.5) that ω_{11}, ω_{12} and ω_{22} are very close, ω_{13} and ω_{23} are somewhat more separated and the separation is the largest between ω_{22} and ω_{13}, and ω_{23} and ω_{33}. In the case of NH neutrino spectrum with very small m_1, one has $\omega_{11} - \omega_{12} \approx \frac{1}{2\varepsilon_{eg}}\Delta m^2_{21} \cong 1.759\ (8.794) \times 10^{-5}$ eV and $\omega_{23} - \omega_{33} \approx (3\Delta m^2_{31} - 2\sqrt{\Delta m^2_{21}}\sqrt{\Delta m^2_{31}} - \Delta m^2_{21}) \cong 1.510\ (7.548) \times 10^{-3}$ eV for $\varepsilon_{eg} = 2.1439$ eV of Yb (for a hypothetical atom with $\varepsilon_{eg} = 2.14349/5 = 0.42870$ eV). In the case of QD spectrum and $\Delta m^2_{31} > 0$, the separations are: $\omega_{11} - \omega_{12} \cong \omega_{12} - \omega_{22} \cong \omega_{13} - \omega_{23} \cong \frac{1}{\varepsilon_{eg}}\Delta m^2_{21} \cong 3.518\ (17.588) \times 10^{-5}$ eV, $\omega_{22} - \omega_{13} \cong \omega_{23} - \omega_{33} - \frac{1}{\varepsilon_{eg}}\Delta m^2_{21} = \frac{1}{\varepsilon_{eg}}(\Delta m^2_{31} - 2\Delta m^2_{21}) \cong 1.082\ (5.410) \times 10^{-3}$ eV. For spectrum with inverted ordering, $m_3 < m_1 < m_2$, the ordering of the threshold energies is different: $\omega_{33} > \omega_{13} > \omega_{23} > \omega_{11} > \omega_{12} > \omega_{22}$. In the case of IH spectrum with negligible $m_3 = 0$, it is not only ω_{11}, ω_{12} and ω_{22}, but also ω_{13} and ω_{23}, are very close, the corresponding differences being all $\sim \Delta m^2_{21}/\varepsilon_{eg}$. The separation between the thresholds ω_{33} and ω_{13}, and between ω_{23} and ω_{11}, are

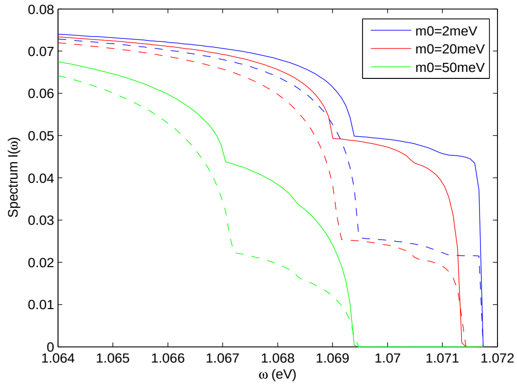

Figure 2: Photon energy spectrum from Yb $^3P_0 \to {}^1S_0$ transitions in the threshold region in the cases of NH spectrum (solid lines) and IH spectrum (dashed lines) and for 3 different sets of Dirac neutrinos masses corresponding to $m_0 = 2$ meV (black lines), 20 meV (red lines) and 50 meV (blue lines).

considerably larger, being $\sim \Delta m^2_{23}/\varepsilon_{eg}$. These results remain valid also in the case of QD spectrum and $\Delta m^2_{32} < 0$.

It follows from the preceding discussion that in order to observe and determine all six threshold energies ω_{ij}, the photon energy ω should be measured with a precision not worse than approximately 10^{-5} eV. This is possible to achieve in RENP experiments since the energy resolution in the spectrum is determined by the uncertainties in the trigger laser frequencies, which are much smaller than 10^{-5} eV.

3.2 Neutrino Observables

The analysis which follows is based on the properties of the dimensionless spectral function $I(\omega)$ which contains all the neutrino physics information of interest.

The Absolute Neutrino Mass Scale. As can be seen from eq. (1.5), the positions of the thresholds ω_{ij} in the photon spectrum contain information about the neutrino masses. Figure 2 shows the Dirac neutrino spectra for three different sets of values of the neutrino masses (corresponding to the smallest mass $m_0 = 2,\ 20,\ 50$ meV) and for both the NO ($\Delta m^2_{31(32)} > 0$) and IO ($\Delta m^2_{31(32)} < 0$) neutrino mass spectra. One sees that the locations of the thresholds corresponding to the three values of m_0 (and that can be seen in the figure) differ substantially. The most prominent kink comes from the heavier neutrino pair emission thresholds. If the spectrum is of the NO type, the measurement of the position of the kink will determine the value of ω_{33} and therefore of m_3. For the IO spectrum, the threshold ω_{12} is very close to the thresholds ω_{22} and ω_{11}. The rates of emission of the pairs $(\nu_2 + \nu_2)$ and $(\nu_1 + \nu_1)$, however, are smaller approximately by the factors 10.0 and 12.7, respectively, than the rate of emission of $(\nu_1 + \nu_2)$. Thus, the kink due to the $(\nu_1 + \nu_2)$ emission will be the easiest to observe. The position of the kink will allow to determine $(m_1 + m_2)^2$ and thus the absolute neutrino mass scale.

