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New results for Higgs plus jet at NNLO

Frank Petriello[*†]
High Energy Physics Division, Argonne National Laboratory, Argonne, IL 60439, USA
Department of Physics & Astronomy, Northwestern University, Evanston, IL 60208, USA
E-mail: f-petriello@northwestern.edu

We report on a calculation of the cross-section for Higgs boson production in gluon fusion in association with a hadronic jet at next-to-next-to-leading order (NNLO) in perturbative QCD. We show explicitly how to employ known soft and collinear limits of scattering amplitudes to construct subtraction terms for NNLO computations, andpresent numerical results for the gluon-fusion contribution to Higgs production in association with a jet at the LHC. The NNLO QCD corrections significantly reduce the residual scale dependence of the cross-section. The computational method that we describe is applicable to the calculation of NNLO QCD corrections to any other $2 \to 2$ process at a hadron collider.

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We describe in this contribution a first calculation of Higgs-boson production in association with a jet at next-to-next-to-leading order in perturbative QCD [1]. This result is urgently needed in order to reduce the theoretical uncertainties hindering a precise extraction of the Higgs properties at the LHC. Currently, the theoretical errors in the one-jet bin comprise one of the largest systematic errors in Higgs analyses, particularly in the WW final state. There are two theoretical methods one can pursue to try to reduce these uncertainties. The first is to resum sources of large logarithmic corrections to all orders in QCD perturbation theory. An especially pernicious source of large logarithmic corrections comes from dividing the final state into bins of exclusive jet multiplicities. An improved theoretical treatment of these terms has been pursued in both the zero-jet [2, 3, 4, 5, 6] and one-jet [7, 8] bins. The second, which we discuss here, is to compute the higher-order corrections to the next-to-next-to-leading order (NNLO) in perturbative QCD. Both are essential to produce the reliable results necessary in experimental analyses. In this contribution we give a brief overview of the calculational framework that was used to obtain the NNLO calculation for Higgs plus jet production, and present initial numerical results.

1. Notation and setup

We begin by presenting the basic notation needed to describe our calculation. We use the QCD Lagrangian, supplemented with a dimension-five non-renormalizable operator that describes the interaction of the Higgs boson with gluons in the limit of very large top quark mass:

$$\mathscr{L} = -\frac{1}{4} G^{(a)}_{\mu\nu} G^{(a)\mu\nu} - \lambda_{Hgg} H G^{(a)}_{\mu\nu} G^{(a)\mu\nu}. \tag{1.1}$$

Here, $G^{(a)}_{\mu\nu}$ is the field-strength tensor of the gluon field and H is the Higgs-boson field. The Wilson coefficient λ_{Hgg} through NNLO in QCD can be found in Ref. [9, 10]. The leading electroweak corrections [11] and mixed QCD-electroweak corrections [12] are also known, as are higher-order QCD corrections in theories beyond the Standard Model [13, 14, 15]. Matrix elements computed with the Lagrangian of Eq. (1.1) need to be renormalized. Two renormalization constants are required to do so: one which relates the bare and renormalized strong coupling constants, and another which ensures that matrix elements of the HGG dimension-five operator are finite. The expressions for these quantities are given in Ref. [1]. We note that the Lagrangian of Eq. (1.1) neglects light fermions, as will the initial numerical results presented. We comment on the phenomenological impact of this approximation later in this section.

Renormalization of the strong coupling constant and of the effective Higgs-gluon coupling removes ultraviolet divergences from the matrix elements. The remaining divergences are of infrared origin. To remove them, we must both define and compute infrared-safe observables, and absorb the remaining collinear singularities by renormalizing parton distribution functions. Generic infrared safe observables are defined using jet algorithms. For the calculation described here we employ the $k_\perp$-algorithm.