The Neutrino Mass Spectrum (or Hierarchy). It is clear that if one could measure at least three among the six thresholds, the neutrino mass spectrum will be determined. As is suggested by Fig. 2, one can distinguish between the NH (NO) and IH (IO) spectra by measuring the ratio

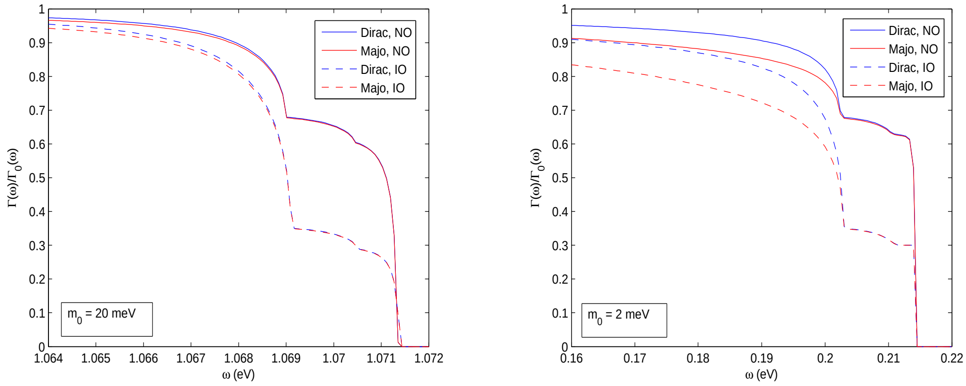

Figure 3: The ratio $R(\Gamma) \equiv \Gamma_{\gamma 2\nu}(\omega)/\Gamma_{\gamma 2\nu}(\omega; m_i = 0) = I(\omega)/I(\omega; m_i = 0)$ in the cases of Dirac neutrinos (black lines) and Majorana neutrinos (red lines) with different values of $\varepsilon_{eg} = 2.14349$ eV (left panel) and $\varepsilon_{eg} = 0.42870$ (right panel) for NH spectrum (solid lines) and IH spectrum (dashed lines)

of rates below and above the thresholds ω_{33} and ω_{12} (or ω_{11}), respectively. For $m_0 \lesssim 20$ meV and NH (IH) spectrum, the ratio of the rates at ω just above the ω_{33} (ω_{11}) threshold and sufficiently far below the indicated thresholds, $\tilde{R}$, is given by:

$$\mathrm{NH}: \quad \tilde{R}(\omega_{33}; NH) \cong \frac{\sum_{i,j} |a_{ij}|^2 - |a_{33}|^2}{\sum_{i,j} |a_{ij}|^2} \cong 0.70\,, \tag{3.2}$$

$$\mathrm{IH}: \quad \tilde{R}(\omega_{11}; IH) \cong \frac{|a_{33}|^2 + 2\,(|a_{13}|^2 + (|a_{23}|^2)}{\sum_{i,j} |a_{ij}|^2} \cong 0.36\,. \tag{3.3}$$

In obtaining the result (3.3) in the IH case we have assumed that ω_{22} and ω_{12} are not resolved, but the kink due to the ω_{11} threshold could be observed.

The Nature of Massive Neutrinos. The Majorana vs Dirac neutrino distinction is illustrated in Fig. 3. The figure is obtained for $m_0 = 20$ meV, $\varepsilon_{eg} = 2.14$ eV (left panel) and $m_0 = 2$ meV, $\varepsilon_{eg} = 0.43$ eV (right panel). The CPV phases set to zero, $(\alpha, \beta - \delta) = (0, 0)$, but the conclusion is valid for other choices of the values of the phases as well. For $\varepsilon_{eg} = 2.14$ eV, the difference between the emission of pairs of Dirac and Majorana neutrinos is very small and thus very difficult to observe. Although its value increases as a function of the lightest neutrino mass m_0, for quite large $m_0 = 100$ meV, the relative difference between the Dirac and Majorana spectra can not exceed approximately 6% at values of ω sufficiently far below the threshold energies ω_{ij} (upper panels in Fig. 4). A possible solution of the problem is the observation of RENP of an atom with smaller ε_{eg}. In the case of a hypothetical atom X scaled down in energy by 1/5 from the real Yb, i.e., with $\varepsilon_{eg} \sim 0.43$ eV, even for a small values of $m_0 = 2$ meV, the Majorana vs Dirac difference is bigger than 5% (10%) for the NO (IO) neutrino mass spectrum (right panel of the Fig. 3).

For the most challenging target to probe information of the CPV phases, the experiment may be relevant only for an atom with small ε_{eg} in the case of large enough m_0. In the Majorana neutrino case (see Fig. 4), the ratio $R(\Gamma)$ is plotted for the four combinations of CP conserving values of the phases $(\alpha, \beta - \delta) = (0, 0); (0, \pi/2); (\pi/2, 0); (\pi/2, \pi/2)$. There is a significant difference between

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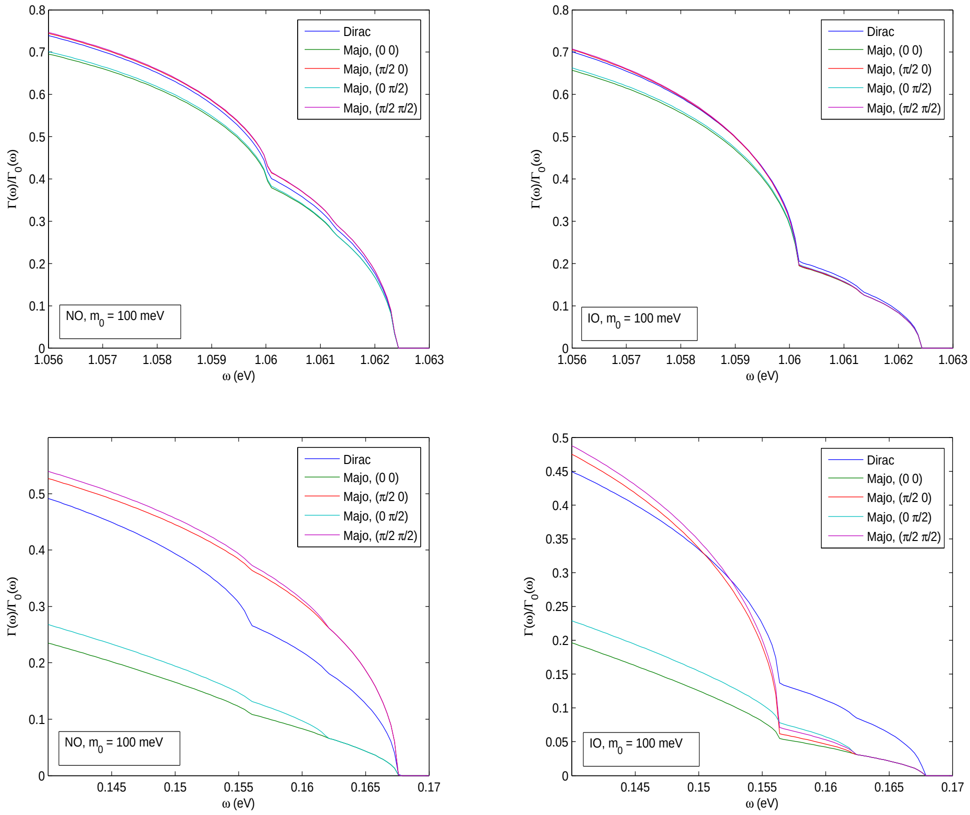

Figure 4: The ratio $R(\Gamma) \equiv \Gamma_{\gamma 2\nu}(\omega)/\Gamma_{\gamma 2\nu}(\omega; m_i = 0) = I(\omega)/I(\omega; m_i = 0)$ as a function of ω in the case of emission of Dirac and Majorana massive neutrinos having NO (left panels) or IO (right panels) mass spectrum, for $\varepsilon_{eg} = 2.14349$ eV (upper panels) and $\varepsilon_{eg} = 0.42870$ eV (lower panels) and four values of the CPV phases $(\alpha, \beta - \delta)$ in the Majorana case.

the Majorana neutrino emission rates corresponding to $(\alpha, \beta - \delta) = (0, 0)$ and $(\pi/2, \pi/2)$. The difference between the emission rates of Dirac and Majorana neutrinos is largest for $(\alpha, \beta - \delta) = (0, 0)$. For $m_0 = 100$ meV and $(\alpha, \beta - \delta) = (0, 0)$. for instance, the rate of emission of Dirac neutrinos at ω sufficiently smaller than ω_{33} in the NO case and ω_{22} in the IO one, can be larger than the rate of Majorana neutrino emission by $\sim 70\%$. The Dirac and Majorana neutrino emission spectral rates never coincide.

It follows from these results that one of the most critical atomic physics parameters for the potential of an RENP experiment to provide information on the largest number of fundamental neutrino physics observables of interest is the value of the energy difference ε_{eg}. Values $\varepsilon_{eg} \leq 0.43$ eV are favorable for determining the nature of massive neutrinos, and, if neutrinos are Majorana particles, for getting information about at least some of the leptonic CPV phases, which are the most difficult neutrino related observables to probe experimentally.

4. Conclusion

The RENP experiments have remarkable physics potential. In these experiments one can determine, in principle, the absolute scale of neutrino masses, the neutrino mass hierarchy and the nature - Dirac or Majorana - of massive neutrinos. They can provide also unique information on the Majorana CPV phases.

It is not clear at present whether the RENP experiments are feasible. However, given the potential the RENP experiments have, their feasibility studies should be further rigorously pursued and supported by the neutrino physics community.

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