Collinear singularities associated with gluon radiation by incoming partons must be removed by additional renormalization of parton distribution functions. We describe how to perform this renormalization in what follows. We denote the ultraviolet-renormalized partonic cross section by $\bar{\sigma}(x_1,x_2)$, and the collinear-renormalized partonic cross section by $\sigma(x_1,x_2)$. Once we know

$\sigma(x_1,x_2)$, we can compute the hadronic cross sections by integrating the product of σ and the gluon distribution functions over x_1 and x_2:

$$\sigma(p+p\to H+j) = \int \mathrm{d}x_1 \mathrm{d}x_2\, g(x_1)g(x_2)\, \sigma(x_1,x_2). \tag{1.2}$$

We write the collinear-renormalized and ultraviolet-renormalized partonic cross section through NNLO as

$$\sigma = \sigma^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \sigma^{(2)}. \tag{1.3}$$

We note that although finite, the $\sigma^{(i)}$ still depend on unphysical renormalization and factorization scales because of the truncation of the perturbative expansion. In the following, we will consider for simplicity the case of equal renormalization and factorization scales , $\mu_R = \mu_F = \mu$. The residual μ dependence is easily determined by solving the renormalization-group equation order-by-order in α_s.

2. Calculational framework

It follows from the previous section that in order to obtain $\sigma^{(2)}$ at a generic scale, apart from lower-order results we need to know the NNLO renormalized cross section $\bar{\sigma}^{(2)}$ and convolutions of NLO and LO cross sections with the various splitting functions which appear in the collinear counterterms. Up to terms induced by the renormalization, there are three contributions to $\bar{\sigma}^{(2)}$ that are required:

- the two-loop virtual corrections to $gg \to Hg$;
- the one-loop virtual corrections to $gg \to Hgg$;
- the double-real contribution $gg \to Hggg$.

We note that helicity amplitudes for all of these processes are available in the literature. The two-loop amplitudes for $gg \to Hg$ were recently computed in Ref. [16]. The one-loop corrections to $gg \to Hgg$ and the tree amplitudes for $gg \to Hggg$ are also known, and are available in the form of a Fortran code in the program MCFM [17].

Since all ingredients for the NNLO computation of $gg \to H+$jet have been available for some time, it is interesting to understand what has prevented this calculation from being performed. The main difficulties with NNLO calculations appear when we attempt to combine the different contributions, since integration over phase space introduces additional singularities if the required number of jets is lower than the parton multiplicity. To perform the phase-space integration, we must first isolate singularities in tree- and loop amplitudes. It required a long time to establish a convenient way to do this.

The computational method that we will explain shortly is based on the idea that relevant singularities can be isolated using appropriate parametrizations of phase space and expansions in plus-distributions. To use this approach for computing NNLO QCD corrections, we need to map the relevant phase space to a unit hypercube in such a way that extraction of singularities is straightforward. It is clear that the correct variables to use are the re-scaled energies of unresolved partons

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and the relative angles between two unresolved collinear partons. However, the problem is that different partons become unresolved in different parts of the phase space. It is not immediately clear how to switch between different sets of coordinates and cover the full phase space.

We note that for NLO QCD computations, this problem was solved in Ref. [18], where it was explained that the full phase space can be partitioned into sectors in such a way that in each sector only one parton (i) can produce a soft singularity and only one pair of partons (ij) can produce a collinear singularity. In each sector, the proper variables are the energy of the parton i and the relative angle between partons i and j. Once the partitioning of the phase space is established and proper variables are chosen for each sector, we can use an expansion in plus-distributions to construct relevant subtraction terms for each sector. With the subtraction terms in place, the Laurent expansion of cross sections in ε can be constructed, and each term in such an expansion can be integrated over the phase space independently. Therefore, partitioning of the phase space into suitable sectors and proper parametrization of the phase space in each of these sectors are the two crucial elements needed to extend this method to NNLO. It was first suggested in Ref. [19] how to construct this extension for double real-radiation processes at NNLO. We give a brief overview of this technique in the next section.

3. An example: double-real emission corrections

We briefly present the flavor of our calculational methods by outlining how the double-real emission corrections are handled. To start, we follow the logic used at NLO in Ref. [18] and partition the phase space for the $g(p_1)g(p_2) \to Hg(p_3)g(p_4)g(p_5)$ process into separate structures that we call 'pre-sectors' where only a given set of singularities can occur:

$$\frac{1}{3!}\mathrm{dLips}_{12\to H345} = \sum_{\alpha} \mathrm{dLips}^{(\alpha)}_{12\to H345}, \tag{3.1}$$

Here, α is a label which denotes which singularities can occur, and dLIPS denotes the standard Lorentz-invariant phase space. At NNLO we can have at most two soft singularities and two collinear singularities in each pre-sector, so as an example there will be an α labeling where p_4 and p_5 can be soft, and where both can be collinear to p_1. Within this particular pre-sector, which we label as a 'triple-collinear' pre-sector, it is clear that the appropriate variables to describe the phase space are the energies of gluons p_4 and p_5, and the angles between these gluons and the direction of p_1.

Our goal in introducing a parameterization is to have all singularities appear in the following form:

$$I(\varepsilon) = \int_0^1 \mathrm{d}x x^{-1-a\varepsilon} F(x), \tag{3.2}$$

where the function $F(x)$ has a well-defined limit $\lim_{x\to 0} F(x) = F(0)$. Here, the $F(x)/x$ structure comes from the matrix elements, while the $x^{-a\varepsilon}$ comes from the phase space. When such a structure is obtained, we can extract singularities using the plus-distribution expansion

$$\frac{1}{x^{1+a\varepsilon}} = -\frac{1}{a\varepsilon}\delta(x) + \sum_{n=0}^{\infty} \frac{(-\varepsilon a)^n}{n!}\left[\frac{\ln^n(x)}{x}\right]_+, \tag{3.3}$$

so that

$$I(\varepsilon) = \int_0^1 \mathrm{d}x \left(-\frac{F(0)}{a\varepsilon} + \frac{F(x)-F(0)}{x} - a\varepsilon \frac{F(x)-F(0)}{x} \ln(x) + \ldots \right). \tag{3.4}$$

The above equation provides the required Laurent expansion of the integral $I(\varepsilon)$. We note that each term in such an expansion can be calculated numerically, and independently from the other terms.

Unfortunately, no phase-space parameterization for double-real emission processes can immediately achieve the structure of Eq. (3.2). Each pre-sector must be further divided into a number of sectors using variable changes designed to produce the structure of Eq. (3.2) in each sector. Following Ref. [19, 20], we further split the triple-collinear pre-sector mentioned above into five sectors so that all singularities in the matrix elements appear in the form of Eq. (3.2). Explicit details for these variables changes, and those for all other pre-sectors needed for the NNLO calculation of Higgs plus jet, are presented in Ref. [1].

Once we have performed the relevant variables changes, we are left with a set of integrals of the form shown in Eq. (3.4). We now discuss how we evaluate the analogs of the $F(x)$ and $F(0)$ terms that appear in the full calculation. When all x_i variables that describe the final-state phase space are non-zero, we are then evaluating the $gg \to Hggg$ matrix elements with all gluons resolved. The helicity amplitudes for this process are readily available, as discussed above, and can be efficiently evaluated numerically. When one or more of the x_i vanish, we are then in a singular limit of QCD. The factorization of the matrix elements in possible singular limits appearing in double-real emission corrections in QCD has been studied in detail [21], and we can appeal to this factorization to evaluate the analogs of the $F(0)$. For example, one singular limit that appears in all pre-sectors is the so-called 'double-soft' limit, in which both gluons p_4 and p_5 have vanishingly small energies. The matrix elements squared factorize in this limit in the following way in terms of single and double universal eikonal factors [21]:

$$\begin{aligned} |\mathcal{M}_{g_1 g_2 \to H g_3 g_4 g_5}|^2 \approx C_A^2 g_s^4 \Bigg[& \left(\sum_{ij \in S_p} S_{ij}(p_4) \right) \left(\sum_{kn \in S_p} S_{kn}(p_5) \right) \\ & + \sum_{ij \in S_p} S_{ij}(p_4, p_5) - \sum_{i=1}^{3} S_{ii}(p_4, p_5) \Bigg] |\mathcal{M}_{g_1 g_2 \to H g_3}|^2. \end{aligned} \tag{3.5}$$

The advantage of using this factorization is that all structures on the right-hand side of Eq. (3.5) are readily available in the literature and can be efficiently evaluated numerically; as discussed the helicity amplitudes for $gg \to Hg$ are known, and the S_{ij} eikonal factors which appear are also well-known functions. Using this and other such relations, the analogs of the integrals in Eq. (3.4) appearing in the full theory can be calculated using known results. Similar techniques can be used to obtain the other structures needed for the NNLO computation of Higgs plus jet production. For a discussion of all relevant details, we refer the reader to Ref. [1].

4. Numerical results

We present here initial numerical results for Higgs production in association with one or more jets at NNLO. A detailed series of checks on the presented calculation were performed in Ref. [1], and we do not repeat this discussion here. We compute the hadronic cross section for the production of the Higgs boson in association with one or more jets at the 8 TeV LHC through NNLO in perturbative QCD. We reconstruct jets using the $k_\perp$-algorithm with $\Delta R = 0.5$ and $p_{\perp,j} = 30$ GeV. The Higgs mass is taken to be $m_H = 125$ GeV and the top-quark mass $m_t = 172$ GeV. We use the latest NNPDF parton distributions [22, 23] with the number of active fermion flavors set to five, and numerical values of the strong coupling constant α_s at various orders in QCD perturbation theory as provided by the NNPDF fit. We note that in this case $\alpha_s(m_Z) = [0.130, 0.118, 0.118]$ at leading, next-to-leading and next-to-next-to-leading order, respectively. We choose the central renormalization and factorization scales to be $\mu_R = \mu_F = m_H$.

In Fig. 1 we show the partonic cross section for $gg \to H + j$ multiplied by the gluon luminosity through NNLO in perturbative QCD:

$$\beta \frac{\mathrm{d}\sigma_{\mathrm{had}}}{\mathrm{d}\sqrt{s}} = \beta \frac{\mathrm{d}\sigma(s, \alpha_s, \mu_R, \mu_F)}{d\sqrt{s}} \times \mathscr{L}(\frac{s}{s_{\mathrm{had}}}, \mu_F), \tag{4.1}$$

where β measures the distance from the partonic threshold,

$$\beta = \sqrt{1 - \frac{E_{th}^2}{s}}, \qquad E_{th} = \sqrt{m_h^2 + p_{\perp,j}^2} + p_{\perp,j} \approx 158.55 \text{ GeV}. \tag{4.2}$$

The partonic luminosity $\mathscr{L}$ is given by the integral of the product of two gluon distribution functions

$$\mathscr{L}(z, \mu_F) = \int_z^1 \frac{\mathrm{d}x}{x} g(x, \mu_F) g\left(\frac{z}{x}, \mu_F\right). \tag{4.3}$$

It follows from Fig. 1 that NNLO QCD corrections are significant in the region $\sqrt{s} < 500$ GeV. In particular, close to partonic threshold $\sqrt{s} \sim E_{th}$, radiative corrections are enhanced by threshold logarithms $\ln\beta$ that originate from the incomplete cancellation of virtual and real corrections. There seems to be no significant enhancement of these corrections at higher energies, where the NNLO QCD prediction for the partonic cross section becomes almost indistinguishable from the NLO QCD one.

We now show the integrated hadronic cross sections in the all-gluon channel. We choose to vary the renormalization and factorization scale in the range $\mu_R = \mu_F = m_H/2,\ m_H,\ 2m_H$. After convolution with the parton luminositites, we obtain

$$\begin{aligned} \sigma_{\mathrm{LO}}(pp \to Hj) &= 2713^{+1216}_{-776} \text{ fb}, \\ \sigma_{\mathrm{NLO}}(pp \to Hj) &= 4377^{+760}_{-738} \text{ fb}, \\ \sigma_{\mathrm{NNLO}}(pp \to Hj) &= 6177^{-204}_{+242} \text{ fb}. \end{aligned} \tag{4.4}$$

We note that NNLO corrections are sizable, as expected from the large NLO $K-$factor, but the perturbative expansion shows marginal convergence. We also evaluated PDF errors using the full set of NNPDF replicas, and found it to be of order 5% at LO, and of order 1-2% at both NLO and NNLO, similarly to the inclusive Higgs case [23]. The cross section increases by about sixty percent when we move from LO to NLO and by thirty percent when we move from NLO to NNLO. It is also clear that by accounting for the NNLO QCD corrections we reduce the dependence on the renormalization and factorization scales in a significant way. The scale variation of the result

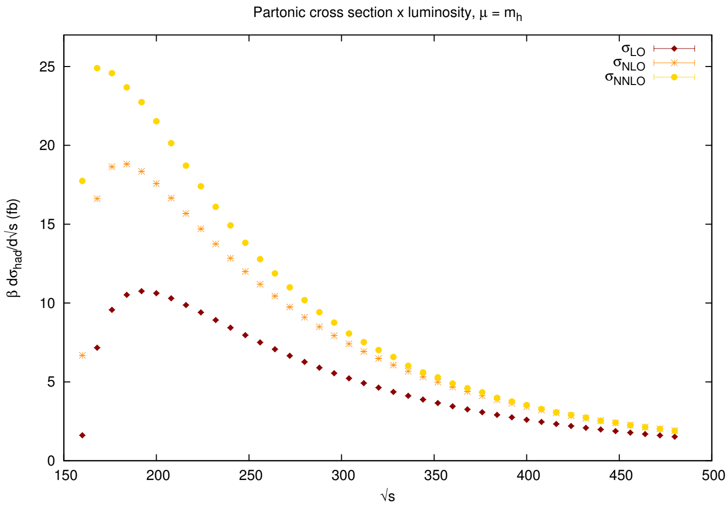

Figure 1: Results for the product of partonic cross sections $gg \to H+$jet and parton luminosity in consecutive orders in perturbative QCD at $\mu_R = \mu_F = m_h = 125$ GeV. See the text for explanation.

decreases from almost 50% at LO, to 20% at NLO, to less than 5% at NNLO. We also note that a perturbatively-stable result is obtained for the scale choice $\mu \approx m_H/2$. In this case the ratio of the NNLO over the LO cross section is just 1.5, to be compared with 2.3 for $\mu = m_H$ and 3.06 for $\mu = 2m_H$, and the ratio of NNLO to NLO is 1.2. A similar trend was observed in the calculation of higher-order QCD corrections to the Higgs boson production cross section in gluon fusion. The reduced scale dependence is also apparent from Fig. 2, where we plot total cross section as a function of the renormalization and factorization scale μ in the region $p_{\perp,j} < \mu < 2m_h$.

Finally, we comment on the phenomenological relevance of the "gluons-only" results for cross sections and K-factors that we report. We note that at leading and next-to-leading order, quark-gluon collisions increase the $H+j$ production cross section by about 30 percent, for the input parameters that we use in this paper. At the same time, the NLO K-factors for the full $H+j$ cross section are smaller by about $10-15\%$ than the 'gluons-only' K-factors, presumably because quark color charges are smaller than the gluon ones. Therefore, we conclude that the gluon-only results can be used for reliable phenomenological estimates of perturbative K-factors but adding quark channels will be essential for achieving precise results for the $H+j$ cross section.

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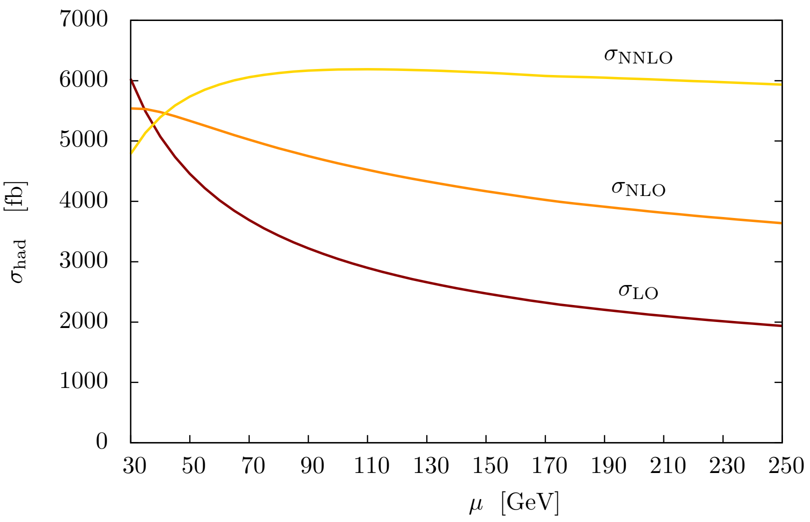

Figure 2: Scale dependence of the hadronic cross section in consecutive orders in perturbative QCD. See the text for details.

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